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DEPARTMENT OF THE NAVY
Contract N00014-70-C-0328
NR 276-021/2-13-70 (462)

DRAFT IN LIEU
OF FINAL
RAC-D7-R
MARCH 1972

AD 739515

Foundations of the Prescriptive Sciences

Volume II

by Nicholas M. Smith
Milton C. Marney

with Appendix
by Donald L. Reisler

DRAFT

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Research Analysis Corporation

394

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* Research Analysis Corporation McLean, Virginia 22101	**2a. REPORT SECURITY CLASSIFICATION** UNCLASSIFIED **2b GROUP** NA

3. REPORT TITLE

Volume 2 of Foundations of the Prescriptive Sciences

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Draft in lieu of client report, March 1972

5. AUTHOR(S) *(First name, middle initial, last name)*

Nicholas M. Smith   Appendix by Donald L. Reisler
Milton C. Marney

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
March 1972	400	99

8a. CONTRACT OR GRANT NO. ONR Contract No. N00014-70-C-0328	**9a. ORIGINATOR'S REPORT NUMBER(S)** RAC-D7-R, Vol-II
b. PROJECT NO. 090.101	
c.	**9b. OTHER REPORT NO(S)** *(Any other numbers that may be assigned this report)*
d.	

10. DISTRIBUTION STATEMENT

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11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Office of Naval Research

13 ABSTRACT

This report, Volumes 1 and 2, is an attempt to formulate adequate conceptual methodological foundations for prescriptive science. Prescriptive science is a mode of rational analysis capable of encompassing valuative as well as factual aspects of optimal decision. The principal problems encountered are essentially metascientific in scope. Major issues are characterised by a degree of generality beyond the immediate concern of any specific scientific discipline.

DD FORM 1 NOV 65 1473

UNCLASSIFIED

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
prescriptive science						
metascience						
philosophy of science						
decision science						

TECHNOLOGICAL SYSTEMS GROUP DRAFT IN LIEU OF FINAL RAC-D7-R Published March 1972

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Research Analysis Corporation RAC

McLean, Virginia 22101

Area Code 703
893-5900

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PART V. METATHEORY AND METHOD

PART IV

PHILOSOPHICAL RECONSTRUCTION

Chapter 9

THE FINITE COGNITIVE AGENT: PHILOSOPHICAL IMPLICATIONS

In the interest of finding an acceptable middle way between the discredited approaches of dogmatic a priorism v. radical empiricism, the best advantage seems to lie toward a system-building effort based on premises that take the character of open policy commitments. That is, premises which so far as possible openly disclose their dependence on existing resources of inquiry for the shaping of an innovative scheme of ideas and for the subsequent conduct of test interpretation. The key consideration here concerns the collection of contemporary scientific concepts and theories utilized in the process of attaining novel intimations which, when posed formally as metatheoretic commitments, will constitute philosophical generalizations.

Informal characterization of the cognitive agent has been our "program of entry" into the continuous process of philosophical reconstruction. A summary of outcomes from preliminary studies of the cognitive agent *in situ* (evolutionary context of interaction) and *in actu* (as an adaptive decision system) will serve this requirement for initial disclosure of resources.

SUMMARY OF PRELIMINARY STUDIES

Figures 9.1-4, reproduced here from Chapters 7 and 8 for convenience, review (1) our dependence on a conception of the cognitive agent as a "constructivist," continuously engaged in reconstituting an original (species-specific) input-output transformation via aesthetic, creative, and control component processes of self-organization (reprogramming, renormalization, reobjectification) and (2) our dependence on an extra-biological extension of evolutionary theory which attributes emergent character to non-substantive conceptual objectifications (constructs, languages, theories, methods) which are "instrumental" to optimization of behavioral response in problem solving and goal seeking activities of the cognitive agent.

For a more explicit account of existing resources, we are now in position to outline a structured array of the characteristics so far attributed only informally on the basis of preliminary analysis. Necessary characteristics of the cognitive agent are those properties held in common with members of all biological categories (infra species) that are capable of modifiable characteristic response: (a) structural features of systems in general, (b) interaction properties of substantive systems, (c) transactional characteristics of metastable systems, (d) communication-control characteristics of fixed-program adaptive systems, (e) response-modifying program characteristics of conditionable behavioral systems. Minimally sufficient characteristics consist in essential, unique properties of the cognitive agent—the differentiae of classification. Ordinary prudence dictates that an arrant provincialism be scrupulously avoided on this point: namely, that we take

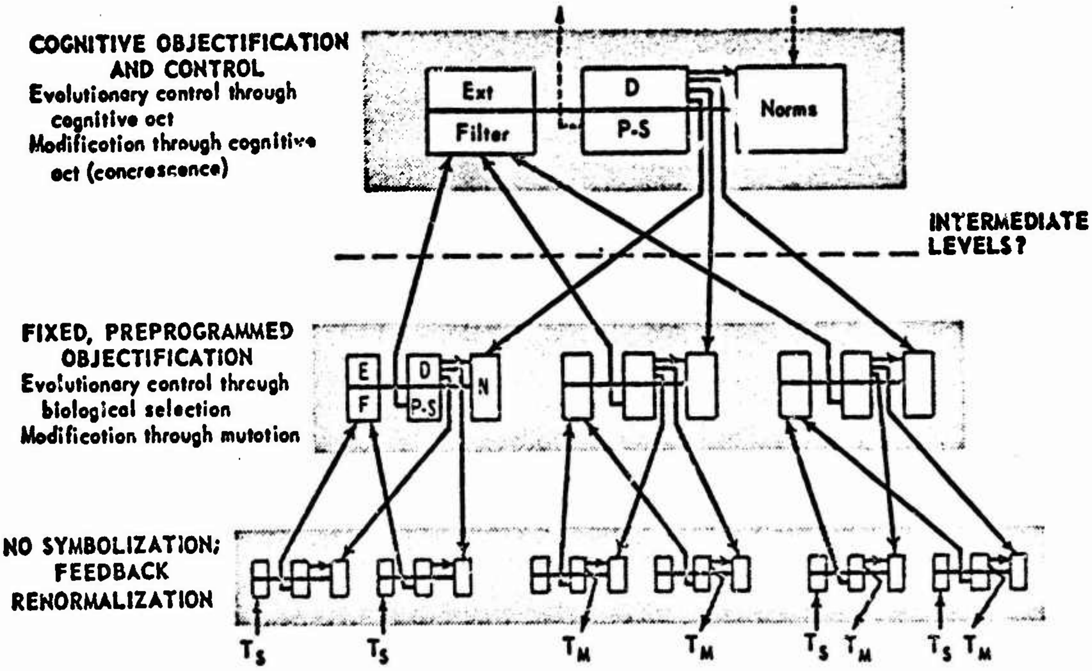

Figure 9.1 --Objectification and Control in a Cognitive Decision System

Ext., extropection or filtered input; D, decision; P-S, problematic situation; T_s, sensory transducer; T_m, motor transducer

Remarks: The operation of decision systems at any level of the hierarchy of Fig. 9.1 may be analyzed in terms of comparisons of filtered input with norms, where violation of any norm instigates a problematic situation to be resolved by a decision procedure involving objectification and selection among alternatives as action-outputs. Extrospection at any level consists of processed input from subsystems; decision at any level consists in the exertion of control on the norms of subsytems. In view of the characteristic regenerative communication-control linkage, the effective hierarchy involved in any decision of a given idiosystem may be much more extensive than the trilevel configuration (subsystem, idiosystem, supersystem) indicated above as the <u>sine qua non</u> condition of meaningful analysis. This fact is suggested by indication of an indefinite number of intermediate systemic levels interposed between the cognitive level of <u>conceptual</u> objectification and the "atomic" level of objectification by sensory-motor transducers. Note that we intend to construe decision at every systemic level as accomplished within an organizational-procedural format that is conformal with the pattern attributed to the cognitive process. There are, however, crucial distinctions between decision processes at alternative levels depending on local systemic complexity and hence on distinct capabilities for objectification. The great disparity of operational means that may be brought to bear is suggested by the distinctions (level-specific) between (a) feedback renormalization, (b) preprogrammed objectification, and (c) objectification and selection as creative conceptualization via symbolic-linguistic cognitive modelling.

Figure 9.2

THE COGNITIVE-SEMIOTIC COMPLEX: FACTORIZATION

1 **ADAPTIVE-CONTROL PROCESS** operating to resolve

2 **PROBLEMATIC SITUATIONS** (disparity between values of actual vs potential states) via

3 **CONCEPTUAL OBJECTIFICATIONS** (symbolic systems as models),

4 **SIMULATIONS** (systematic manipulation of symbolic systems), and

5 **NORMATIVE COMMITMENTS** providing criteria for control of

6 **SELECTION** among objectifications and alternative

7 **MODIFICATIONS OF CHARACTERISTIC RESPONSE** contributing toward

8 **OPTIMAL ORGANIZATION** as a terminal objective under

9 **CONSTRAINTS** imposed by finite resources, species-specific norms of sub-systems, and given context of interaction.

Figure 9.2

Figure 9.3

COGNITION: MINIMAL CONDITIONS

Principal Capabilities (conceptual)	Subsidiary Capabilities Presupposed (communication-control)
OBJECTIFICATION	PRE-PROGRAMMING (SPECIES-SPECIFIC) signal detection information processing autonomic control heuristic info-processing
SIMULATION	
SELECTION	IMMEDIATE REPROGRAMMING attention sensory-motor control conditioned learning heuristic self-conditioning

MEDIATED REPROGRAMMING

Figure 9.3

EMERGENT SYSTEMS, NOOSPHERE

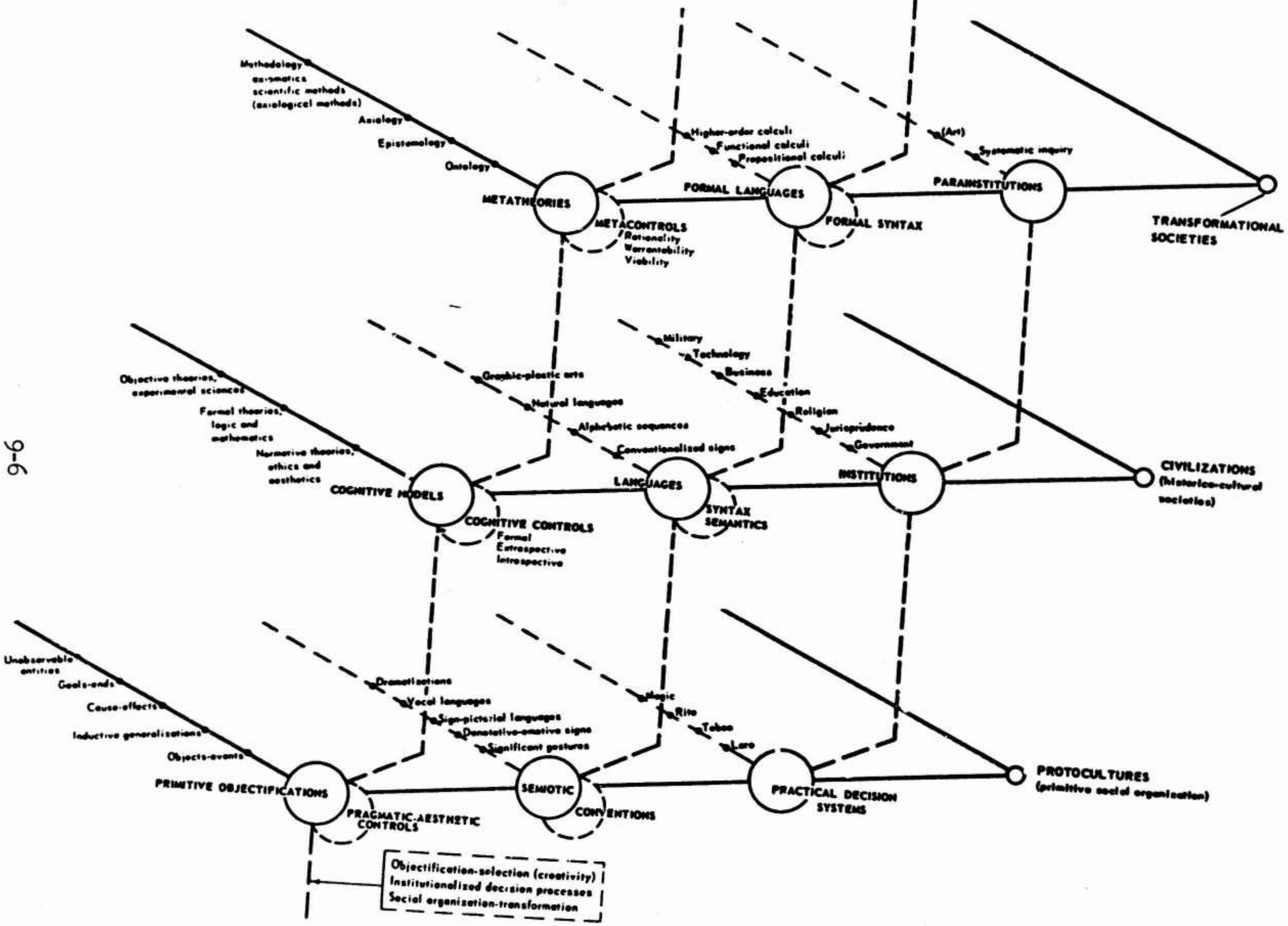

Figure 9.4

care to define "cognitive agent" in a manner that does not prejudicially restrict the term to the domain of human intelligence, or even to <u>animate</u> systems, in view of contemporary work on machine-intelligence. With this proviso, minimally sufficient characteristics are identified with noetic-semiotic capabilities of a self-conceiving (self-mapping or modelling) system. Further properties of significant interest that are admitted by previous attributes are here termed "corollary" characteristics: viz., reflective-mediative operational characteristics of a self-organizing system and, finally, characteristic extremalization tendencies of a self-realizing system.

The advantage of explicit communication of these premises should perhaps be placed in balance with an equally explicit standing injunction against "psychologizing" which is met with throughout contemporary philosophy. In advancing a structured characterization of the cognitive agent, we shall be following an old (an often excoriated) approach which begins with attention to the perennial influence of psychological theories on anthropological philosophy. Historically, the cognitive agent has been variously characterized as (a) sensory encoder with fixed regimen (Protagoras), (b) deterministic mechanism (Democritus and Leukippus), (c) introspective subject (Augustine), (d) infallible "perceptron" (Descartes), (e) <u>tabula</u> <u>rasa</u> passive receiver (Locke), (f) habituated sensory associationist (Hume). Whenever men have seriously addressed the question as to how—i.e., in what modes of thought—we might hope to erect a warrantable and comprehensive <u>system</u> of explanation, the question has always led back to this issue of the fundamental characteristics of the human organism as cognitive agent. We renew this old course, untroubled by the charge of psychologism, simply

because we do not believe there exists any effective rebuttal to Hume's definitive contention on the primacy of the cognitive process (or a characterization of it, at least) as determinative of all that follows in systematic inquiry. It will be evident that in taking this course we are essentially elaborating the more recent conception of the cognitive agent as "voluntarist-activist" which was initiated by William James [1] and clarified by John Dewey.[1] C. W. Morris [2] has traced the impact of this view in the following brief account of early investigation of phenomena associated with psychological "set," roughly put, the tendency of the human (1) to respond only to that in experience which corresponds with organic <u>purpose</u>, (2) to notice initially in experience only what is of immediate <u>interest</u>, (3) to impute as factual in experience primarily that which will be <u>important</u> if it is a fact.

> The emphasis upon action implicit in the growth of modern biological science had taken at times an abortive form, <u>as if the organism merely responded mechanically</u> to an environment which itself owed nothing to the organism. Such a position could not long stand in the face of the facts which crystallized in voluntarism as a biological and psychological principle. For American thought, William James had marked the emphasis in pointing out the way attention helped to constitute the object of perception. John Dewey had isolated the basic point in his 1896 article, "The Reflex Arc Concept in Psychology:" the stimulus is actually a stimulus to the organism only in virtue of the implicit response or interest which sensitizes the organism to those features of the world capable of furthering the release of the response itself.

With fuller development of the crucial emphasis on semiotic capabilities and their significance, works by Cassirer [3] and by Mead [4] in the 20th century advance the originative ideas of what we here term a "constructivist"

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1. Cf. Ref. 2, passage cited.

view of the cognitive agent. Our elaboration then takes the form primarily of an attempt to delineate, in terms of the contemporary scientific vocabulary of communication and control processes, the evolutionary aspects of cognitive creativity. The result is a natural extension of philosophical behaviorism inasmuch as G. H. Mead's The Philosophy of the Present [5] expressed the general intent of showing that "social and psychological process is but an instance of what takes place in nature, if nature is an evolution."

In the following summary, each major entry under a given category of characteristics begins with a predicate adjective intended as a covering term for the group of structural or functional properties detailed thereunder. The entire array may be thought of as a cumulative expansion of the term "cognitive agent," progressing from elemental toward holistic (organizational) properties.

Outline

INITIAL CHARACTERIZATION OF THE COGNITIVE AGENT

NECESSARY CHARACTERISTICS

EXISTENT: STRUCTURAL FEATURES OF A SYSTEM

Dyadic Configuration: initial partition of a universe into an individual entity-with-complement as a nexus of relations.
Hierarchical Structure: the result of recursive partitioning and concrescence of distinct partitions.
Elemental and Holistic Terminations: partitioning (analysis) and concrescence (synthesis) as finite recursive operations terminate respectively with infimum v. supremum levels of hierarchical structure.
Irreducibility: a system consists of parts-as-related by a protocol or rule of composition. The whole is not identical with any sum (concatenation) of parts.

Outline

INITIAL CHARACTERIZATION OF THE COGNITIVE AGENT (Cont.)

OBSISTENT: INTERACTION PROPERTIES OF A SUBSTANTIVE SYSTEM

Sensitivity: subject to change of state under perturbation.
Normativity: possessing internal measures essential to stability and hence to enduring actuality.
Selectivity: not all state-variable permutations admissible; specifically, states entailing violation of norms are unrealizable.
Reactivity: characteristic response tending to maintain norms in the sense of minimization of action over all reactions.

ORGANISMIC: TRANSACTION CHARACTERISTICS OF A METASTABLE SYSTEM

Matter-Energy Transactions
- ingestion, metabolism synthesis (organization up-grading)
- storage, retrieval (delayed utilization)
- support, actuation, maintenance, growth

Information Transactions
- catalytic positive feedback
- homeostatic chemical releasor-suppressor mechanisms (negative feedback)
- self-replication

Idiosyncratic Characteristics of Dynamic Stability: maintenance of "self-determined" states in the sense of reactions in part independent of externally imposed conditions.

SENTIENT: COMMUNICATION-CONTROL CHARACTERISTICS OF A PREPROGRAMMED ADAPTIVE SYSTEM

Perception and Propriorception
- afferent-efferent subsystem specialization
  - signal detection
  - information processing
  - sensory pattern formation
  - pattern indexing and storage
  - pattern retrieval and permutation

Autonomic Control
- sensory-motor automation (reflex)
- appetition and aversion
- homeostasis via channelled feedback (versus diffused chemical transmission)

Outline

INITIAL CHARACTERIZATION OF THE COGNITIVE AGENT (Cont.)

Information-Process Control
- fixed search strategy
- pattern recognition (perceptual judgment)
- precursor-pattern extrapolation (expectation)

HEURISTIC: RESPONSE-MODIFYING PROGRAMS OF A CONDITIONABLE BEHAVIORAL SYSTEM

Perceptual Inhibition (attention selectively maintained)

Sensory-motor Programming Strategy
- sensory exploration
- guidance-control via error reduction

Heuristic Programming
- quiescence-pattern hunting by random trial
- immediate reinforcement of satisfactory behavior and strategy
- homeostasis via habit fixation and extinction

Heuristic Communication
- emotive gesturing
- total-system simulation (role taking)

MINIMALLY SUFFICIENT CHARACTERISTICS

Note: Properties of this class cannot, in principle, be ascribed on the basis of experimental observation. Unlike the type-systems previously characterized, the cognitive agent must be regarded as a "black box" with respect to internal behavior imputed to be dependent on semantic interpretation and valuation. An attempt to factorize minimally sufficient and corollary characteristics therefore necessarily constitutes the rudiments of a theory of cognition.

SEMIO-NOETIC: PSYCHO-SOCIAL CAPABILITIES OF THE SELF-CONCEIVING SYSTEM

Conceptual Objectification (concept attainment)
- inductive generalization (abstraction and concrete reduction)
  - idealized entities (unobservables)
    - self v. other as objects
    - self v. other as subject-object pair
    - ego v. alter ego as subject-subject pair

Outline

INITIAL CHARACTERIZATION OF THE COGNITIVE AGENT (Cont.)

Conceptual Objectification (concept attainment) (Cont.)
- relations
  - spatial and temporal
  - antecedence and consequence
  - cause and control
  - quantity and quality
  - equivalence and preference
  - precedence and succession
  - means and ends (virtual acts and anticipated goals)

Denotative Signification
- overt gestures as socially significant symbols
- sign conventionalization
  - vocal, pictorial, and graphic natural languages

COROLLARY CHARACTERISTICS

CREATIVE-RATIONAL: MEDIATED BEHAVIORAL CAPABILITIES OF THE SELF-ORGANIZING SYSTEM

Formalized Objectification (Linguistic Models)
- objective formalization: construction of a novel system of conventional symbols such that linguistic objects and operations are representative of perceptual-conceptual aspects of a problematic situation—in the context of some reduction of experience.
- normative formalization: institution of logical-pragmatic-aesthetic commitments as criteria controlling the admissibility of cognitive models, decision procedures, and problem solutions.

Simulation: manipulation of linguistic models in imaginary trial-error exploration; assessment of (virtual) outcomes from alternative conditions, plans, strategies, and decisions.

Selection: decision to reconstruct behavioral repertoire utilizing the operations, programs, strategies represented in the linguistic model which are associated with preferable outcomes under the criteria instituted.

Outline

INITIAL CHARACTERIZATION OF THE COGNITIVE AGENT (Cont.)

CONATIVE-AESTHETIC: EXTREMALIZING TENDENCIES OF THE SELF-REALIZING SYSTEM

Optimal Control

Tactical tendency toward decidability, i.e., acquisition of control principles ensuring decisions admissible with respect to a presently given hierarchy of system norms—thus, a drive toward maximal immediate effectiveness in problem solving.

Maximal Freedom

Strategic tendency toward maximal adaptive range, i.e., preservation of capability for creative concrescence of acquired norms and subsequent reorganization of the total portfolio of cognitive models such that:

(1) decidability becomes attainable in previously obstructive situations,
(2) committed cybernetic capacity is markedly reduced (cybernetic "elegance"),
(3) the scope of possible environmental interactions (and hence the viability) of the system is increased—thus, a drive tending toward optimal adaptive response in goal seeking.

Maximal Realization

Holistic tendency toward optimal tradeoff, i.e. provision for "legislation" over antithetical requirements of optimal control v. maximal freedom—thus, a drive tending toward optimal organization as the supremum of extrinsic value. This norm, connoting dynamic coordination of creative, rational, and aesthetic component processes requires assignment of priority alternatively to needs for the efficiency of rigid programming (decidability) v. a costly but providential flexibility in reorganization (freedom). Maintenance of stationarity for a measure of optimal organization defined on the product of freedom and decidability, $\delta(F \times D) = 0$, is then instrumental to maximal realization as a singular terminal value, hence an intrinsic value. The holistic tendency of the cognitive agent is toward maximal realization in three distinct senses of "realization":

(1) apprehension of the implications of a world-view attained via description, prediction, explanation, and adjustment via prescriptive control of self and environment, i.e., knowledgable accomodation of "reality" in the interest of survival;

Outline

INITIAL CHARACTERIZATION OF THE COGNITIVE AGENT (Cont.)

Maximal Realization (cont.)

(2) transactional gain in terms of total human interests—physical, psychological, social, i.e., cultural enhancement of the quality of life during survival;
(3) actualization of potentialities inherent in individual and institutional capabilities for emergent self-transformation, i.e., enlargement of the range of human experience and the meaning assignable to "existence."

Object-Theoretic Conclusions as Philosophical "Determinants"

In what was perhaps the crucial point of Chapter 6, Renewed Enterprise in Systematic Philosophy, we maintained that inescapable presumptions—of ordinary language, of technical concepts, of current conclusions in specialized sciences—necessarily influenced the selection of philosophical primitives. With disclaimer as to the accessibility of absolute foundations we began an attempt to isolate fundamental intimations, not as "self evident" insights bearing a priori certitude, but as trial-generalizations, extrapolations of what we presume to know thus far. Our characterization of the cognitive agent therefore represents a repository of innumerable presumptions of this sort. If it were necessary to depend explicitly on details of this characterization, the very worst effects of "psychologizing" would probably be unavoidable. Fortunately, we find it sufficient to our purpose at present to utilize a single primary premise as an object-theoretic conclusion determinative of many of the major features of our ultimate philosophical position. This primary commitment predicates the finite character of the cognitive agent. While a number of technical concepts and a collection of

subsidiary theoretical commitments will later be drawn from the preliminary-characterization (above) in elaborating a philosophical system, the primary commitment to finitism, by entailment, shapes the general features of a philosophy of evolutionary systems.

FINITISM—ITS PHILOSOPHICAL IMPLICATIONS

By the term "finitism" we formalize our recognition of limitations on the range of semiotic freedom and on the span of cybernetic control of the cognitive agent.

Primary Commitment. All cognitive agents are finite decision systems. That is, all cognitive agents: (a) detect perturbations of their environment within the constraints of a finite number of modalities with finite channel capacities, (b) proce s data internally at finite rates, (c) possess finite memory store and limited information retrieval programs, (d) communicate by means of finite sequences of symbols transmitted at finite rates, and (e) endure over finite lifetimes.

The two-part thesis of this section is (1) that the finite character of the cognitive agent places necessary restrictions on the nature of admissible conceptual objectifications in general and (2) that finitism entails subsequent commitments to relativism, reductionism, provisionalism, operationism, and meliorism collectively as characteristics of the only type of philosophical system that lies within the competence of cognitive agents so constituted as in our foregoing preliminary studies. Since the conditions on admissible conceptualization imposed by finitism are more restrictive than those presently accepted in the formal sciences, we anticipate that formal systems of logic and mathematical analysis, as well as philosophical systems, may be

subject to normative modification. Further, as to consequences in the physical sciences, we shall later attempt to show that certain of the profound "discoveries" of invariance principles (construed as objective properties of the natural world) are to be more appropriately attributed to normative requirements of cognition. This is to say that such principles may be construed alternatively as idealized properties required of any cognitive model whatever in order to satisfy the most general criteria of admissibility appropriate to a finite cognitive agent.

Our present interest, however, is limited to immediate tasks. The first of these is to give the general argument which carries our primary commitment (finitism) into a collection of derivative commitments. The line of argument, while straightforward in its main outline, can be endlessly complicated by detailed consideration of the interaction terms of relation, i.e., by systematic attempts to show the import of each distinct commitment for each distinguishable compartment of a philosophical position: epistemology, ontology, axiology, methodology, praxiology. We hew to a straight line here, leaving refinements to be developed in all that follows.

Derivative Commitments: Rationale

The basic rationale for derivation of subsequent primitive commitments can be compressed into the following compound statement:

(1) that semiotic and cybernetic characteristics of the finite cognitive agent entail (a) the relativistic status (conditional, not absolute) and (b) the reductionistic structure (homomorphic, not isomorphic) status of all conceptual objectifications as symbolic representations relevant to an individual-environmental dual system of interactions;

(2) that the open-endedness of continuing interaction precludes the possibility of (a) complete characterization of experience, (b) infallible prediction of future states, or (c) incorrigible prescription of anticipatory human response in terms of cognitive models so constituted;

(3) that the only significant (realizable) aim open to cognitive attainment must therefore be predicated on iterative reconstruction of modes, concepts, theories, and criteria of admissibility which are (a) provisional in regard to their initial status and their coverage of human concerns, (b) operational in regard to meaningful interpretation and practicable trial in test implementations of description, explanation, and prescription, and (c) meliorative in regard to systematic improvement in the measures of warrantability and comprehensiveness with which emergent conceptual systems successively serve aesthetic and pragmatic human aims.

In all of this, no doubt, we move very far indeed from the absolutist stance—and the heady optimism—of classical idealism and rationalism. Significantly, each of our derivative commitments is cast as a constraint on the competence of the cognitive agent regarding ideally coherent, holistic organization of thought and experience. In reaction to the initial proposal of each one of these constraining commitments in the history of inquiry, some charge of radical pessimism has been made—as if the human condition were hopeless without access to absolute foundations for knowing, valuing, acting. A bare denial of absolutism however, entails no vitiation of the cognitive enterprise but, rather, merely a requirement to work creatively

within limitations that are in fact sufficiently innocuous to permit continuing cultural extension of a range of freedom and a span of control that was already considerable even in the first appearance of Hominidae.

In anticipation of the direction our efforts will take from this view, the following list associates with each of these constraining commitments the type of philosophical accomplishment that we find reasonable to attempt within the terms of the given constraint.

CONSTRAINT	FEASIBLE ACCOMPLISHMENT
Relativism	Establishment of conditional freedom and relative decidability via a general system schema with extended canons of rationality.
Reductionism	A strategy of reduction permitting, in principle, the attainment of concrete-universal representations in addition to abstract-universal representations.
Provisionalism	Connectivity over the range of compartmentalized (disciplinary) theories via convergent embedding of distinct cognitive models dependent on a unitary paradigm for formal, objective, and normative inquiry.
Operationism	Programmable tests for "rational" admissibility of cognitive models in general, subject to a holistic collection of criteria: formal, empirical, pragmatic, aesthetic, and evolutionary.
Meliorism	Iterative improvement of cognitive organization via introduction of a normative-theoretic mode of inquiry sensitive to valuative as well as formal and factual aspects of optimal decision and optimal organization.

Derivative Commitments: Discussion

A second immediate task is to give each of these commitments a brief discursive treatment at least sufficient to allow appreciation of its content and its future role as a component of a systematic position.

Relativism. The term "relativism" is notorious for the confusion it so readily breeds. This is due to the fact that it has connotations which are relevant to several compartments of investigation (epistemology, ontology, axiology, methodology) and it has, as well, at least two distinct levels of interpretation. Basically, its reference has been to conditionality—of judgments, procedures, concepts, commitments—in short, to the non-absolute character of almost any one of the significant aspects of the conceptual process or its output.

On the elementary level of interpretation, where relativism is taken to mean simply relationism, it is a doctrine unlikely to be objected to by anyone. That the theorems of a formal system are valid only with respect to the logic and the axioms selected, that the operational decisions of a social organization are explicitly conditional on prior policy decisions, that the meanings of ordinary language terms are contexturally dependent, that the statutes of civil law are relative to the value-commitments of particular societies—none of these senses of conditionality pose significant difficulties of acceptance. A debatable issue arises only with reference to the more profound sense of conditionality associated with mutually conditional, mutually constitutive entities or operations; specifically, with the appearance of indeterminability or underspecifiability that is inherent in the logical "circularity" of mutually determinative processes. This is the sense in which relativity came to prominence with recognition of the indeterminability of simultaneity in modern physics; and it is in this sense that we shall use the term "relativism" to denote the following doctrinal extension of the earlier physical principle. This generalization is designed specifically to cover the implications of the finite character of the cognitive agent:

> Cognitive agents with their conceptual objectifications comprise mutually constitutive pairs (subject-object dyads) in which the pair-elements are definable only in terms of mutually determinative processes and interaction-properties. Characterizations of cognitive subjects and conceptual objects are therefore "relativistic" in the sense that characteristics of individual components (subject or object) as "independent" existent entities are <u>indeterminable</u>.

A more cryptic expression of this commitment would be simply to assert that a cognitive subject with a collection of conceptual objects comprises a system. Subject-object pairs (in the sense above) satisfy the formal properties of (a) dyadic configuration, (b) hierarchical structure with supremum and infimum termination, and (c) <u>irreducibility</u>—the inherent property of any complex of mutually constitutive entities with mutually determinative processes.

The cognitive agent, in generating an external object, e, via conceptual objectification, concommitantly generates a complementary dual-object as an element of a self, s. The class E of all e is the external world—i.e., the world of "reality;" and the class S of all s is the self. No "self" can exist independently of an external world. If <u>cogito, ergo sum</u> holds, then it must also hold <u>a fortiori</u> that an external world—as a totality—exists. Self-awareness presupposes awareness of externality. No cognitive subject could be a self-conceiving system in the absence of peremptory sensations. The self and the external world, each as a totality, have the strongest warrant as to existence, i.e., the warrent of interdependence. Any doubt as to warrant or applicability applies to a particular component pair (e,s). The existence of the total classes E,S is indubitable—though this is very far from saying that their existence is unconditional. Within E,S however, components e_i, s_i may be objectified in any number of ways.

The "peculiarity of the scientific world-view," in Schrödinger's phrase, at once a strength and a weakness of objective inquiry, entails a simplifying neglect of the presence and effect of the cognizing subject. In this traditional mode there is no attempt to structure E and S in complementary pairs; rather, the external components are taken ostensibly as independent. The merits of this simplification are seen in the impressive accomplishments of the formal and physical sciences, its limitations in the course of diagnosis we have undertaken regarding the present status of the behavioral sciences undertaken on this approach. The necessity for explicit accomodation of the duality of objectifier and objectification will become apparent when we later deal with the interdependence of science and axiology. In the interest of accomodating the valuative aspects of rationally admissible conceptualizations in general, we shall undertake systematic cognizance of the conclusion that (1) conceptualizations of the self and of the external world are simultaneously generated in the cognitive act, and (2) the self and the external world represent counter perspectives for dual modes of treatment of a single flux of interaction.

On this basis we pose the following principal tenets of relativism and introduce discussion of the significance of this commitment in a number of distinct philosophical roles:

(1) that all (a) philosophical systems, (b) formal-factual-valuative theories, and (c) programs of practical judgment and action—as regarding their foundations—are "afloat" in virtue of their sensitivity to the mutual conditionality of thinke and things-as-they-are-thought-to-be, i.e., no absolute foundations re conceivable;

(2) that a finite cognitive agent has recourse only to indefinite (underspecified) conceptual objectifications of self v. generalized "other" and is therefore incapable of attaining either complete decidability (programmed control) or holistic comprehensiveness (unconditional freedom) in the representation of experience and the choice of action based on cognitive simulation;

(3) that self-corrective and self-amplifying improvement of cognitive organization—with "improvement" defined in terms of self-instituted and self-modified criteria of optimally—must characterize the cognitive enterprise as a process of evolutionary realization rather than a process of discovery or revelation;

(4) that no meaningful reference can be made to <u>the</u> kinds of things that exist, or the way things <u>really</u> are independent an attendant type of cognitive agent as mutual determinant of "things" (structures, contents, processes, process-criteria) <u>qua</u> "reality" so constituted by the admissibility tests of that type;

(5) that neither apodictic (necessary) factual knowledge nor invariant (immutable) substantive goals can be predicated as attainable directives to human belief and behavior.

Ontological relativism, then, refers to the doctrine that <u>existants</u> (existing entities) are mutually constitutive with confidence-bearing concepts. The crux of this notion is to be found in its denial of the possibility of meaningful reference to "the way things really are" independent of any context of interaction between object and objectifier. It is illustrative to note that recent innovations in foundations of quantum theory

involve the assumption that the properties of matter are incompletely defined alternative potentialities that can be realized only in interactions among systems. Thus, at the quantum level of resolution, an object does not have any intrinsic properties as characteristics in isolation. Instead it acquires properties mutually and indivisibly with the complex of systems with which it interacts. The ontological commitment here involves the notion that any object—if it is to exist—must exist as a distinct something; and that this "something" can be definitive only in virtue of a characteristic response in interaction, the most rudimentary interaction being that between observer and the observable.

It seems quite natural to view our own commitment as an extension of this idea. The fundamental import of ontological relativism may be brought out by explicit insistence on the contextural dependency of reality. Because a given object may interact on different occasions with different systems that bring out different potentialities, any object may be construed as subject to continual transformation, each transformation representing a conceptual construct bearing its own particular warrant of confidence with regard to the adequacy of the expectation it provides from the viewpoint of a given interacting observer-objectifier.

It is essential here to recognize the misleading effect of the phrase "the thing in itself." Such a purported description can have no meaningful reference. No thing-in-isolation "exists." To exist is to be an element of a system of mutually constituitive elements which—by whatever lengths of inferential chaining—can be mapped onto the elements of some conceptual objectification. Any admissible existential statement must necessarily be

a testable statement; and tests for existence, which involve perceptual and conceptual operations leading to the fixation of some expectation, entail the connectivity of each existing object with some complementary objectifier. Bishop Berkeley's esse est percipi (to be is to be perceived) indeed puts the case too restrictively. His emphasis, however, must be accommodated to this extent at least: to be is to be objectifiable. Existence may be attributed to just those entities which are the referents of "admissible" conceptual objectifications. The institution of adequate criteria and test procedures for determining admissibility is clearly the crucial matter in inquiry. On this issue hinges the whole question of what it means to be rational. The determinative effect of rational canons goes very deep indeed, for the "kinds of things there are" and the "way things are" can never receive any specific characterization except in terms of specific kinds of tests brought to hear in the selection of admissible conceptualizations.

On this view, a number of specialized interpretations of relativism arise. (1) Epistemological relativism and (2) axiological relativism, as regarding the nature and extent of knowledge and value, the sources and methods of knowing and valuing, the validity and warrantability of predictive-explanatory and prescriptive theories, respectively assert the non-absolute character of conceptual foundations and, particularly, the dependence of formal, factual, and valuative admissibility on appropriate tests for alternative conceptualizations. (3) Relativity of method, which we shall refer to under the term "procedural relativism," concerns the doctrine that no absolute frame of reference exists for object-theoretical formulations. This essentially subsumes the insistence of Einstein's principle:

that acceptable physical laws must not be sensitive to aspects of formulation which are properly conventional. Procedural relativism very generally concerns the invariance of transformations in an object-space.

To establish the issue, consider an object-space purportedly containing an absolute origin, O. A situation referred to some distinct point, X, is to be transformed in order to refer to some other point, Y. (Y moving with respect to X complicates the transformation, but it does not change the concept of the problem.) On this supposition, an unambiguous procedure can in principle always be prescribed in the operative syntax which transforms first to the absolute origin, thence to the final point, Y. What happens now when commitment to the existence of any such absolute reference turns out to be unwarrantable? Any transformation from X to Y is now path-dependent, or procedurally dependent. The object-theory in use becomes procedurally ambiguous. Recognizing the inadmissibility of any such absolute spatial frame of reference, Einstein proposed a means of avoiding ambiguity: physical theories must be limited to those whose forms are invariant under velocity transformations. We make here a slight extension on Einsteins's proposal. Treating the issue as a completely general requirement for unambiguous procedure, we shall propose the cognitive formulations must be invariant under transformation with respect to all significant variables, where a "significant" variable is one whose range of variation is sufficient to render the procedural ambiguity of a transformation detectable in the presence of concommitant uncertainties. For example, the Newtonian laws of motion were long acceptable because they were applied to problems involving very small differences in velocities. Hence they were for all practical intent invariant in this range.

One can employ non-invariant theoretical forms provided the scope of application is appropriately restricted. It is safe to say that almost all theoretical models fail to satisfy the invariance-criterion with respect to at least some of their variables; but not all such failures are obstructive in the sense given to significant variables.

Summarizing with regard to relativistic commitments, the major import is that the quest for certainty is being given over. It has gradually been accepted in the physical sciences that empirical confirmation of predictive-explanatory theories can establish only the sufficiency of a cognitive model. There exists no means of demonstrating that any theory will be consistent with, or even accomodative of, future data; nor can it be demonstrated that alternative theories of greater adequacy and scope are prohibited. The relinquishment of the quest for certainty has very frequently been viewed with repugnance, particularly in regard to value-commitment and valuative judgment. This repugnance arises, we believe, from a mistaken supposition that stable ethical principles cannot be achieved on a relativistic basis. Such a conclusion too heavily discounts the capabilities of the cognitive agent for creative extension of control principles.

To accede to relativism is, admittedly, to regard the domain of cognitive freedom as open-ended; and the possession of many degrees of freedom via human intelligence is, first of all, a problematic situation. Whenever freedom exists, some general principles must be instituted at a super-ordinate level of cognitive organization in order to achieve uniqueness of decision and specificity of action. Such general principles, that is, principles at the metatheoretic level,are major objectives of philosophical

reconstruction, constituting—in a manner of speaking—"hypothetical" absolutes required for practicably effective action via cognitive control of behavior. As Kent [6] has pointed out, a reinstitution of absolutes invariably represents the objective prompted by the adoption of relativism in general. Relativism, he maintains, whether anthropological or physical, is a response to intractable variety, complex variation, and mutual-causal relation. It attempts both to allow for diversity and to transcend it by calling out a new absolute which will resolve ambiguities introduced by multiplicity and change. In effect, we revoke absolutism at the object-theoretic level in order to reinstate it more defensibly at the meta-theoretic level of generality.

Reductionism. No other term that we shall employ will be more immediately open to misinterpretation than "reductionism," which is used here to cover the following considerations:

(1) Instrumental Limitations.

The operations of cognitive modelling (objectification, simulation, and selection) limit linguistic representations to a one-to-one correspondence between the symbolic elements of a model and only a selected subset of the perceptual-conceptual interaction characteristics of cognitive agent and environment. It is only as a working supposition enabling successive refinements that the interaction of elements of the model, within the scope of a homomorphism, may be treated as analogous to real world interaction.

(2) Limitations of Reductionistic Abstraction.

Representations constructed as formal abstractions are susceptible

to specious decomposition of the organized wholes which are psychologically primitive in perception and conceptualization. By "specious" decompositions we refer to arbitrary partition of interaction properties of a conceptual system in a manner such that recomposition fails to yield an equivalent of the original concrete object of attention. The ontology of the formal sciences is monistic, i.e., only formal objects are predicated. Idealized separability of the elements of a complex conceptual objectification is always possible on this basis, and therefore specious decomposition is always a costly possible consequence.

(3) Requirement for "Holistic" Representations.

Characterization of the organized <u>systems</u> which are native to human conceptual objectification is possible <u>in principle</u> by way of abstract formulations; but this course is blocked in practice by the near-incredible complexity of the consequent task of composing innumerable isolated relationships into a coherent overall representation. Any given cognitive model would be more appropriately regarded as requiring embedment in a "portfolio" of models being utilized by a decision maker. The necessary reduction of experienced interaction must be carried out under a strategy which ensures decoupling of a total system configuration only at junctures of least interaction, preferably negligible interaction. At best a cognitive model specific to a local decision problem in the context of a reduction will be constructed with a view to improving the adequacy and coherence of an entire portfolio of models as a holistic representation.

Obviously we are faced here with unfortunate terminological similarities. "Reductionism" is the only term which good sense would demand for assignment to the commitment outlined above. Yet the position we designate as "reductionism" directly opposes that type of over-simplification ordinarily referred to perjoratively as "reductionistic" representation. Examples of formalistic and empirical over-simplifications typical of reductionistic representation are: a system of interactions viewed as nothing but a collection of elements, organismic input-out transactions viewed as nothing but an abstract inventory-control process, a human decision maker viewed as nothing but a conditioned organism, social values viewed as nothing but intersection sets of individual values.

The burden of discussion here is to make clear the important distinctions that are likely to be obscured by similarity of terms. The tenor of the following comments can be given as, first, an accommodation of the effects of instrumental, semiotic, and cybernetic limitations in cognitive modelling: thus, every cognitive model is undeniably constructed in the context of a <u>reduction</u> of some phenomenal domain of interaction. Second, such models nevertheless need not be <u>reductionistic</u> (disregardful of essential systemic relations) in virtue of the possibility of preserving (a) multiform structure, (b) polytypic content, and (c) polymodal processes via representation in terms of a portfolio of embedded models—which would constitute a <u>concrete</u> reduction as against a mere collection of abstract representations.

The conventional decomposition of objectifier-objectification pairs into two distinct classes, subjects v. objects, tacitly assumes that the interaction between them can be ignored or made arbitrarily small, in short,

that pure objectivity can be achieved. Our contention is that the effect of the cognitive agent as a determinant of the objects of perception and conceptualization can be ignored only at the cost of locking inquiry into a circuit strictly bounded by fixed strategy and program.

The process of objectification (conceptualization) takes place within a cognitive agent situated in exposure to a flux of uncontrollable, peremptory perturbations. The cognitive subject is not a passive observer but plays an active role in constituting the object of perception. Subject-object pairs are therefore primitive dyads whose formation serves to give meaning—the only determinable meaning—to both members simultaneously. They are mutually defined each in terms of the other. Thus the character of objects is interdependent with the nature of the species that objectifies them. Sensory as well as cognitive apparatus and cybernetic characteristics play a significant role. In illustration of this point, consider a TV receiver as a crude analog of the cognitive agent. A picture (an objectification) results from the combined effect of three determinants: (1) a policy decision as to channel selection (reduction of the universe of experience), (2) a flux of external perturbations (source of peremptory signals), and (3) internal information processing and response in terms of a characteristic organizational format (production of a perceptual construct). The limitations of this analogy are obvious inasmuch as a TV receiver has no means of determining whether there is anything "out there" as a correspondent of the construct. No recourse is available to procedures that would provide an "objective" perspective. Yet this elementary similarity holds: that a conceptual objectification emerges partly on the basis of uncontrollable

external perturbations and partly in virtue of the contribution of an organizational process internally characteristic of the cognitive agent.

A rudimentary but crucial type of contribution of the cognitive agent arises from the fact that the flux of external perturbation is filtered by a selective detection process which admits only certain definite components. Two principal mechanism are instrumental to the production of filtered input. Filtering of the first type is due to the characteristic response of sensory transducers as detectors of fixed design reacting only to certain stimulus characteristics. The visual subsystem, for example, responds to electromagnetic radiation only within a bandwidth that is sharply bounded in both the direction of the infrared and the ultraviolet; the auditory subsystem detects dynamic pressure changes only within a limited frequency range. Further, the various sensory transducers produce subsidiary objectifications only in terms of fixed formats, e.g., arrays of dots fused as lines or regions, rapid sequences of static representations fused as continuous motion. The cognitive agent is physically incapable of observing directly many of the broad categories of phenomena which may be imputed on the basis of supplementary instrumentation and inferential chaining of concepts and theories. A second type of filtering is due to prejudgmental effects of prior conceptualization, that is, the theory-laden character of observation. The cognitive agent is psychologically prohibited from perceiving "all that is there" in any instance of perception in virtue of the selectivity instituted by anticipatory interest and attention. Habitual constructs, models, theories furnish the prior categories in terms of which ongoing experience is interpretable, and the cognitive agent is

literally incapable of perceiving any "thing" that is not of a kind for which categories of descriptors have been prepared by commitment in advance of perceptual judgment. The case is similar for inferred properties of objects. Prior to the proposal of Uhlenbeck and Goudsmit in 1925, spin was not attributatable as a behavioral property of electrons, no matter what types of experiments were performed. Thereafter, experimenters throughout the world found perceptual evidence supportive of such an interpretation.

This instance brings up the important question as to the conditions under which new categories are introduced. Under a given theoretic orientation, i.e., given prior categories and rules of correspondence for the interpretation of experiential data, we view any datum that is inconsistent with the regnant theory as anomalous, rejectable on policy. However, if it develops that inconsistent data are persistently encountered, the stress of cognitive dissonance assumes importance. Prior categories are failing in their role of ordering and organizing the conduct of judgment. It is at such a point that a new way of "looking" at things may acquire the status of an explicit goal.

It is in this connection that we are led to advocate (1) a systems approach in cognition and (2) an attempt to establish connectivity over embedded collections of reduced models in the interest of holistic representations. This approach presupposes that a composite is formed—whether by concrescence of elementary objects of by partitioning of a universe—according to some definite rule of composition that is unique to the resulting system. Independent characterization by parts does not unambiguously determine the state or the nature of the whole. It is the

interaction characteristics of the parts-as-related by a protocol or rule of composition that is determinative. However, cognition does not ordinarily take place in the context of a total problematic situation involving the whole of reality, E, but rather in the context of a reduction, ϵ, relevant to a limited and specifically determined subset of decisions. Cognitive models, in general, are reductions that have been constructed under a controlling compromise between practicability v. realism. The collection of reductions, $\bar{\epsilon}$, which constitutes E are therefore not necessarily consistent, i.e., E is not well structured but consists instead of some mixed collection of overlapping models, loosely coupled models, and disjoint models. Increase in the coherence of $\bar{\epsilon}$ and extension toward systematic organization within E then represents an idealized goal of rational inquiry.

One may now objectify subsets within ϵ. A particular member of ϵ, e_k, has a set of <u>compositional</u> properties—which we may somethimes refer to as "inside" properties. All other properties in ϵ are "outside" (interaction) properties which may modify e_k but are not determinative. That is e_k results from a partitioning of ϵ into complementary sets (e_k) and $(\epsilon - e_k)$. The full definition of e_k consists of both its inside and its outside properties. For example: an automobile may be defined by the phrase "a type of wheeled, self-propelled vehicle." However, the full significance of this concept can be given only through specification of interaction characteristics in terms of a system of roads, service stations, repair facilities, drivers, and other vehicles. The roads in, say, the state of Washington, are not constitutive of an automobile in New York; but they do modify it since they extend or limit its range.

A second subset, e_s, of ϵ may be similarly structured. The inside properties of e_k lie in the outside properties of e_s, and vice versa. The <u>interaction</u> of e_s on e_k is defined as the change in $(\epsilon - e_k)$ brought about by the removal of e_s from E.

This construction presupposes that every thing is what it is in virtue of its interaction with every other thing. On this view, no adequate understanding of complex systems can be anticipated on the basis of abstract representation alone. Arbitrary decomposition and representation of specially selected features of a concrete system in terms of an abstract formal system necessarily entails the discard of interaction linkages. Our contention is that a fruitful alternative is feasible: namely, the embedding of cognitive models in successive contexts of reduction, such that the universe of discourse is manageabley restricted while interaction features are retained. This strategy admits of representations that are "holistic," at least in the minimal sense that no significant aspect of systemic structure or function is actually disregarded. It is not to be supposed that either formal abstraction or concrete reduction must ultimately predominate in analysis. These modes aim at two distinguishable versions of generality, i.e., universal interpretability v. comprehensive relevance; and balanced interplay between the two is the most promising means of intellectual advance.

<u>Provisionalism, Operationism, and Meliorism</u>. Derivation of this group of commitments does not involve the crucial level of problematic situations encountered in regard to relativism and reductionism. These subsidiary positions follow rather straightforwardly from the open-endedness of

experience, the incompleteness of descriptions, the fallibility of predictions, the corrigibility of prescriptive judgments. Provisionalism, in particular, requires little amplification. This commitment will be immediately understood so far as it represents a proposal (1) to hold even our foundational concepts initially as trial-formulations and (2) to regard evidences of their warrantability and serviceability as certifications *pro tem*. Only one further extension of meaning remains to be made clear: that the term "provisionalism," in our usage, will carry the additional connotation of *provident* coverage of the widest human concerns. To make provision for the pluralistic interests and aims of the human in the cognitive enterprise, to maintain the flexibility of modifiable cognitive organization: these intentions also are to be associated with a commitment to provisionalism which, in its bare essentials, affirms merely the necessity to begin "where we are" in mounting a program of reconstruction continually aimed at improvement.

Operationism, too, is open to immediate appreciation of content in terms of the following injunction, now a familiar constraint: that mode and method, concept and theory, all must admit of specifiable interpretation and practicable testing, subject to a battery of criteria for rational admissibility. The principal thrust of modern analytical philosophy has been directed toward thorough establishment of this constraint as the *sine qua non* of meaningful inquiry. In the main, we shall depend informally on the conditions for meaningfulness emphasized first by C. S. Peirce, later developed in American pragmatism, in both logistic and linguistic schools of analysis, and more recently in the philosophical "operationalism" of P. W. Bridgman.

In connection with programmable testing and operational interpretability within the assured capability of the finite cognitive agent, however, there arise futuristic problems of such depth as to evoke the necessity of reconstruction at the level of foundations of mathematical analysis. In cybernetic terms, the human brain and nervous system has finite storage capacity, finite rates of information processing, finite lifetime, and finite repertorie of programs referencing finite linguistic resources. All conceptualization takes place in the context of the finite mapping and modelling agency that we loosely term "the human mind;" and it must be expected that these characteristics will play a fundamental role in determining the extent to which <u>decidability</u> can be achieved via operational tests for rational admissibility.

The issue arises most clearly in questions of decidability in formal systems. Follwoing the account given by DeSua [7], the Church-Turing modification of Kurt Gödel's theorem on undecidable propositions asserts that there exist—in any formal system presupposing the Peano axioms for arithemetic—well-formed formulae which cannot be proven (in a finite number of operations) to belong either to the class of theorems or of non-theorems with respect to the given axiom system. Alfred Tarski extended this result in showing similarly the existence of well-formed sentences whose status cannot be decided in any number of operations. These results do not, of course, preclude the achievement of conditional decidability in a system limited to sentences of finite length and to finite strings of logical operations—provided sufficient computational capacity is available. It is obvious, however, that whenever the computational capacity of a finite

cognitive agent is exceeded, some measure of decidability must necessarily be lost. The significance of computation for operational interpretability and programmable testing on the part of the finite cognitive agent therefore brings into issue the process of counting, which is the intrinsic basis of all quantitative conceptual constructs.

The process of counting presupposes a space-time manifold and a cognitive agent (perhaps extended by a machine) capable of constructing a one-to-one correspondence and registering a cumulative index. Counting is accomplished in terms of specifiable events in some object-space. Events simultaneously develop a measure of space and of time in that manifold as well as a registry-measure instrumental to the cognitive agent. The point of this observation is that these measures must be consistent. The relation known as "time" does not allow more than a finite number of events to be associated with a finite number (of unit-counts) in a particular space-time manifold: it requires both space and time to count.

Thus the capacity of the finite cognitive agent itself determines the order of those measures definable on the space-time-event-counting process. An "order" of counting refers to the span of events associated with counting up to some maximal capacity—a finite number within the set, $\aleph_0$. In order for a substantive object-space to be accessible to the cognitive agent for direct interaction (i.e., testing of constructs, observing and predicting future states of a substantive system, prescribing courses of action influencing future states) the counting events and the space-time metrics must be of the same order as that of the cognitive agent. Objectifications whose metrics are not of the order of the cognitive agent are inaccessible

for subject-object interaction; hence they cannot represent substantive properties of the empirical world.

Objectifications associated with a countability of higher order than that of the cognitive agent can represent formal processes only and are meaningfully subject only to formal tests for ambiguity. Since the testing of formal constructs involves extrospective interaction only with symbols, which in turn can represent constructs of manifolds at lower (or higher) orders, these processes may be accomplished within the manifold of the cognitive agent as if they were occurring at the required order—provided that the interaction with the substantive symbols is controlled by the specifications of the appropriate manifold. Symbols representing constructs of a given manifold may be assembled meaningfully in the same sentence with symbols representing constructs of the next higher manifold only if there are operators within the sentence which develop each term of the sentence to the same order. We shall later develop this statement as a commitment to the cognitive control principle termed "ontological parity."

In order for even a conditional measure of decidability to be achieved, the temporal process of computation must be limited to a finite number of events corresponding in order to that of the cognitive agent. Hence all objectifications representing substantive (experiential, "real world") constructs are limited to properties which can be warranted in a finite number of operations. In general, this limits models of the real world to the class of finite models: models which are expressible in finite strings of symbols and whose consequences are testable within a finite number of operations. "Finite" models entail also a limitation to discreteness, since

continuity is meaningful only with respect to a countability of an infinitesimal order and therefore cannot represent an empirically testable property. Continuity is meaningful only in conjunction with a formal operator that, in a manner of speaking, "operates it away," i.e., the formal operator and operand (continuum) together yield measures whose order corresponds to that of the cognitive agent. The "existence" of continuity may be posited as an internal property of a formal model, permitting transformation or connectivity among discrete substantive elements, provided that it operates away in statements having extrospective ontological status and provided the operator which accomplishes this reduction of continuity is consistent with the recursive process used originally in extensional specification of the continuum.

Thus we are led to a general premise that is complementary to the theorems of Gödel *et. al*. Whereas those theorems develop restrictions on decidability in a formal system, the complementary premise places restrictions on the freedom of the cognitive agent to create operationally warrantable objectifications. *Neither unconditional decidability nor unconditional freedom is attainable by the finite cognitive agent*. This result is immediately suggestive of the limitations on physical measurements imposed by the Heisenberg Uncertainty Principle. Our view is (1) that the interdependence of limitations on decidability and freedom *extends* the Heisenberg commitment of irreducible indeterminacy in objective v. normative measures associated with observations on conjugate physical quantities (position and momentum); (2) that the effect of this extension is to recognize a more general indeterminacy in objective v. normative measures associated with the

selection of cognitive models in terms of conjugate criteria of admissibility (decidability and freedom); and (3) that the larger import of a generalized uncertainty principle is (a) that objective and normative modes of inquiry are conjugate perspectives; (b) that no purely objective characterization of "the way things are" and no purely normative stipulation of criteria for admissibility of "the way things are thought to be" can be definitively established independently; and finally (c) that "optimal" organization of cognitive models in a systematic structure controlling knowledge, valuation, and action cannot be defined on any basis other than the stationarity of a product-measure of decidability and freedom.

Since the logic of finite operations is considered to be basic to testable conceptualizations, an attempt has been made to construct the rudiments of a type of analysis that would not involve covert assumption of continuity (or infinite processes). This exploratory work is reported in an appendix, "Geometry Over a Finite Field," prepared in collaboration with D. L. Reisler. The Galois field, as an underlying structure for development of finite analysis, has been chosen over other alternatives because it generally enables one to avoid stoppages in computation due to ambiguity or exhaustion of resources. In addition, its unique sums and inverses avoid the necessity of *ad hoc* prescriptions when a procedural impasse is encountered. However, such arguments for dependence on a Galois field are heuristic. In order to justify this approach it must be shown that the resulting finite system permits interpretation and performance of the operations of conventional mathematical analysis—at least in principle. It is certainly more efficient to perform most calculations with the aid of continuous mathematics,

and we are not advocating any rigoristic sacrifice of such an advantage. Any use of continuous mathematics, however, should properly follow from an operationally impeccable finite mathematics. Once this has been done, the mathematics of continua may be used confidently in facilitation of computation. Nevertheless, it can never be universally applicable but must be justified in terms of correspondence with conclusions assured by the intuitive priority of finite mathematics.

Early attempts to develop the operational repertoire of finite analysis have led to the development of numerous concepts that behave locally as do their counterparts in continuous mathematics. Such notions as inner product, norm, metric, complex number, for example, have all found realization. The global implications of these constructs, however, are significantly different in the context of a finite field. Of particular interest in this respect, an argument will be given (cf. Appendix) that continuous passage to the classical limit entails the loss of certain important properties of an ideally acceptable cognitive model. In order to reinstitute these crucial properties, uniformities regarded as "laws of nature" must be postulated. Certain of the invariance principles postulated as supposedly "objective" properties of the natural world may be more adequately construed as categorical features of rationally admissible conceptual models and therefore as consequences of the finite character of cognitive decision processes. Intimations of this order, belonging properly to futuristic topics in relativistic and quantum physics, indicate just how deep lying are the potentialities for reconstruction inherent in a thoroughgoing operationism.

Meliorism, in contrast, is a commitment which seems to display its significance prima facie, as if its connotations were lying together on the surface of commonsense notions concerning "improvement" of the human situation. Such meanings, however, must lie together most uneasily in view of the complex interplay of ideas on human nature and its improvability stemming variously from Platonism, early Christianity, medieval theology, secular humanism, the Protestant work-ethic. and contemporary idealization of expectations regarding social-political and scientific-technological "progress." To attempt to work through to a consistent position in commonsense terms is not a feasible aim at any length less than that of a major investigation. Counter currents of theism and atheism, predestination and perfectibility, idealism and pragmatism, determinism and vitalism have roiled the passions of centuries in such a cause. We shall stake a commitment to meliorism, in our usage of the term, at the more elemental level of implications derived from characteristics attributed to the finite cognitive agent.

In its negative aspect, meliorism follows simply as a special case of the bare denial of absolutism. The relativity, reductivity, and provisionality of conceptual objectifications holds for value-concepts as well as for substantive contructs. Key terms of normative import, e.g., "good" and "rational," are not associable with any definitive meaning independent of the cybernetic characteristics, objectives, norms, constraints, and the psycho-social and biological domain of interaction specific to the given cognitive system (individual or cultural). Not "the good" but the admissible, the preferable, the optimal with respect to specified (but modifiable) criteria constitutes the operational directive or normative judgment. Not man as "the rational animal" but man as an

animal endowed with capability for increasing approximation of optimal systems control becomes the claimnant of an epistemological warrant. Here the "bootstrap" character of the cognitive enterprise enters with additional force. The very criteria that make aesthetic and rational selection operationally meaningful must themselves be instituted on the basis of creative and selective processes that are provisional. Underlying this exclusion of absolute value-concepts, however, the characterization of the finite cognitive agent as capable only of conditional decidability and conditional freedom nevertheless yields an obverse aspect of meliorism: evolutionary realization of novel forms of cognitive-cultural organization marked by superior adaptive capability, and hence viability.

This more significant positive aspect of meliorism is entailed by just the bare rudiments of a world-view that are to be found in the earlier adoption of an evolutionary paradigm (Chapter 7) and the subsequent construction placed on cognitive-cultural development (Chapter 8). The synoptic hypothesis implicit in such a view is: that the processes of transformation operating throughout the cumulative domains of the geosphere, biosphere, sociosphere, and noosphere are selective, norm-directed processes admitting, in the long range, only of (a) termination of lineages or (b) improvement in the organization of systems belonging to the lineages of any ontological category whatever. On this basis we outlined a developmental process that yields, in fine, an iterative improvement of the individual cognitive system in terms of broadening scope of interaction and increasing range of adaptive response. This developmental pattern holds, in the large, over cultural and conceptual systems. A major conceptual system (in science, ethics, law, religion)

does not reside with any single individual but is lodged within a culture or cultural sector, subject to one special condition--the possibility of communication via symbolic structures more enduring than the individual cognitive agent. Man could not communicate his thoughts and feelings and therefore could not participate in a social world without the semiotic capability to map, to model, to simulate--to objectify--his conceptualizations in fixed and reasonably durable linguistic constructions. Cultural development is possible only because communication across generations is possible. The individual need not originate all his concepts, commitments, theories, but can acquire prepared ideas and practices by way of culturally determined education. Even an enduring cultural inheritance, however, is in constant danger of losing its meaning. Its existential status is operational not substantive; its significance is referential rather than immediately affective. The relationship of symbolic structures to experience never ceases to require reinterpretation and reconstruction. Thus an evolutionary production of emergent novelty characterizes the transitions by which the developments and advances of one human generation are acquired, reinterpreted, and passed on <u>with modifications</u> to successor generations.

In regard specifically to the notion of an evolutionary development of natural science, S.E. Toulmin [8] amplifies this point in the following way:

> "The carrier of scientific thought, at any particular stage, is the relevant 'generation' of original young research workers. Each new generation <u>re-creates</u> for itself a vision of nature, which owes much to the ideas of its immediate masters and teachers, but in which the ideas of the preceeding generation are never replicated exactly. (Perfect replication is the mark of Scholasticism.)

> The operative question for any adequate philosophy or logic of science accordingly is: What criteria does each new generation of scientists rely on, in deciding which aspects of their elders' theories to carry over into their own ideas about nature, and which to abandon in favor of current variants and innovations?"

On this question of criteria of selection specific to sciences as a cultural component, an evolutionary paradigm yields the following answer. That in a sufficiently advanced stage of human development, where creativity and rationality locally override sheer biological mutation and instrumental adaptation, man as the one accultural (purposefully culture-acquiring) animal selects with regard to the criterion of <u>optimal organization</u>; and in the case of scientific advance, does this by way of a second-order optimization of cognitive organization--the operational agency of all his conceptualized aims. An intellectual version of meliorism is clearly implied in all of this. However, our characterization of an evolutionary cognitive agent did not project any abstract consideration of man <u>qua</u> thinker but, rather, addressed the rational-aesthetic-creative complex of the whole man, engaged in multifarious interests and socio-noetic aspects of a total environment. Under the heading of "extremalizing tendencies of the self-realizing system," our attribution of paired criteria (optimal control and maximal freedom) as complementary aspects of optimal organization actually entails a much more broadly generalized version of meliorism.

It is technically advantageous to deal with extremalization processes in terms of <u>criteria</u> for quiescence of a goal-seeking system; and it is even more advantageous, computationally, to deal with explicit <u>objective functions</u> (value functions). However, there is some clarity to be gained at this point in using still a third alternative form of specification:

the attribution of <u>motivating drives</u> to a selective system. In order to account for the modifiability of the behavior of the cognitive agent, we earlier posited the dual criteria of optimal control and maximal freedom as fundamental determinants of characteristic response. We shall present these commitments now in the guise of their equivalents, a set of corresponding psychological "drives" of the cognitive agent. These drives are construed as analogous to commonly accepted biological drives for reduction of physiological stress and appetitive satisfaction:

(1) <u>Tactical Drive: Categorical Aversion to Ambiguity</u>.
This aversion causes every instance of ambiguity (cognitive dissonance) to be taken as a problematic situation, the resolution of which is accomplished by one of two operations: (a) reprogramming or (b) renormalization. Reprogramming entails modification of habitual behavior and redistribution of the resources under control of the cognitive agent. Renormalization requires the readjustment of values (norms), policies, goals. Since ambiguity prevents the attainment of unique decisions, this primitive commitment implies a drive toward decidability.

(2) <u>Strategic Drive: Localized Preference for Cybernetic Elegance</u>.
"Cybernetic elegance" is associated with minimal allocation of information processing capacity to immediate requirements of internal cybernetic operations (systemic control). Minimization here is with respect to the total portfolio of programmed responses to problematic situations. This drive constitutes an aesthetic goal-orientation and its satisfaction requires adaptation by systematic reorganization of an existing conceptual

system. Minimal allocation of capacity to cybernetic requirements frees more capacity for use in formulation of additional programs, thus increasing the range of external conditions over which programmed responses are attainable. This drive is associated with the essential criterion of "simplicity" in pattern formation and recognition (here considered as an internal optimization process).

As we have noted earlier, optimal control and maximal freedom are inherently antithetical. Accordingly, the aversive and "appetitive" psychological drives posited above as corresponding to these opposed criteria cannot, in principle, be pursued simultaneously. This observation, of course, has even broader application. Multiple objective functions, multiple criteria, multiple drives--in general--do not admit of simultaneous extremalization except by fortuitous coincidence of solutions to distinct programming problems. Singularity of the value-criterion is a condition of unambiguous determination of optimal response or decision. It was in recognition of this fact that a singular criterion for optimal organization (the intrinsic value "maximal realization") was earlier posited as a principle legislating over the assignment of priority to improvement of control v. increase of freedom. In a comparable role we shall pose the following psychological drive--not, indeed, as "legislating" over subsidiary drives, since the triggering and extinction of drives requires no such mechanism--but as <u>subsuming</u> aversion to ambiguity and preference for elegance, as serving to render these subsidiary drives operable in the face of continued failure and stress:

(3) <u>Entrepreneurial Drive: Global Preference for Holistic Coherence</u>.

Essentially this drive is representative of the restless, exploring tendency of the cognitive agent to enlarge his domain of interaction toward the limit of a universal scope. Far from being associated with solution of inescapable problems, such a tendency <u>generates</u> a global problem which could be but consciously is not avoided: the idealized demand for coherent systemization of all perceptual-conceptual relationships. "Optimal" cognitive organization connotes a balanced capability for (a) effective tactical action in attainment of immediate goals and relief of immediate stress, combined with (b) maintenance of a strategic posture productive of long-range viability with respect to a given reduced context of interaction. The drive for holistic coherence amounts to a self-imposed requirement that this tenuous balance shall be maintained even in the face of a continuous, purposeful widening of the scope of interaction--which must necessarily open the cognitive agent to new evidences of multiplicity, incessesant change, disunion, incomprehensibility. The payoff of this high-risk tendency, however, is correspondingly high. For the satisfaction of this overall conative (goal-seeking) drive can be approached only by (a) reobjectification and (b) reformalization. "Reobjectification" refers to the creative act of modifying the primitive concepts and commitments in terms of which observation and theory-construction, factual judgment and valuative judgment, have previously been carried out--as well as the criteria and

procedures of tests for admissibility of such judgments. "Reformalization" entails comparable modifications in the structure or operation of the abstract formal systems (mathematical and logical) previously used as the relational paradigms for substantive and valuative object-theories. Such modifications may range from the elemental to the organizational: for example, the posited existence of the neutrino v. the annunciation of a quantum theory, the prohibition of a particular criminal act v. the assertion of "the rights of man," the resolution of a mathematical paradox v. the achievement of a calculus of variations. When taken cumulatively, with a view to the ideal of establishing coherence among all aspects of a holistic individual-universe system, such modifications admit of <u>emergent</u> realization of more and more of the novel potentialities regarding what man and universe can <u>come to be</u>.

This "perfectioneering" drive toward a coherent universe of discourse in which a systematic relationship would bind all perception and conception, all interests and activities, all values and goals, typically exhibits some appearance of counter-productivity, or at least utter impracticability. It necessarily entails the persistent questioning of conventional wisdom, the relaxation of traditional--even hallowed--constraints of conservative social and rational practice, the trial-denial of habitual belief and expectation. This much <u>non-rational</u> (imaginative) behavior is the price of participation in creative reobjectification and reformalization. The basic activity is that of seeking answers to questions no one (of practical mind) is asking. It is only this type of activity, however, that can lead to breakthrough, to the injection of the novel construct, the novel

format of organization, the novel method of inquiry, the ultimate novelty of a new world-view that opens possibilities for newly effective pursuit of immediately significant pragmatic and aesthetic aims. In a situation where the improvement of systems control is blocked by intractable ambiguities, where the capacity of the human decision maker for adaptive response to cultural or phsyical evolutionary change is overloaded, it is only the slowly maturing payoff of reobjectification and reformalization of the whole system (or portfolio) of cognitive models that can render these immediate aims operable again in terms of new cognitive resources and capabilities.

We may therefore associate the psychological drives (above) with extremalizing tendencies of the cognitive agent as given by the following correspondence:

PSYCHOLOGICAL DRIVES	CHARACTERISTIC GOALS	EXTREMALIZATION-CRITERIA
Ambiguity categorical aversion	Decidability	Optimal Control
Cybernetic Elegance	Potentiality free capacity for adaptive reorganization	Maximal Freedon
Holistic Coherence	Extensionality universal domain of interaction	Optimal Organization

It may be helpful now to review these psychological drives, taking them respectively as aligned with a multiplicity of simultaneous aims in cognition. A fundamental dictum of pragmatism is the assertion that thinking is for the sake of acting. It is to the purpose of achieving human ends in the face of present stresses and frustrating failure that the cognitive process moves into action. This begins with issues

expressible in such questions as: What is to be expected? What is going to happen? What is to be done? Thus objectification and classification of things, correlation of events, symbolization of concepts and interpretation of experience in terms of meanings constitute the elemental tasks of pragmatic thought. Whenever such activities lead to expectations which lend themselves to successful attainment of human ends, such expectations establish habits of behavior which are a basis for programmed, uniform approach to problem situations of a given kind. The efficiency and precision of prepared (programmed) response is all-significant at this level; ambiguity and the paralysis of decision that ambiguity entails are potentially disastrous. Unstructured sensation applies pressure on the cognitive agent to produce constructs and categories that provide at least a primitive system of prediction and explanation. Particular objects and events must be successfully subsumed under a class or kind in order that they may be confronted thereafter with expectations which prove to be appropriate. Unobjectified sensory experience, perceived only in terms of chaotic perturbations unassimilable under existing programs of habitual response is unpredictable, uncontrollable, and therefore dangerous; it places the cognitive agent in a threatening situation. Thus, aversion to ambiguity and a concommitant drive toward decidability characterize the elemental version of rationalization which affords an ability to map, to model, to simulate behavioral problems in advance of the tensions which call for an immediately successful action. Decidability is the proximate aim of pragmatic thinking because it is the primary condition of successful thinking for the sake of survival.

In attempting to attain a well-adapted repertoire of programmed behavior, however, the cognitive agent is constrained to operate within

the limitations of a given sensory and cybernetic system design. He does not have the freedom to create arbitrarily many different categories and modes of objectification. It is in this problem context that a further cognitive aim of thinking for the sake of thinking is superimposed. A decidedly aesthetic dimension of the motivation to think emerges, often quite unrelated to the tensions of immediate desire for material ends which pragmatic throught addresses. The central issue here is expressible in the question, What is to be believed? That is, what is the "best" form of representation or mode of simulation for experience? The choice, as determined by an optimization principle of parsimony, simplicity, or in our terminology "cybernetic elegance," is forced by the overwhelming strategic advantage of minimizing the information processing capacity devoted to cybernetic control in employment of a given conceptualization. There is an obvious premium on those programs that can minimize the requirements for attention and computation and thereby free more capacity for acceptance of a new order of problems. A program that requires less capacity allows the cognitive agent to do more within the limitations of his species-specific constraints. A highly complicated, extensive program might well solve a given problem but, by exhausting all available capacity and energy, leave the cognitive agent powerless to respond to concommitant problems or wider interests of equal significance. If several conceptual schemes and their related programs can be combined or consolidated under one generalized regimen, the immediate increase in free capacity can be devoted to enlargement of the span of control and the scope of knowledge. Above all, the contribution of cybernetic elegance is toward added potentiality for meeting crucial demands for

adaptive modification in the face of drastic environmental change. The significance of "potentiality" here is that it clarifies the sense in which a drive for cybernetic elegance entails the cognitive aim of thinking for the sake of survivability, for the sake of adaptive range, and hence viability.

Even the most strenuous efforts to maintain the adaptedness and the adaptability of cognitive-cultural organization, however, strike ultimately against the limitations of a given system design. With accumulating environmental change, massive problem situations arise which call for a type of transformation which the given design--in virtue of incompatibility or inconsistency among multiple commitments, goals, strategies, and programs--does not permit. The system, as it is, cannot get there from here. That the encountering of critical blockages is inevitable, one can conclude from such fundamental analyses as Whitehead's account (Adventures of Ideas, 1932) of the necessary discordance, the limitations undiscoverable in advance, in any complex scheme of ideas or social organization. That such blockages are capable of obliterating existing systems can be appreciated in the deadly portent of such assertions as Lincoln's "This nation cannot continue to exist half slave and half free." Cognitive-cultural organization perennially encounters crises of tension between the effects of creativity and rationality, between the counter demands of freedom and control, played out in both social and individual contexts. In order for a cognitive system to remain viable, it must operate within a structure that does not admit of complete freedom. A system that is completely free can be nothing more than a dissociated aggregate without internal determinants or constraints on characteristic interaction. The components of a system must be coordinated

by constraints if they are to function as a whole. Conversely, a system that is completely determined is an automaton that can never adapt or evolve. It is irrevocably limited to its original programs. Its structure does not admit of concrescence or a broadening of its scope to accommodate new experience. It is literally trapped in a fixed mode of behavior; it can only carry out those functions predetermined by fixed design. Hence, an adaptive system must exist in some intermediate mode that is characterized by both freedom and determinism, with neither completely dominating.

The creative capability of a system introduces new freedom and the control capability forecloses freedom and institutes determinative decision and action. Thus, the overall entrepreneurial task of the cognitive agent is self-transformation and self-organization, providing new degrees of freedom and new options, while also permitting improvement of the controls constraining the system and thereby enabling it to exploit additional freedom. In the face of these antithetical demands, the supreme strategic issue will always consist in the fateful choice of some particular balance between the flexibility which can be provident with respect to viability in the long range and the stability that is required for immediately effective response in action. By attribution of an "entrepreneurial" psychological drive for holistic coherence, we accommodate the demonstrated capability of the cognitive agent to maintain this tenuous balance even when the design of the given conceptual system will no longer do, even when the condition for success is redesign for survival in a new mode. The motivation for the kind of thinking that matters here, i.e., reobjectification and reformalization, is still further removed from immediate problems. It is dependent upon the

idealized and perfectionistic interests ordinarily associated with "pure" inquiry, and necessarily so. The maturing of fundamental rearrangements of ideas is a slow process. If it were not motivated continuously by aesthetic, exploratory interests, the necessities of action in a crisis could not be met, as they must be, by alternatives prepared <u>in advance</u>. It is of tremendous import that systematic explanation and prescription, undertaken under the aims of pure inquiry, may later be applied toward solution of problems of the most utterly practical type: life or death for men and societies, extinction or emergence of novel forms of organization. The fertility of pure inquiry in this respect is somehow amazing, despite our realization of the fundamental character of its typical questions: What is conceivable? What potentialities of existence, of knowledge, of action are realizable under a more nearly complete and coherent organization of experience? The fortuitous way in which purely theoretical constructs are repeatedly found to "fit" critical needs is hardly understandable except under the view that the preoccupation of pure inquiry with what can conceivably <u>come to be</u> represents a third distinctly prescient motivation in thinking. We describe it as <u>thinking for the sake of new modalities of survival</u>; thinking, that is, for the sake of realizing (actualizing) successively more durable and more satisfactory forms of organization.

The multiple aims ascribed to cognition may now be presented in alternative ways that bring out a number of related aspects of significance. Reproducing the theme specially emphasized in the foregoing review, we may think of the cognitive agent as being motivated by a

cascade of connected objectives: (a) survival as a system, (b) survivability of the system of given design, and (c) system design for survival in a new modality, i.e., survival of a prototypical system. Additional connotations of the notion of "cascaded" objectives are given by the terms of the following parallel triads: (1) adapted-adaptable-emergent structure and function; (2) tactical-strategic-entrepreneurial drives; (3) reprogramming-reorganizing-reobjectifying operations; (4) homeostatic-morphostatic-morphogenetic processes; (5) pragmatic-aesthetic-evolutionary criteria. Each of these triads contributes toward explication of what has been heretofore a long-standing conundrum for intuitive judgment: that cognitive processes somehow serve, simultaneously, quite disparate human ends; that they somehow support the cognitive agent in an equilibrium of tension between two orientations that philosophers have termed "the mighty opposites," being and becoming. In summary, cascaded psychological drives permit cognitive agents (1) to maintain existence and (2) enhance the quality of existence even while they are (3) modifying the very terms of existence by successive realization of possibilities regarding what can, conceivably, come to exist.

It might be thought that the attribution of psychological drives necessarily involves the philosophical systematist, finally, in the reprehensible version of psychologizing that we have been at pains to avoid: that is, prejudgment regarding matters of fact which only empirical inquiry could properly establish. This is not so. One firm conclusion can be drawn from the considerable furor over injection of hypothetical constructs and intervening variables as unobservables in behavioral inquiry--and here biological or psychological drives are examples par excellence as, indeed, are forces in general. The conclusion

is that "unobservables" cannot, in principle, be assigned the status of empirical matters of fact. They are not the kind of constructs that can be directly confirmed or disconfirmed on the basis of empirical evidence. Rather, they constitute either formal or valuative elements of a conceptual format, a way of representing the world (an objectification) to which we commit ourselves provisionally by policy-decision, under the proviso that the admissibility of any such format shall hinge on (1) operational interpretability and testability in the distinct senses appropriate to formal, factural, and valuative elements respectively and (2) theoretical "gain" in the sense of attaining generalized correlations of observables that are not otherwise achievable. With the concept of drives we therefore infringe not at all on the province of the experimental psychologist, but merely pose an alternative way of assigning meaning to his results.

All of this can perhaps be more readily appreciated when it is recalled that the attribution of psychological drives is equivalent to positing optimal control, maximal freedom, and optimal organization as idealized criteria of extremalization processes characteristics of cognitive behavior. Such idealizations patently admit no trace of any misleading confusion with matters of fact, which is an admitted liability for the notion of psychological drives. It is precisely for this reason that our primary formulation of characteristics of the cognitive agent has been given in a criterion-terminology, i.e., in terms of a succession of generalized norms associated with levels of a hierarchical adaptive process in which response at each level is the result of selection under successively more abstract criteria.

Only in the permissible sense of posing this alternative conceptual format--a normative theoretic format--have we engaged in "psychologizing" and the point of all the foregoing illustrative treatment of psychological drives has been simply a clarification and amplification of the manner in which the finite character of the cognitive agent entails a philosophical commitment to meliorism. This characterization provides the fundamental justification for meliorism in its positive sense as a commitment to "improvability" of the human condition. With respect to the negative sense of meliorism, namely, that _only_ improvability rather than closure or absolute attainment in knowing, valuing, acting is open to the cognitive agent, Brown [9] in his recent _Laws of Form_ states our case:

> We cannot escape the fact that the world we know is constructed in order (and in such as way as to be able) to see _itself_.
>
> ...But in order to do so, evidently it must first cut itself up into at least one state which sees, and at least one other state which is seen. In this severed and mutilated condition, whatever it sees is _only partially_ itself .... In this condition it will always partially elude itself.
>
> ... We, as universal representatives, _can_ record universal law far enough to say
>
> > and so on, and so on you will eventually construct the universe, in every detail and potentially, as you know it now; but then, again, what you will construct will not be all, for by the time you will have reached now is, the universe will have [emerged] into a new order to contain what will then be.
>
> In this sense, in respect of its own information, the universe _must_ expand to escape the telescopes through which we, who are it, are trying to capture it, which is us.

That part of the world (the cognitive agent) which is capable of seeing the world, capable of mapping the whole onto a part, is capable *a fortiori* of seeing itself as a subsystemic whole and mapping that whole in turn onto a part of itself. From this inward-spiraling sequence arises the peculiar difficulty of philosophies of organismic systems and of social-behavioral system sciences. But from the self-corrective, self-transforming procedures of this same sequence arise, as well, the outward-spiraling compass of the cognitive process as a principal exemplar of the general process of evolutionary realization, by which the world has come to be what it is and is now unfolding what it can become.

Chapter 10

EVOLUTIONARY SYSTEMS PHILOSOPHY

The best hope for a supportable advance in systematic philosophy rests with careful self-cognizance regarding the operations we perform in philosophical construction. Explicit awareness is crucial with respect to commitments which institute entrepreneurial control over constructive specification and the selection of criteria for an "improved" philosophical system. In view of the laborious preliminary studies demanded by this reflexive mode of inquiry, one might be tempted to conclude that self-cognizance is more properly the despair of philosophy than its best hope. An important part of our responsibility will be to show that, in fact, a very good return indeed can be realized from preparatory studies so obviously costly of time and effort.

EVOLUTIONARY REALIZATIONISM--A PRÉCIS

In this chapter we move at last onto more conventional ground. The order of business is an attempt to formalize the structural outline of a distinctive philosophical system--so far as a coherent and comprehensive scheme of ideas can be drawn from intimations gained in prior studies. The suggestion of double reference in the chapter title is intentional. We do intend the scheme of ideas as a philosophy of evolutionary systems. Also we do presume that the philosophical scheme, if

it is to prove at all durable, must itself comprise an evolving system. From this section forward in subsequent chapters, the objective will be to elaborate at successive levels of metatheoretic, theoretic, and applied development, attempting to expose the philosophical system to tests for its serviceability in description, prediction, explanation, and control of experience.

Our reference to entry onto more conventional ground means that this précis will be given in terms of philosophical commitments proper, that is, in terms of concepts and assertions recognizable as primitive components of a philosophical position. The significance intended for such terms as "proper" philosophical commitments and philosophical "primitives" is perhaps not at all clear. The key notions here are simply (1) the formality with which commitments are posed when they are intended for concerted development and (2) the emphasis on systematic extension of initial technical concepts. The construction of a philosophical system can by no means approximate the kind of closure that is possible in axiomatic construction. Yet there is a similarity to this extent: in contrast with the dialectical development of entrepreneurial commitments, the selection of components for a philosophical position fixes or "freezes" a system design pro tem. The premises with which we shall now be dealing are to be maintained with sufficient conservatism that we can examine their system-capabilities at length, moving by this means from synthesis to analysis, from system construction to implementation--until blocked by encounter with limitations of the system.

On entering this phase we engage in tasks with which systematic philosophy traditionally has begun. Even here, however, the regimen of philosophy in the reflexive mode dictates further detailing of

commitments--again, a matter of sharpening the specification of certain aspects of philosophical construction that have too often been treated covertly.

The Total Structure of Commitments

Table 10.1 presents a categorization of principal commitments and primary concepts by phases of the process of reconstruction. At earlier critical points of development we have found it helpful to preserve continuity by the use of a resume that is at once a summary and a preview. So with this array of Table 10.1, the list of entrepreneurial and organizational premises may be regarded merely as reviewing our conception of "what is to be done" in philosophical reconstruction and the a priori policy determinants which constrain the project of system-philosophy within the region of competence of the cognitive agent. On the other hand, programmatic and operational premises deal with what is to come: (1) the placement of a modal commitment which will be both formative and regulative of (2) the operations that traditionally comprise the opening moves of philosophical construction, i.e., the positing of technical primitive assertions.

Modal Commitment

The multifaceted structure that we term a "modal" commitment initiates the formal phase of a trial-synthesis. The ordinary connotations of the term associate it with a manner of proceeding, a characteristic way of approaching a task, a regimen controlling the performance of tactical operations. A more precise development of its connotations might be based on our prior reference to the "cognitive-semiotic modality," specifically, the behavioral capability for symbolizing and controlling action by construction and manipulation of conceptual models. Obviously

PHASES OF RECONSTRUCTION	PRINCIPAL COMMITMENTS	PRIMARY CONCEPTS
Entrepreneurial	Identification of Problematic Situation	fact-value dualism
	Estimated Adequate Means of Response	philosophy in reflexive mode
	Selection of Terminal Goal	philosophical reconstruction
	Prolegomena	characterization of cognitive agent
Organizational	A Priori "Determinants" of Philosophical Position	
	(Finitistic aspects of conceptualization)	
	Relativity	mutually constitutive constructs
	Reductivity	individual-universe partition
	Provisionality	semiotic freedom
	Operational Testability	cybernetic control (decidability)
	Meliorative Utility	optimal organization
Programmatic	Modal Commitment	
	(Evolutionary Process as paradigm of conceptualization)	
	Substratum Existence	individual-universal nexus of interaction
	Generative Process	conceptual objectification
	Emergent Existants	conjugate holons (system-antisystem)
	Selection Process	evolutionary tests for holistic admissibility
	Holistic Criterion	maximal realization
Operational	Philosophical Commitments Proper	
	Ontological	enduring entities (self-object duals)
	Epistemological	canons of rationality (cognitive controls)
	Methodological	objective-normative complementarity
	Axiological	categorical hierarchy of norms
	Praxiological	convergent embedding of system models

Table 10.1 Philosophical Reconstruction: Principal Commitments and Primary Concepts

a "modal" commitment in this context should be expected to characterize the mode of conceptualization in general. Such a commitment constitutes <u>the</u> initial maneuver in formalized thought (analytical as against dialectical) inasmuch as it purveys a supreme program-directive. This is to say that a modal commitment is <u>constitutional</u> in the sense that it is both productive and regulative of a philosophical "style." Generally, any such regimen comprises an implicit background feature of a way of thinking, remaining to be ferreted out by inference as to those assumptions that have been made without conscious realization that any have yet been made. The mere attempt to be explicit here is therefore an innovative feature of philosophical reconstruction.

To engage in cognition is to opt for the employment of one particular modality among several that are open to human intelligence; and to put forward a modal commitment is to be explicit about "what goes on," operationally, under that particular modality. It is in this sense that we undertake continuation of David Hume's reorientation of philosophy toward concern for the nature of ideas and the operations that we perform in reasoning. In summary, a modal commitment is a deposition concerning the nature of things thinkable or conceivable, as well as the nature of the process of conceptualization. The subject matter of a modal commitment is an account of the origin of ideas, concepts, constructs, or "things" in the most general sense of that term. Table 10.2 outlines the key features of our modal commitment, and we abbreviate the full development of this programmatic primitive by means of the following statements:

(1) that the bare concept of an organismic <u>system</u> constitutes a schema or format characterizing the structure of conceptual objectifications in general;

EVOLUTIONARY PARADIGM (Factorization)	CONSTRUCTIVE SPECIFICATION (Conceptual-Evolutionary Process)
Substratum Existence	Cognitive Agent in Context of Interaction
Generative Process	Conceptual Objectification
Emergent Existants	System and Anti-System (Primal-Dual)
Selection Process	Canons of Rational Selection
Holistic Criterion	"Optimal" Cognitive Organization (successive dominant)

Table 10.2 Modal Commitment: Evolutionary Process as Paradigm of Conceptualization

(2) that every novel objectification initially appears as a component of an emergent dual system subject to selection in the context of an evolutionary system;

(3) that an evolutionary paradigm is therefore appropriate for constructive specification of the process of conceptualization.

Freedom and Control: Systemic Structure. A conceptual objectification is the emergent result of a creative act which externalizes (maps onto symbols) a novel organization of concepts and concommitantly internalizes (maps onto images) a complementary component of self-organization--on a trial basis subject to admissibility under cognitive norms (controls) instituted *a priori*. In order to clarify this cryptic description, a number of considerations need to be brought to bear simultaneously.

The general context of conceptualization is the primordial partition comprising an individual-universal dyad, with interaction between complementary components characterized, in part, by unstructured interaction in the sense of sheer perturbation. Insofar as the individual cognitive agent is capable of (a) identifying apparent correspondence relations among repetitions of perturbation, (b) synthesizing concepts of "things" which as unitary wholes may be imputed to be distinguishable and stable features of an external "world," and (c) mapping these "created" things (constructs) onto symbolic elements[1] of cognitive models, he has the means of (d) formulating *characteristic* correspondences between perceptual

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1. No prejudgment is intended as to the substantive character of symbolic "elements." Mere sequences of sensations, subjective images, covert behavioral syndromes--in addition to significant gestures or formalized linguistic objects--are capable of bearing meaning. The account of conceptualization above applies not just to relatively sophisticated cognitive-semiotic behavior, but to infant behavior as well.

versus conceptual entities (objectives, events, processes), and (e) preparing--on this basis--<u>anticipatory</u> responses which admit of (f) successes in the purposeful activities of goal-seeking behavior.

The process of conceptualization is carried out by a finite command and control organization (a cognitive agent) capable at best of limited semiotic freedom and conditional cybernetic control. Undecidability, as to selection among alternative expectations and consequent actions, indicates the presence of a range of freedom that must be foreclosed by a higher-order selection (decision) principle in order to yield any definite response whatever. The provisional-experimental character of conceptual objectification entails the necessity of normative control over selection <u>among</u> objectifications in advance of taking the risks of actual trial-and-error behavior. Two considerations are particularly significant here: (1) Distinct dimensions or types of freedom must be foreclosed at successive levels of decision by principles serving as decision operators characteristic of each particular level of the problematic situation--for example, the successive questions: which act, which cognitive model, which strategy of reduction of experience, which goals, which terminal values? (2) The requirement for institution of principle beyond principle entails a hierarchical structure of cognitive control, each level composed of more abstract and more general principles than those occurring at lower levels of the normative echelon. Since decision principles at each level must be successively more general they must decrease in number. A finite hierarchy of norms, extending from the level of elemental natural norms to the level of a singular holistic criterion must therefore stand in one-to-one correspondence with the structure of embedded decision levels (operational, tactical,

strategic, organizational, entrepreneurial). The connectivity established between embedded levels of decision by this hierarchical coupling of decision principles ensures that any sufficiently "complete" conceptual objectification--i.e., any holistic representation of a decision situation--will necessarily have a structure characterized by the essential compositional property of an organismic system: finite, irreducible hierarchical structure with infimum and supremum terminations. Hence "system" is not a concept which applies to certain objects or constructs and not to others. It is the concept-schema or format constitutive of all conceptualizations, of all conceivable "things."

It will no doubt be immediately acceptable, intuitively, that all things *qua* conceptual constructs presuppose this underlying systemic structure. Nothing could be clearer than the fact that conceptualization has the effect of organizing or systematizing, a universe of experience out of a previously "given" flux of unordered, sporadic sensory perturbations. By conceptual objectification the world of perceptual and conceptal objects (as against a given "substratum existence") is continually being made and remade, congnitively, as a system of interacting subsystems. What is not so obvious, though ultimately paramount significance, is the attending consideration: that conceptualization simultaneously has the effect of organizing, systematizing, synthesizing a correlative *self*-system which is at once a *determinant and a consequence* of the parallel organization of a world-system. Despite the apparent threat of a conundrum here, there is in fact no real obstruction. Cognitive agent and conceptual objectification respectively comprise *mutually constitutive*, *complementary* components of a subject-object dual system--an element of an aggregated dual system comprising a

self-with-external-world. By conceptual objectification a "self," as well as a "world," is continually being made and remade cognitively in terms of progressive individualization, personalization, self-realization. A holistic representation of any conceivable thing whatever, when given in the context of this duality, will therefore have a structure further characterized by additional interaction properties of an organismic system: dyadic configuration and idiosyncratic response (in part). Against (1) the reductionistic abstraction of "things in themselves" as outputs of the cognitive process presupposed by naive realism, and against (2) the holistic abstraction of "a unitary system encompassing all things" as the output presupposed by absolute idealism, a commitment to the modal complementarity of all "existants" occupies the difficult middle ground given by the following concrete generalization. "Things" are realizable only as mutually constitutive component systems and--in the case of special interest here--"things" are <u>conceivable</u> only as mutually constitutive components of subject (self)-object dual systems.

The distinction thus introduced with respect to things that exist by thinking (cognitive agents) versus things thought to exist (external world) is not the irremediable splitness of mind-body dual<u>ism</u>. It is, rather, a "distinctiveness within unity" in virtue of the dual<u>ity</u> of objectifier and that which is objectified, i.e., in virtue of their mutual determination, their simultaneous realization, and their relation as complements.

Such a commitment follows from the primordial consideration that, in cognition, the world is being regarded by a part of the world which must necessarily be (held as) distinct from that complement which is being regarded. But far from severing all relation between part and

whole, the introduction of just such a distinction is the necessary condition for existence of that relation which is most significant in cognition: namely, a homomorphism, a mapping of some reduction of the whole onto a part. It follows immediately on this view of cognition that the world will be generally characterized by "becomingness." Just so far as cognitive modelling leads to conceptualization of newly distinguishable things as components of an external world, the cognitive-agent-with-conceptual-model now constitutes a realization of a feature of the world which was not "there" previously in any sense except that of potentiality for subject-object interaction. On this basic pattern, emergent events necessarily proliferate in a way that ensures that the world--as partitioned into subject-object dual systems--will forever remain _under_specified by thought, though increasingly precise specification continue indefinitely. An implicit aspect of a modal commitment to _dual system_ as the basic concept-schema is therefore the posit that an evolutionary process is the appropriate paradigm for the process of conceptualization.

Evolutionary Paradigm. In traditional metaphysics the opening move is usually the proposal of a set of categories. Essentially, categories provide an originative factorization of concepts in terms of which all further investigation will be undertaken. In a sense they have status similar to that of the undefined terms of an axiomatic system. The significant difference is that categories possess the holistic character of psychologically primitive _concretions_, admitting of the unfolding or unpacking of meaning and implication, in contrast with the elementalist character of logically primitive abstractions, which constitute modules for construction of successively more complex objects

by formal definition. A sharper appreciation of the special role of a modal commitment can be gained by contrasting our approach at this juncture with the traditional posit of a set of categoreal concepts and a synoptic view of "the way things are."

As suggested by the correspondence of entries in Table 10.3, it is a relatively straightforward exercise to generalize further on the basis of an initial factorization of evolutionary process, to secure idealized notions for a cosmological theory that might be thought to be implicit in any view of cognition which ensures "becomingness" as a property of all things conceivable. To begin with some such view of the world-as-object, an organization of formal constructs capable of unfolding all of the specifiable entities and processes realizable in experience: this approach has an undeniable appeal. We believe ourselves to be late comers on the evolutionary scene. We recognize the ultimate requirement of explaining not only our own substantive emergence but that of antecedent entities reaching back toward the limits of our imagination. We are under compulsion to see ourselves come to be, spatially and temporally. An initial, all-encompassing commitment that admits of anything at all plausible on this order will perhaps always exert its attraction. The hard conclusion which must be faced, however, is that this version of initial commitment is indefensibly premature and prejudgmental in philosophical construction. A cosmological theory is, first of all, simply the most general of object theories. Despite any satisfaction we may take from a loose impression that a formal-ideal overview of the world can be used to generate the actual trajectory of a substantive evolutionary history, no such objectivist construction can properly hold the status of a foundational commitment. Impute what

EVOLUTIONARY PARADIGM (Factorization)	METAPHYSICAL CATEGORIES (Implicit Cosmology)
Substratum Existence	Individual-Universal Nexus
Generative Process	Actualization Matrix
Emergent Existants	Conjugate Holons
Selection Process	Extremal Principle
Holistic Criterion	Maximal Realization

Table 10.3 The Objectivist Alternative of Traditional Metaphysics

timeless, immutable, universal properties we will to cosmological primitives, their admissibility as confidence-bearing constructs will still be subject to prior metatheoretical commitments and these, in turn, will be derivative from some characterization of the cognitive process. Cognition, as process, has a primacy that is categorical. As Hume observes, everything will finally be "judged of by its powers and faculties." So even with its own output. From this consideration stems the basic requirement for philosophy in a reflexive mode. Cosmological hypotheses constitute, at most, retrodictive theories which may tend to vindicate primitive commitments; but the foundations of the cognitive enterprise can never be other than contemporaneous with the ongoing deployment of the cognitive modality. Our "beginnings," in this sense, must always be found where we are.

The content of a modal commitment, in contrast with that of a traditional metaphysics, does not comprise any categorization of reality, such as that abortively explored in Table 10.3 (The Objectivist Alternative of Traditional Metaphysics). No cosmological theory, no synoptic hypothesis about the character of the world as object may legitmately be attempted in advance of ontological, epistemological, and axiological commitments--since these are construed as *directives* for the formulation of object theories. Instead, a modal commitment consists in an initial factorization of a primitive process of *realization* (as against an objective "reality"). This factorization, in its essentials, distinguishes the following formal, functional, and normative aspects (precisely the non-substantive aspects) of that mode of realization by which all existants, as cognitive objects, are presumed to come into being:

(1) characteristic <u>structure</u>: configuration of system as a hierarchical interaction-dyad;

(2) characteristic <u>processes</u>: iteration of (a) the generative process of conceptual objectification and (b) the regenerative (control) process of selection among alternative objectifications;

(3) characteristic <u>norms</u>: a holistic collection of criteria for admissibility of objectifications.

As expanded in Table 10.2, this factorization decomposes the process of conceptualization into (1) the ongoing activity of an individual cognitive agent, as correspondent to a universal complement, situated in a context of interaction imposing (2) a flux of perturbations that is encoded or patterned in accordance with species-specific modes and characteristic degrees of decidability and freedom to yield (3) dual systems as objectifications (subject-object, self-other, system-antisystem) subject to (4) selection among alternatives for admissibility with respect to (5) a holistic set of norms subsumed under the supremum-criterion of optimal cognitive organization (maximal realization).

It is to this complex, in effect, that our modal commitment assigns the ordinary language term "cognitive process." The service of this complex is to denote the schema which will be entailed, as a whole, in comprehending the meaning of a reference to any "thing" whatever (formal, substantive, or valuative) as existent, actual, real, conceived, perceived. The most notable feature of this commitment, of course, is that the process of conceptualization is construed as an emergent process, an <u>evolutionary</u> process of realization. Since conceptualizations ultimately are subject to tests for admissibility with respect to the total hierarchy of cognitive controls, this hierarchy produces an effect directly analogous to evolutionary selection by an

"environment"--in the sense that it selectively admits alternative conceptual objectifications, permitting only relatively durable conceptualizations to "emerge" as successively dominant (locally optimal) forms of cognitive organization. Two important consequences of this commitment, at the level of object-theoretic construction, can already be anticipated:

(1) that successive modifications of the selective system of decision principles can be viewed historically as constituting an extra-biological lineage of emergent rational prototypes--the warrantability of each prototype depending upon the adaptive range (hence, viability) it confers on the particular psycho-social-biological systems that utilize that novel mode of adaptive control;

(2) that the formalism of mathematical duality will admit of interpretation in explicitly psychological terms, i.e., in terms of a correspondence between perceptual-conceptual entities and operations of cognition and formal entities and operations of mathematical analysis (conjugate spaces, adjoint functions, system-antisystem formulations, and primal-dual modes of analysis in decision making).

The overriding significance of this modal commitment, however, rests with its programmatic character, with its overall effect as a directive to the operations of philosophical system-building. In subsequent chapters it will be seen that philosophical commitments (in the traditional sense of technical premises) are, in every instance, shaped by intention to accommodate this view of cognition as a process of evolutionary realization.

Philosophical Commitments

While there exists no generally accepted paradigm for the structuring of a philosophical position, it has long been recognized that systematic treatment of issues or "problems" associated with the fundamental concepts of existence, knowledge, value, and action constitute an appropriate framework. Technical divisions of philosophical inquiry, appearing as headings in the following outline, are compartments that

have been constructed around just these issues. It seems reasonable to employ this compartmentalization as a plan of presentation, even though its use immediately requires a qualification: that strong <u>interdependence</u> holds among the components of a philosophical system--in contrast with the independence of component axioms of a formal system. Being, becoming, knowing, valuing, and acting do not admit of isolated treatment. Yet there are distinct features of the primary theme of evolutionary realization that can be pursued effectively by divisions.

It would accord well with the ideal of elegance, of course, if the essential content of the foundational premises in each division could be compressed into a single characterizing statement. However, no way has been found to achieve any such encoding ideas that is not seriously misleading. (This failure is perhaps no loss at all to communication, since every attempt seems to yield a result so cryptic as to rival the sayings of an oracle.) Leaving the content of philosophical commitments to full development at the length of successive chapters on metatheoretical and methodological topics, we provide here a mere directory. With each technical division of inquiry we associate, in Table 10.4, a designation that is at least <u>descriptive</u> of the philosophical commitment--the philosophical position taken--in that sector; and we append to this suggestion of content brief references to (1) the central problem and response, (2) the subsequent section in which commitments central to that position have been developed. Finally, the rudimentary flow chart of Fig. 10.1 traces the stages by which the project of philosophical reconstruction moves toward modification of inquiry and practical action.

TECHNICAL DIVISION	PHILOSOPHICAL COMMITMENT	REFERENCES (1) Central Problem and Response (2) Developmental Section
Ontology	Provisional Pluralism	(1) Reality: imputation of existence on the basis of testable objectification in the context of a reduction; general theory of specification and composition of "things" as enduring objects. (2) Metatheoretical Innovations--Ontological (section immediately following)
Epistemology	Conceptual Relativism	(1) Knowledge: theory of cognitive control; selection among alternative cognitive models by canons of rationality. (2) Chapter 11. Metatheoretical Innovations II --Epistemological
Axiology	Evolutionary Realization	(1) Value: adjoint functions as value functions, mathematical formulation (perturbation threory); principle of invariance; hierarchy of norms as regulative of adaptive response. (2) Chapter 13. General Value-Decision Theory and sections following.
Methodology	Modal Complementarity	(1) Action (inquiry): objective v. normative method; complementarity and unification via formal duality; analogical conformity; uncertainty principle. (2) Chapter 12. Methodological Unification
Praxiology	Organizational Meliorism	(1) Action (practice): optimality in practical decision via convergent embedding of decision models; resolution of conflict; measures of effectiveness; adaptive range as measure of viability. (2) Chapter 14. Optimal Organization

Table 10.4 Philosophical Commitments--A Directory

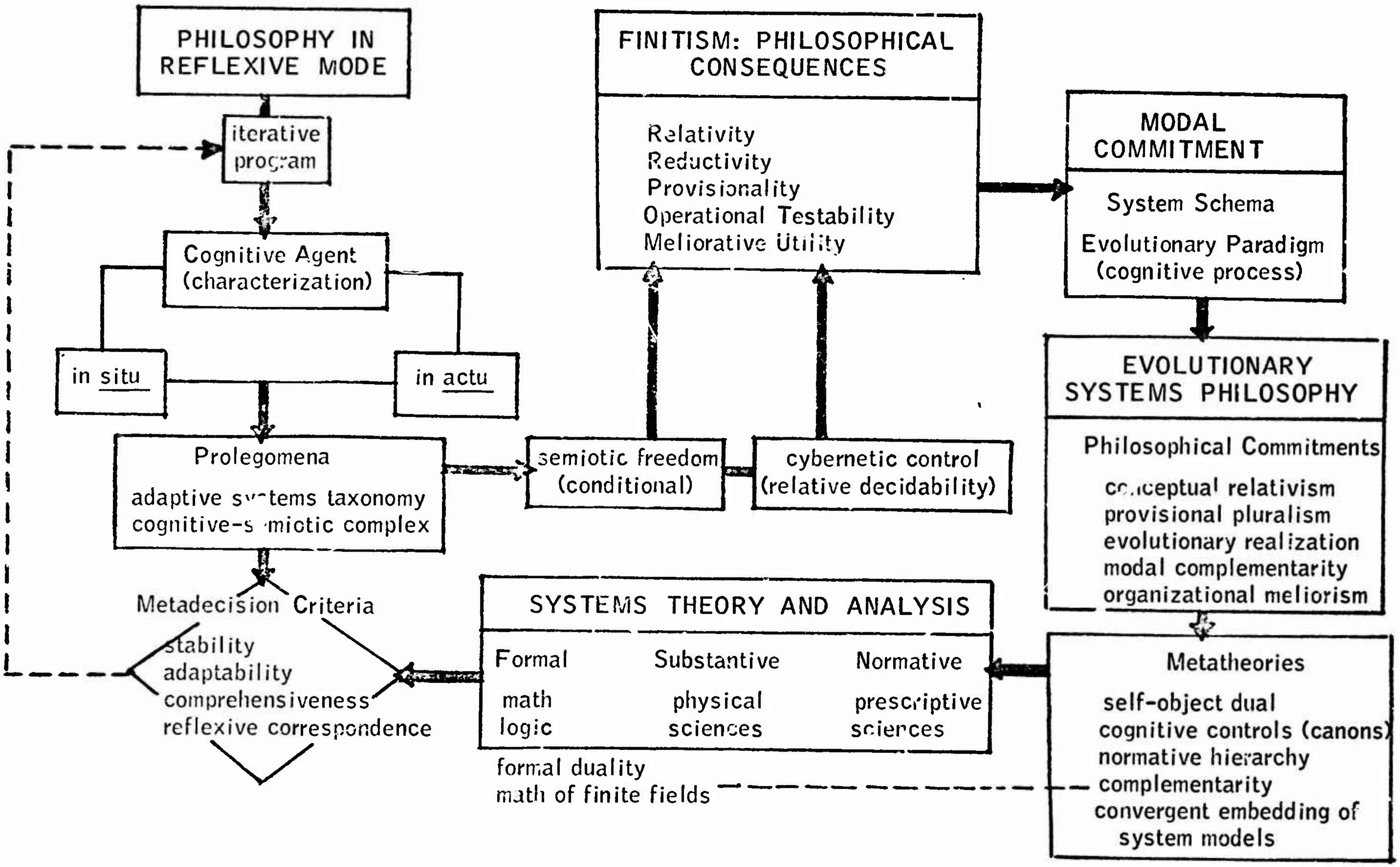

Figure 10.1 Flow Chart: Philosophical Reconstruction

METATHEORETICAL INNOVATIONS—ONTOLOGICAL

AN ANTHROPOCENTRIC PREFACE

In large measure, the most intractable problems of Western philosophy can be subsumed under the counter themes of disparity between idealism and naturalism: the dichotomies of mind versus body and fact versus value which have issued from radical alternative attempts to view the world _either_ as independent of, _or_ as dependent on, the conceiving subject. It is clear enough that the cognitive agent, existing somehow in tension between the obsistent character of sensory experience and the insistent character of ideas, holds the key to anv possible resolution or synthesis. It is for this reason that we have not hesitated to make the task of characterization of the cognitive-semiotic process central to philosophical reconstruction and to run close risks, knowingly, with regard to the dangers of psychologistic commitments. Because _interdependence_ of subject and object, fact and value, mind and body is a fundamental ground that stands under necessity of vindication by the total outcome of reconstruction, we shall do well to introduce the treatment of a theory of objects in general (ontology) with special consideration of the (psychologically) crucial types of entities that we term "self" and "object."

The Self-Concept

The act of conceptualization is two-directional. An act of objectification is also an act of subjectification, since both are necessary for meaningful interaction. The concept "self" can have meaning only with respect to a set of objects related to it by the conceiving act. An isolated "perceptron"--without a history of discriminatory

interactions with something "other" than self--could not conceivably attain the Cartesian premise, Cogito, ergo sum. The existential warrant of a self can have only the same weight as that of the class of external objects related by the self.

A theory of "reality" developed from a purely objectivist viewpoint will necessarily miss this essential duality in concept attainment. Further, any satisfactory theory of conceptualization must entail a process of cybernetic optimization internal to the cognitive agent. If a creative act involves the modification of objectifications, and if self and object always occur as a complementary pair, the act of re-objectification will involve a corresponding partial metamorphosis of the ego. The subjective self--as a connected sequence of self-object interactions--will not be connected to this part of the self (as re-subjectified) whenever the external world is re-objectified. Fortunately, only part of the objective world is re-objectified at any given time and a thread of continuity can be maintained with respect to a single ego. However, there can be no self awareness of a creative act, since no corresponding component of the self exists with respect to a novel objectification during the process of its construction.

This viewpoint introduces an hypothesis concerning the elemental psycho-biology of the cognitive agent: that cognitive behavior is characterized by a tendency toward minimization of the internal cybernetic capacity committed to storage and operation of the repertoire of programmed responses. Related to this drive toward cybernetic freedom is the tendency of the self-component of the cognitive self-object dual toward egocentricity, that is, toward one self. Since progressive development of cognitive organization toward comprehensive scope provides a

contiguous array of overlapping operational programs, it thereby provides a basis for identification of the self component of the self-object dual as a connected set constituting <u>the</u> (integrated) self. Conversely, the concept of self provides a corresponding basis for a connected set of objectifications as a concept of the external universe.

In any discussion of tendencies or drives, this classical issue is inevitably raised: Does the self possess freedom of the will? The viewpoint implicit here is (1) that freedom is attributable only with respect to levels of decision and (2) that it can extend only from a here-now situation over a limited region of space and future time.

The traditional antinomies associated with free will versus deterministic hypotheses and therefore emptied of meaning with regard to remote states (timewise or spacewise); they are equally subject to undecidability in view of cumulative uncertainty or lack of specificability as to predesignation of remote consequent states. From the perspective of a hierarchical range of decision levels, furthermore, the traditional grounds of debate are weakened by indifference considerations. Suppose that, at a given level of decision, there exists a policy which selects a unique course of action at the next lower level. The decision maker will be construed as engaging in a normative control process in applying this higher policy within his span of control. To an external observer the behavior of the cognitive agent may certainly be interpreted as governed by a deterministic characteristic response. But, alternatively, the cognitive agent may be viewed as executing a free choice under some self-instituted criterion. Indeed, the external observer may take this view even for the "behavior" of inanimate objects, e.g., the "free" fall of an object in a gravitational field characterized as satisfying

the normative principle of least action. It is fruitless to debate the question as to whether an object system is "truly" free when the conclusion can be manipulated by relative reference to hierarchical levels of organizational control. Whenever a normative theory may usefully be employed, the object system--animate or inanimate--may legitimately be imputed to possess some degree of freedom over some range, even if this is nothing more than an undertermined multiplier in an abstract system model. The application of a higher-order norm as an optimizing principle will absorb this degree of freedom via selection of a unique act within the permissilbe (free) range. The institution of normative principles at successive levels of decision on the part of a free-will agent leads necessarily to a pyramidal structure terminating in some <u>unique</u> criterion (supremum principle), since principles at successive levels must increase in generality in order to legislate over conflicting lower-order principles. Responses selected under such a format of self-organization and self-control will therefore always be interpretable, in principle, as deterministic under <u>sufficiently detailed</u> redefinition of elemental specifications for initial conditions and uniformities ("natural" laws). This is true simply because "determinism" and "uniqueness of controlling principle" are synonymous.

The hypotheses of determinism versus free will--on the basis of formal considerations alone--have identical claim as formulations that can be brought under indifference conditions. On the practical ground of feasibility and fruitfulness in behavioral inquiry, however, distinction is easy and preference between them is obvious. As the objectivist view in science has rightly maintained, it is not <u>necessary</u> to consider normative aspects in experimental study of behavioral systems. Yet the

evidence is now undeniable that an adamantly value-free mode in behavioral research is infeasible. The characterization of whole organisms, not to mention societies of organisms, which is admittedly possible in principle in terms of strictly observable measures, is blocked in practice by the near-incredible complexity of the task of composing innumerable isolated relationships (of a "sufficiently detailed" characterization) into a coherently organized specification of the whole. An alternative theoretic approach which imputes, to selective systems in general, self-determined (free will) institution of normative principles of control has a claim to feasibility which the overburdened enterprise of objective inquiry cannot match. The advantage of a normative characterization of the behavior of adaptive systems (as goal-seeking self-systems) is that sufficiently "complete" system specification is far more readily attainable than by use of the abstract deterministic format. The task of specifying innumerable independent cause-effect relations and intractable details of elemental interactions reduces to a manageable project of specification in terms of (1) a single value-function with constraints and (2) a unitary system model connecting decision parameters (policies, strategies, programs) relevant to adaptive self control (optimization) of overall system performance.

The Object-Concept

Self and object, like primal and dual, have similar formal properties rather than one-to-one conformality. Although the structure of self and of universe do not have one-to-one correspondence, the formal generalities recognized as applicable to the external world will have subjective counterparts. (A generalization of the Heisenberg Uncertainty Principle for subjective constructs will ultimately have great significance

in this sense.) As an immediate example, consider a cognitive criterion that we shall later develop under the term "ontological parity"--literally, equality of mode of existence--which constitutes a generalization of present day physical conservation principles. If the verb "is" or the formal symbol for equality "=" be taken as denoting existential equivalence, then the elements A and B must have the same determinants in "A is B" or "A = B." In whatever mode of existence an object system endures, it will evince some corresponding conservation principle. The meaning of "exists" connotes some finite endurance, otherwise there can be no conceivable means of warranting the existence of the object. The point of these observations is that every object must be construed as a factorization of some subject-object relation, rather than as a thing-in-itself, a completely closed system. A meaningful notion of "closed system" admits only of isolation with respect to some existential property, not with respect to all existential properties. For, in the latter case, there could be no sensible reference to a closed system. All conclusions would, by stipulation, be untestable.

Put in another way, existence is determinable via interaction; and in the absence of interaction there can be no confirmation of existence. The being of any object, of any "thing" whatever, is determinable in its becoming. This is the basis of Heisenberg indeterminacy. Manifestation of existence admits of some change in the properties of the existant. Otherwise there can be no interaction, hence no awareness of existence.

We have endeavored to establish three points:

(1) Attribution of any property of existence presupposes the stability of some "holon" (some entity as a unitary whole) throughout the course of events from antecedent state to

consequent state such that a basic conservation principle is established with respect to that property.

(2) Determination of measures for existential properties will necessarily involve some degree of uncertainty, since to quantify them is to be involved in interaction with them, thus admitting of a change with respect to a given property. The concept of closure for identification of a holon must therefore be modified to mean <u>quasi</u>-closure, i.e., the holon is assumed stable between our interactions with it. Equivalently, the given property is assumed to endure between the interactions establishing antecedent and consequent states.

(3) The entity of closure (the holon) may be instituted either extrospectively (as an external object) or introspectively (as a component of the self).

Here we encounter the central problem of the concept of the enduring object. The meaning of "enduring object" will later be developed in terms of distinct sets of inclusive versus exclusive "descriptors." One set, which we shall refer to as "identifiers," consists of descriptors of identity that permit the formation of classes or "species"[1] of composite percepts. Every realization of a set of this type will have measures which, respective to each descriptor, are identical; thus a set of identifiers establishes an equivalence class. But if every member of an equivalence class has measures which are the same in every respect, it is impossible to discriminate among members of the class. So far as any test is concerned, all members, being indistinguishable, are the same member; that is, each equivalence class--so far forth--could have only one member. Objectifications constructed on the basis of identifiers alone are deficient in virtue of their reductionistic character, as will soon be discovered by the child who has attained,

1. The term "species" is used here to suggest the task of taxonomic classification, which we readily recognize as a difficult problem in biological systematics, and its counterpart at the level of initial concept attainment--where the constructivist aspect of the cognitive process is obscured by our lack of awareness of commitments in "perceptual systematics."

say, the capability of identifying Santa Claus but erroneously supposes that the Santa Claus figures encountered in several locations are the same individual. (An alternate mode of naive objectification would be ineffectual in virtue of massive redundance: namely, that each instance of a composite percept be regarded as establishing a singleton class as a species sui generis. This mode, if it is ever employed in fact, is presumably discarded in the early infancy of the cognitive agent.) An adequate mode of objectification clearly will entail the employment of descriptors of a second kind that we shall call "discriminators." A set of discriminators permits mutually exclusive specifications, hence unique distinctions, among class members that are equivalent under a given set of identifiers. As a rudimentary example, one may define a set of identifiers characterizing ball bearings of a given size and then discriminate individuals on the basis of temporal and spatial measures.

The crucial service of the enduring object concept is that of instituting a serviceable and warrantable reduction of demands for holistic characterization and response (information processing and cybernetic control) imposed on the cognitive agent by continual engagement in interaction with a changing environment. The role of the enduring object is that of composition or synthesis. As enduring objects, whole sequences of interaction events are efficiently represented in terms of transformations (of discriminators) which are invariant with respect to a sufficient subset of identifiers. In the ball bearing example (above), our experience may consist in a sequence of interactions such that from event to event a projection of a particular geometrical figure endures as the identifier, differing only as to translations in space (change of discriminator). In employment of the concept of an enduring object,

the cognitive agent constructs a contiguous transformation in the discriminator--a transformation which is conservative with respect to the identifier. When this basic format is applied to more difficult versions of concept attainment covering the evolution of adaptive behavioral systems, it will be seen that being and becoming, for any enduring object whatever, stand in complementary relationship.

Ontological Innovations

The following assertions can be presented as an outline of innovations which, having been merely broached in discussion so far, now require development in a general theory of the specification and composition of "things."

(1) A "thing" is an operator which maps a function (defined over the whole of a peremptory perceptual space) into a set of properties admitting of tests by a cognitive agent.

(2) Several operators independently factorize the functional whole into orthogonal properties.

(3) An individual thing is denoted by a collection of quantifiers over the independent properties such that each individual thing has some difference in its quantifiers.

(4) Every quantifier has a conjugate.

(5) For every thing, there exists a conjugate (or anti-thing) which is producible by replacing each quantifier by its conjugate.

(6) The collection of sets comprising all combinations of quantifiers and conjugates is "complete," i.e., any situation which can be described in terms of the original holistic function can be described equivalently as a combination of "things."

(7) There are indefinitely many ways to factorize--hence to discriminate things. We seek that complete set which is optimal with respect to cybernetic representation and processing. That is, we seek to factorize along "planes of natural cleavage," along surfaces of weak interactions (such surfaces expressed, as suggested below, in a form of the holistic function achieved by an appropriate transformation).

$$f(X) = O_x f(X_1)t$$

	t_1	t_1	+	δt	t_1^1
	X_1	X_1	+	δX_1	X_1^1
	X_2	X_2	-	δX_2	X_2^1
$H \rightarrow J$					
	X_3	X_3		δX_3	X_3^1
$B_1 \rightarrow B_1$	p_1	p_1			p_1
	p_2	p_2			p_2
	p_3	p_3			p_3

(8) The operation of producing a thing-quantifier is such that the quantifier constitutes a measure subject to some operational test procedure. The set of quantifiers that individualize a thing must all be testable measures of the same class.

(9) Formalization in terms of holistic functions (e.g., the Schrodinger wave function, or the ordinary probability function) having degrees of freedom corresponding to measures that are <u>not</u> extrospectively testable may, nevertheless, be admissible--provided that such degrees of freedom are formally "absorbed" by application of the operator.

(10) Truth-measures for the warrantability of distinct ontological types of object-constructs are determined by the characteristics of the relevant thing-generating operators.

A gradual transition is obvious in the summary list of assertions above, a transition from initial emphasis on the concept of existence to emphasis on conditions for warrantable knowledge of existence. This is evidence of what was forewarned: that the interdependence of philosophical commitments will not allow any strict compartmentalization to hold. This transition from ontological to epistemological topics will be seen in larger scale as the following theoretical development brings out the necessity for a corresponding renovation of the criterion "true." This renovation is undertaken in the opening sections of Chapter 11 under the topic of epistemological innovations.

THEORY OF THE THING

Whenever we are able to fit perceptual data into a category holding over a class or kind, we thereupon consider an experience "explained," in the simplest sense of that term. However, in order to have any categorization of experience in the first place, there must be some "theory of things" to provide concepts and relations. Thus, the question naturally arises: How do we originally construct the theory required for organization of experience? The motif of

philosophy in the reflexive mode is again realized. The provisional and iterative aspects are dominant. When first encountering experience, one must introduce—perhaps at random—some theory, no matter how rudimentary. Then this theory and its concepts can serve as a provisional basis for interpretation. When new experience is met, the theory is checked and validated or found lacking and modified, then the process is repeated. In this way the basic theory is under constant revision. Sensory maturity corresponds to the stage when an individual has developed a theory which explains most experience. Thus, an adult can quickly and efficiently interpret experience by searching through his vast collection of constructs and conceptual relationships.

For example, the experienced driver can "understand" highway lights at night in terms of his experience with traffic patterns, road structure, etc., whereas the neophyte must truly struggle to "find" meaning in the seemingly disjoint maze of lights.

Let us further examine the process of construct introduction, especially that corresponding to quasi-permanent objects. Whenever we introduce a construct that corresponds to a substantive objectification, there are certain characteristic attributes that this construct must possess. If we are to speak of a substantive object (a "real" thing) there must be a quality of permanence or endurance so that an identity can be established. If a single sensation is received and never repeated, then we do not, and cannot, speak of an associated object. Conversely, if we receive a sequence of sensations, all correlated with a single objectification, then we speak of the enduring object, the thing. However, it is not necessary that every aspect of the thing endure—if it were, there would be very few things. All we require is that some

"important" identifying aspect or property remain so that identification is still possible. To illustrate this, consider the Nile River which is today considered to be the same river as bore Cleopatra's barge, although it is filled with different water, flows along banks with different earth, cuts different channels and has constantly changing currents. Despite these differences, there remains enough to identify and establish a connection with the ancient waterway. It is this connectibility that is crucial. If we can devise a transformation that relates the sequence in a way that permits identification, then the "object" has endured. Of course, there must be a simplicity or reasonableness to the connection. Thus we accept the aging of individuals but not the sudden and discontinuous transformation into a new being. Julius Caesar grew older but never became Brutus. In a real sense, the transformation involves a pattern recognition, or more precisely, a pattern creation to link the flickers of sensation.

Thus we see that it is not necessary for any aspect to remain unchanged, only that some identifying characteristics can be connected, past to present, by a transformation of sensations as if a unitary object endured throughout the interval between observations.*

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* In this way, we handle the philosophical puzzle about the existence of the tree outside the window when we turn away and then back again. The two flashes of experience are connected by a simple transformation that considers the tree to have endured between observations. Note that this connection is not logically necessary because we can never demonstrate that the sensations wouldn't have drastically altered just when our backs were turned. However, it is a simplifying assumption of tremendous elegance that leaves more cybernetic capacity for cognition. Thus we invent a "continuity" to connect.

This connection can be vague and the identity questionable if the time interval is long. For example, that overweight, bald man can only be connected weakly to the bright young schoolmate whom you remember

There is an aspect of this general problem that must be discussed at this point. The only way one system can "know" of another is by interaction which necessarily involves change. Thus, there must be change for there to be awareness in a system. Time is a concept that counts the number of fundamental changes or events and we operate by making reference to this "objective" measure. Therefore one is tempted to declare that all systems develop in a pulsed or discrete manner because elemental interactions are discrete. Further, since sense organs can only transmit signals at finite rates, there appears to be no possibility of continuity. However, this conclusion is of partial validity because it assumes an external observer's viewpoint. To an individual, time appears continuous because awareness occurs only during change; hence one is continuously aware. This double description will be repeated throughout our analysis and is a reflection of a basic complementarity.

Thus the outside or extrospective description of a CA is one of discrete discontinuous evolution, whereas the introspective version is of continuous change and development. These two descriptions are <u>not</u> in contradiction but, rather, offer complementary descriptions of fundamental events. One can only gain full understanding by considering both perspectives. There is no "real" answer, only a description from a given vantage point or reference frame. We are again encountering the philosophical relativism that so thoroughly permeates our discussion.

Let us elaborate upon the extrospective description of discrete changes. It is important to realize that the pulsed interaction is

not the result of inattention on the CA's. but a fundamental characteristic of our sensory equipment. We cannot have a continuous perception because our transducers only transmit pulses of information. It is our cybernetic and interpretation center that connects the points and smoothes them into a continuous line. Consider the behavior of the human eye in conjunction with moving pictures. The pictures are flashes but we see continuous motion because the mind connects the flashes by an identity preserving transformation. Again there is no "proof" whether there was a different signal between received flashes. We view this discrete interaction that is smoothed out to a continuous form as a conceptual paradigm for all such extrospective interaction. The continuity is a transformation that takes us from one discrete impression or framework to another. This interpretation will also have important implications for the idea of mathematical continuity.

Specification of a Thing

The idea of some "aspect of a system or thing" is still vague and we must examine it more carefully. By aspect of a thing we refer to some property (construct-concept in a theory) that permits us to specify the object. Thus, we seek a formalization of the specification of an object. In this discussion we shall understand the term object to have its most general interpretation. There are formal and valuative objects as well as substantive objects. Thus, in mathematics, a function F(x) such that, ..., is a formal object just as a large chunk of granite is a substantive object.

The only way to specify a substantive object is to indicate what the response would be to a measurement interaction. These reactions,

whether potential or actual, were traditionally considered to be manifestations of inherent and intrinsic properties of matter. We now realize that the properties do not inhere solely within the "external system," but arise within the context of an interaction. There is a primitive and irreducible dyad. A theory groups interactions and reactions together by the introduction of concepts and theoretical interrelationships that connect them. Thus different measurements can examine the same general characteristic (constructed) and the broad concept subsumes the entire class. Energy is an example of such a concept. We shall use the term descriptor to refer to these concepts that are used to characterize the "behavior of objects." For example, in a census which views people as objects to be characterized, typical descriptors might be sex, age, weight, etc.

However, a different context or realm of objects, requires different descriptors. Thus if the objects under discussion are elementary particles, then the above descriptors are devoid of meaning and are replaced by charge, spin, mass, etc. Thus the characterization is strongly dependent upon the context which is determined—and simultaneously determines—the descriptors. (We shall return to the idea of levels of objects later on.)

Our earlier discussions have shown that the same general considerations are also relevant for formal and valuative objects and we shall use the term descriptor in this larger sense. If we are dealing with formal objects, then the descriptors are correspondingly chosen to operate in the formal domain. For example, the objects of a vector space could have descriptors such as norm, inner product with some fixed vector, etc. Therefore, we shall restate our objective

for this section as a study of the relationship between objects and their descriptors.

It is important in any discussion to be clear as to what are the objects, and what are the descriptors. If we enumerate a set of descriptors then they are the only ones to be considered. If U is a system of objects x,y,z,..., with descriptors $\{D_1, D_2, \ldots, D_n\} = \mathcal{D}$, then we cannot introduce another $D_i \notin \mathcal{D}$ to be used to distinguish two objects in U. In U—i.e., from the perspective of a U inhabitant—there is only $\mathcal{D}$, and the objects that are differentiated by D_i are viewed as the same object. This is not the result of stubbornness or refusal to face facts, but the result of a lack of capability or resources for resolution. For example, two stones of the same size and weight that appear identical under ordinary light are indistinguishable as to material of constitution. If however, one introduces ultraviolet illumination and then observes a difference, he has expanded his set of descriptors $\mathcal{D}$ and changed U into U′. The fellow without the ultraviolet light isn't being foolish—he literally cannot distinguish the two. Note, the observer in U could still see two stones because he had the descriptor "position." However, if a list of properties were constructed that only included appearance under visible light, size, weight, etc., but not position, then only the list, one would believe there to be but one stone—and he would be correct. In his system there is one store. It is important to realize that we do not generally have access to this privileged vantage point and must learn to operate within a system—we cannot go outside ourselves for purely objective assistance.

Let us formalize these ideas of object sets and descriptors. Much of the discussion will seem to be set-theoretical but the formulation

is more general. It is only that much foundational work in mathematics deals with the problem of set membership and set existence which is closely related to the idea of object specification [1-5].

Let us consider a collection of objects, U, containing objects denoted by x,y,z,... . These may be formal, substantive, or valuative objects. The nature of the descriptors would be different, but the general method of specification is common to all things. How we got the collection is a nontrivial problem that cannot be simply answered.

Mathematical Digression

The early set theorists, e.g., Cantor, assumed the validity of the axiom schema of abstraction which declared that given any condition T, a set $\mathfrak{J}$ could be defined as the set of all x satisfying condition T, in symbols, $\mathfrak{J} = \{x: T(x)\}$. Our earlier considerations regarding tests for admissibility would have led us to reject such an unqualified schema as potentially ambiguous. And, the various antinomies demonstrated the unacceptable nature of the schema. To resolve the paradoxes, two general approaches were followed. Russell and Whitehead chose the more profound route and introduced restrictions upon the conditions for establishing sets. Their theory of types provided a hierarchical structure that stressed the relation aspects between a set and its elements. If x is an object of level or type i, then it can "belong" to a set of type i + 1. In this way, the vicious circle difficulties are avoided because self-referential inclusion cannot arise [6].

However, the general theory of types is a complicated and difficult structure which does not satisfy most mathematicians and a different route was chosen. Zermelo and then Skolem and Fraenkel, chose to sidestep the whole problem by letting the sets and objects be undefined

and the $\in$ (or membership) relation be primitive [2,3]. Zermelo first introduced the concept of "definiteness." A predicate P is definite if by using the basic relations, axioms and logic of a system, P's validity or invalidity can be decided without arbitrariness. Then the axiom of separating out states: if a predicate P is definite $\forall x \in M$, where M is a preexisting set, then $\exists M_p \subset M \ni M_p = \{x: x \in M \text{ and } P(x)\}$. This retains the flavor of Cantor's axiom of abstraction but avoids the contradictions by assuming the preexistence of the set M. Hence, Zermelo introduced a formally consistent schema, but it fails to satisfy certain fundamental questions, viz. where did M come from and how was it generated and how do we recognize whether an element is contained within a set. There have been subsequent refinements to this theory, but they avoid the questions just enumerated. Fraenkel clarified the concept of definiteness and introduced equality as a primitive. Von Neumann changed the emphasis from sets and their members to functions and their domains. He introduced a greatly weakened version of the theory of types by a simple stratification of his objects into two interpenetrating layers. Bernays served to bring the Zermelo-Fraenkel and von Neumann formulations into closer harmony and provided a unified theory [7,8].

However, these men all avoided the difficult problems of set membership and how do we establish the initial set or universe of discourse.

Since we believe that mathematics is a rational system that is generically similar to other such system, e.g., physical science, we believe that it has concepts that change and evolve. They are not absolute. Even the forms of logic are not immutable and are open to alternative description and interpretations. Thus, if the concepts and

foundations of mathematics are subject to change and revision, then one must expect a set of admissibility criteria. In this way, as in physical science, one can impose certain a priori constraints upon the acceptable constructs. These constructs are the invention of man, not the discovery of some objective fact or truth. Mathematics is not exempt from the philosophical analysis that rendered the objects of physics less independently existent. Thus, for us, mathematical discovery is as much creative invention as it is anything else. Thus, the mathematician is <u>not</u> free to create systems with arbitrary properties, but must subject these systems to examination in order to check for consistency and elegance. The objects of mathematics are formal objects so they need not satisfy the entire battery of tests, but they must satisfy the formal tests listed elsewhere. Consistency is necessary for mathematical existence; that is why the antinomies were such a threat to the very structure of the theory.

At this point, let us pause to note that mathematics too is context dependent, provisional and relative, just as are all other conceptual systems. The resolution of the paradoxes was not unique; we have described two quite different formulations, each of which is satisfactory. Both yield a set theory that is adequate to serve as a base for the development of mathematics. This does not mean that one system is right or better than the other. Just so long as both are consistent and lead to the requisite conclusion, they are equally valid. They represent different perspectives or reference frames, but we do not possess the capability of "going outside" them both to ascertain the truly correct one. Such a concept is meaningless.

One characteristic of mathematical reasoning that we wish to

question is the unqualified use of definition. There are two basic ways in which a new symbol or object is introduced, although in a fundamental sense they differ only in degree, not kind. The first corresponds to grouping known and given quantities (objects) into some explicit form and giving this complex a symbol. Thus, in physics, $\alpha = \hbar/me$, the fine structure constant. The second kind corresponds to a definitional complex that introduces new concepts. Newton's laws of motion simultaneously introduced and defined the constructs of mass, force, and acceleration. The three are equally primitive and only gain their full meaning in terms of each other. Hence, the expression $F = ma$ literally creates the constructs F, m, and a.

Let us point out that there is a creative act or accomplishment that takes place in both cases. It requires creativity to combine symbols or objects in a new or innovative way. Certainly mere combination of symbols can serve to illuminate a pattern in a set of data. Perhaps the choice of coordinate axes, or the choice of independent variables can make the difference between solving a problem and not doing so. Consider the startling gain due to studying the behavior of gas pressure with temperature and volume changes. The now classic gas laws ensued.

However, it is a different kind of creativity that is responsible for the existential definition, such as Newton's Laws. We have chosen to designate this second kind as definitional in order to emphasize the difference. There is indeed a sense in which both introductions of new objects are definitional in character but we do not wish to obscure the differences.

In either case described above, one must ask whether the definition is possible, whether the objects so combined are actually combinable.

Also, there must be a way—at least in principle—that an unknown object can be tested or evaluated to determine if it is or is not of the kind referred to by the definition. One can build a force meter to determine, and measure, the status of a "force."

The same is true of conceptual or formal objects. They must be formally operational and formally testable. The rigorous mathematician adds an existence proof to a definition to say, "and there exists such and object." The Russell paradox clearly demonstrates the failure of unqualified definition. One must check all definitions to determine if they are meaningful, i.e., if they can be said to introduce objects. In short, we cannot merely collect and connect words or other symbols, and expect that a meaningful object has been defined. Hence, definition which is existential in character must operate within constraints. It cannot operate without qualification and controls.

Elementary Objectifications

One of the major difficulties, as it always is in such discussions, is where to begin. The Zermelo-Fraenkel set theory posits a preexisting set which is a prior given collection of objects. However, it is given in a purely ad hoc fashion and calling it primitive with a restriction against any further examination, is unsatisfying. We shall seek to establish somewhat more firmly—though certainly not absolutely—the nature or source of this primitive set.

One of our primitive commitments is to a study of the whole man, not a fragmented version, but all aspects. Thus we shall maintain that one can locate the source of the primitive objects, the so-called elemental objectification. We have fused the subject-object pair into an irreducible dyad and we shall look to this interaction. The trans-

ducers or sense organs of the cognitive agent are not passive instruments but play an actual role in the reception of information. Every sense organ makes a selection from the available input and only receives a certain portion. The eyes do not "see" infra-red light; they only respond to the visible portion of the spectrum. Furthermore, after this initial filtration, the signal is then classified into preexisting categories. The eye sees lines, shapes, curves, etc. and any signal is required to accommodate itself to one of these formats in order to be further processed.

We wish to emphasize that these categories or formats are not independent of past experience; quite the opposite for they are strongly dependent upon our earlier experience. A child has to learn to see things and shapes, and this it can be a long and difficult process [9]. However, we are claiming that certain characteristics of the resultant categories are due to the nature of the sensory equipment. These characteristics then lend necessary aspects to the consequent "objects" and kinds of things. To the individual, these aspects are considered universal properties of the external world because everything has them. However, these universal properties are actually species-specific and are due to our transducers leaving an inevitable mark upon all input. An analogy can be made with an individual who always wears red sunglasses; his every observation (and objectification) will be strongly influenced. "All things are of a redish hue." One is reminded of Eddington's icthyologist who searched the sea with a net with 2 inch squares and then announced, "No sea creature is less than two inches."[10] We are not claiming an idealist position, only that all objectifications are influenced by our transducers.

This then is the primitive level of objectification: the level of categories of our sense organs. The "objects" or "things" of this level, e.g., lines, are then combined, according to some rule of composition or systhesis into objects of a higher level. Again we are involved with a hierarchical structure whose various levels are interdependent and mutually derivative.

At any given level of objectification, whether it be pure tones of hearing, elementary particles of physics, or traffic flow patterns through urban areas, one has a corresponding collection of descriptors. These descriptors serve to allow a differentiation between objects; in fact, it is only with such descriptors can there be a meaningful difference between objects. For the accoustical objects, descriptors might be: frequency, intensity, timbre, etc., whereas for elementary particles they would be charge, mass, spin, parity, etc.

It is important that one does not mix the levels of the descriptors and the objects. For example, one does not ask the color of an electron. Color is a descriptor from a different level than is the object, electron, and only ambiguity can result from mixing descriptors and objects from different levels. It is important to keep clear at just what level the discourse is being conducted. It is only in this way that one can determine what the objects are. For example, if considering international relations, then the objects are nations, and the descriptors are chosen correspondingly. If, on the other hand, we are considering interpersonal relations, then the objects are individual people.

Let us also point out that descriptors from one level become objects at another level. (We are using the term "object" in a very broad sense. An object is the focus of attention; hence, a concept can be an object

as much as can a tree.) Thus electrons are described by spin, charge, mass, etc. and these descriptors themselves become objects of study.

One can formalize these statements and it is instructive to do so. Let $E^{\alpha} = \{x,y,z,\ldots\}$ be an universal set of objects at the α level*. Let $D^{\alpha} = \{C_i\}$ be the associated set of descriptors that characterize the objects of E^{α}. If E^{α} contains more than one element, then there must be at least one descriptor that yields a difference for different elements. Let us denote by $C_i(x)$ the value of descriptor C_i for object x. Then, the above statement becomes: If $x,y \in E^{\alpha}$ and $x \neq y$, then $\exists\, C_i \in D^{\alpha} \ni C_i(x) \neq C_i(y)$. Let K_i be the number of distinct values of C_i and let λ_{ij} be the j^{th} value of the i^{th} descriptor. For example, if C_1 is the sex descriptor, and male $\equiv 1$, female $\equiv 0$, then $K_1 = 2$ and $\lambda_{11} = 1$ and $\lambda_{12} = 0$.

Let us define a set as follows:

$$A^{ij} = \{x:\ x \in E^{\alpha} \text{ and } C_i(x) = \lambda_{ij}\}. \tag{1}$$

In the above example, A^{12} would contain all the females from E^{α}.

The following theorem is obvious from the definition of A^{ij}.

Theorem (1). $\forall\, x,y \in A^{ij},\ C_i(x) = C_i(y)$.

Let us also define a companion set to A^{ij} by

$$\overline{A^{ij}} = \{x:\ x \in E^{\alpha} \text{ and } C_i(x) \neq \lambda_{ij}\}. \tag{2}$$

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*The existence of a universal set at each level is to be compared with with the Russell-Whitehead result calling for a universal set of each type [6].

In this case, $\overline{A^{12}}$ contains all nonfemales from E^{α}.

<u>Theorem (2).</u> $A^{ij} \cap \overline{A^{ij}} = \emptyset$.

<u>Proof.</u> $x \in A^{ij} \Rightarrow C_i(x) = \lambda_{ij} \Rightarrow x \notin \overline{A^{ij}}$. Conversely,

$$y \in \overline{A^{ij}} \Rightarrow C_i(y) \neq \lambda_{ij} \Rightarrow y \in A^{ij}.$$

<u>Theorem (3).</u> $A^{ij} \cup \overline{A^{ij}} = E^{\alpha}$.

<u>Proof.</u> In this proof we shall assume the validity of the law of the excluded middle by assuming that every $C_i(x)$ is decidable. Thus $\forall\, x \in E^{\alpha}$, $\forall\, C_i \in D^{\alpha}$, $\exists$ unique $\lambda_{ij} = C_i(x)$. Now for the proof. $\forall\, x \in E^{\alpha}$, $(C_i(x) = \lambda_{ij}) \vee (C_i(x) \neq \lambda_{ij})$; therefore $\forall\, x \in E^{\alpha}$, $(x \in A^{ij}) \vee (x \in \overline{A^{ij}})$.

The descriptor C_i and the descriptor value λ_{ij} are said to partition the set E^{α}. We do this all the time whenever we decompose or partition a collection according to some descriptor value. We can generalize this partition by using all K_i value of C_i. This is done by defining a collection of K_i subsets by

$$A^{ij} = \{x : x \in E^{\alpha} \text{ and } C_i(x) = \lambda_{ij}\}.$$

A simple example would be to decompose a set of objects according to color, all blue objects together, all green objects together, etc.

<u>Exclusive Versus Inclusive Descriptors</u>

We have introduced certain general ideas about the process of set determination or specification. To amplify this somewhat, let us introduce additional terminology. If a set S is to be specified, then there must be one—or more—descriptors such that they must be satisfied by

an object iff it belongs to S. Thus, $\forall$ x, $c_1(x), c_2(\cdot), \ldots$, must yield definite values if x is to be in S. Conversly, if x does not lead to these values, then $x \notin S$. These descriptors which are satisfied by the elements of S are called the "inclusive" descriptors for it is they that determine whether an x is included in S.

However, if the set S is to have more than one member, there must be some means available to differentiate among the different members. The descriptors that yield different values for the different members of S are called "exclusive" descriptors. For example, if S = {x: x is a Jones}, then "last name" is the inclusive descriptor and "first" and "middle names" are the exclusive descriptors. This again emphasizes that in order to specify a set one must specify how membership is gained and then specify a descriptor that distinguishes among the various members, otherwise the set can have but one member.

Let us repeat that if two objects are declared to be different, then there must be at least one descriptor that differentiates them. The relativism of our philosophical position can again be seen by the following. The set D^α is not absolute, but open to modification. Hence if x and y require descriptor C_i to be distinguished, then $D^\alpha - C_i = D_1^\alpha$ is a set of descriptors that can not tell them apart. Hence, if D_1^α is used, x and y are the same because there is no difference. Hence the objects of the set E^α are dependent upon the descriptors in D^α. A star becomes two stars when a new, more powerful telescope is able to resolve the image into two points of light. Hence we see the relationship between the objects constituting E^α and the descriptors of D^α. In fact, the membership relation for inclusion in E^α is determined by the values of the $\{C_i\}$. For example, one might define membership in E^α as

$$x \in E^\alpha \Leftrightarrow \forall\, C_i \in D^\alpha,\ C_i(x) = \lambda_{ij},\quad j = 1,2,\ldots,K_i.$$

There is a converse problem to that described above. If we require all descriptors in D^α to give equal results for two objects to be equal (i.e., $x = y \Leftrightarrow C_i(x) = C_i(y)\ \forall\, C_i \in D^\alpha$), then we obtain undesirable conclusions. For example, if E^α is a collection of everyday things, such as tables, chairs, etc., and if D^α contains descriptors corresponding to location, then this table here is not equal to that table there. Strictly speaking, this conclusion is correct because we are committed to a relational interpretation and the table stands in a different relation to its surroundings when located across the room. However, it is often convenient—and contextually unambiguous—to surpress the difference brought about by some descriptors. We generally consider it to be the same table whether here or there. We are indifferent to some of the descriptors in D^α. In fact, we are establishing a class of objects that we shall consider to be the same object. This class we shall call an indifference class, because we are indifferent among its members. Thus we make a division of the set D^α—actually a partition—into relevant or important descriptors and irrelevant descriptors. Such a partition is strongly context and purpose dependent. Some descriptors that are irrelevant for one purpose are truly significant for another. In short, the indifference set is both context and purpose dependent.

If we are at a given level α, with objects E^α and descriptors $D^\alpha = \{C_i\}$, then the objects $x \in E^\alpha$ are considered atomic. They can be specified, but not decomposed or analyzed into constituents. Thus each $x \in E^\alpha$ is completely and exhaustively specified by the set of values $\{C_i(x)\}$. This exhaustive specification is, of course, from the point of view of the α level. Thus, if we embed the system into another level

$\alpha + 1$ that gives structure to each x, then the members $\{C_i(x)\}$ are no longer a complete specification. Each x is now viewed as being composed of given "amounts" of a,b,c,..., where a,b,c,..., $\in E^{\alpha+1}$. However, the specification of the amounts of atomic material is insufficient to determine uniquely the composite object x. We require as well a rule of composition or synthesis to establish the configuration corresponding to x [11]. It is not enough to tell what pieces constitute a system, one must also tell how they are put together. Consider the following example. Let an object x of E^{α} be the U.S. Cabinet. Let the objects a,b,c, $\in E^{\alpha+1}$ be the citizens of the U.S.A. Then the constituents of the U.S. Cabinet are twelve men; however, to fully specify the system x, one must describe how they interrelate, what office each holds, to what group he holds allegience, etc. We see again how important it is to include mention of the level of discourse whenever confusion can arise. Further, one must also include mention of the level of the objects and the descriptors to be unambiguous. Also, the property of being atomic is relative to the level under consideration, not absolute. (As an aside, we can point out how important it is to choose our objects well and in accordance with ones needs for the given problematic situation: when faced with international problems, don't study local municipalities, study nations.)

The rule of composition is an object from still another level because it—the rule viewed as an object—is not atomic and not derived from amalgamation of atoms. Thus there is a trilevel structure to everything that is repeated throughout the entire heirarchy. There are systems at level $\alpha + 1$ that are the "stuff" of the systems, and the rules from level $\alpha - 1$ that describe how the atoms are put together to form these systems. Given our commitment to finitism, we are led to ask about

the ceginning and end of the hierarchy. There should be a level that is so far forth atomic, say level N, and also a level that is the most general, say level 1. Thus the general paradigm for systemic configuration of atoms governed by rules of composition is repeated for N - 2 levels. Furthermore, our commitment to provisionalism forces us to conclude that a level can never be considered absolutely atomic nor absolutely the most general. The irrefragibility or generality of a level is dependent upon there being no intrusion of new experience or new developments. Just as the Dalton atom dissolved to become the quantum electron proton system, other "atoms" seem destined to be analyzed into complexes of "elementary particles."

There is yet another aspect of the problem of specifying or identifying a thing. Earlier we discussed the concept of an enduring object and the implications of that discussion must be considered here. An object $x \in E^{\alpha}$ is fully specified by the values $C_i(x)$; however, we are often able to recognize x by only knowing some of these values. We recognize a friend after a brief glance at his face even though a full specification would require a vastly more thorough analysis. Furthermore, we recognize this person even if time has elapsed since we last saw him. In a real sense, he is no longer the same person, he is older, his hair is grey, etc., but we are able to connect our sense impressions. We introduce a transformation that links the observed values $\{C_i(x)\}$ today with the values remembered from long ago. We again see that the transformation enables us to consider the object as enduring. In a certain sense, it is the connecting transformation that is the enduring object. Furthermore, the transformation concept refers to objects that are "displaced" in ways other than temporally. We recognize a "square"

even though we only observe a trapazoid. We can transform the trapazoidal shape into a square configuration by a simple rotation corresponding to our walking over and standing directly over it[12].

The problem of pattern recognition can be well understood in terms of the transformation concept. We recognize a pattern when we devise a transformation that changes the observational results to a well-known (i.e., something from prior experience) configuration. The transformation allows us to relate the unfamiliar with the familiar. Hence transformation operates within the context of an experience base. The rotation of an elliptical disk into a circular one does little for the individual who has never seen a circle.

Enduring Object

We are now able to return to the problem of an enduring object using the resources that have just been developed. We introduced the indifference set to be a subset of descriptors such that objects that are differentiated by these descriptors are considered equivalent. This book on the table and this book moved to the chair are—strictly speaking—different books. One book is older than the other and certainly stands in different relation to the objects in the room, etc. However, we suppress this differentiation and consider the book to be one and the same. We are defining the book to be an entire class of different—but not in an "important" way—books. The enduring book is actually a collection of snapshot books that are connected by a suitable transformation.

When we say that this book on the table is transformed into this book on the chair the following must be recognized. A transformation changes some aspects (descriptor values) and preserves others. If the preserved aspects are significant for identification, then the object

endured throughout the transformation—it is the same object. In fact, we declare that the object is the transformations that preserve the appropriate measures.

Any object is characterized by many descriptors and we do not require full knowledge of them all to permit identification without exhaustive search through the complete range of descriptors accomplished with the aid of memory. Any CA, as his experience grows, develops a resevoir or memory. After he has observed and successfully identified a particular class of objects (or object) he learns to make the identification on the basis of a smaller number of descriptors. I can recognize my house with a brief examination, perhaps simply by checking for the broken door in the cellar, without a complete search. Then after having made an identification on the basis of memory and a few descriptors, one makes predictions as to values of other descriptors. These predictions are derived from the memory bank and are made with a certain level of confidence, never certainty. To be certain, one would have to examine all descriptors in order to make a complete identification. In general, we do not examine all descriptors and our statements and predictions are made with a given confidence, not certainty.

We have declared that the descriptor set is partitioned into two subsets, those that are significant for identification and the so-called indifference set. The choice of descriptors to be retained for identification is strongly context and purpose dependent. Those factors which are needed to specify an object for one purpose may prove unnecessary or even destructively supernumary for another purpose. The precise decomposition is determined by the contextual conditions but it is not uniquely or unambiguously given. It may well require a trial and error procedure

to develop a fruitful decomposition. In some special cases, it can actually require the introduction (invention) of a new descriptor to obtain the desired descriptors. An example of the wide range of situations and the corresponding range of descriptor values is given by the lifetimes of objects. The objects of high energy physics can exist for as short a time as 10^{-23} sec. whereas the objects of geology can be as old as 10^{9} years or 10^{16} sec. This is a range of over 10^{40}.

The procedure whereby one selects the necessary descriptors is basically similar to that followed in the construction of any descriptive schema. One introduces provisional decompositions and studies their observable consequences to determine if sufficient agreement is found between experience and the "theories" observable consequences or predictions. Then an iterative process is followed until a suitable system is developed.

Let us briefly consider a counterargument to the general position we are presenting. One could maintain a modified realism that there are real things existing independently and prior to any knower. Thus the task of inquiry is to discover that these things are and what their properties are. The historical fact that many presumed discoveries have been subsequently disproved forces that realist to adopt a position of provisionalism. One is seeking knowledge of the way things are but conducts his research by provisionally adopting concepts that appear to be correct. However, he must then check the validity of these ideas against experience to insure that no error has crept into the system. The concepts are viewed as best available estimates of the true versions and are thus subject to revision whenever experience demands

it. Since experience is inexhaustible, one can never achieve final validation and the concepts remain estimates that are open to modification and improvement. Thus, one concludes that the knowledge of a provisional realist is never certain and forever open. The degree of truth or validity is measured by the agreement of predictions about future experience and the concepts are under modification throughout history as one continues the endless search for increasingly durable concepts.

To an observer who seeks to compare this position with that which we prescribed, there is no meaningful distinction. Both require provisional acceptance of concepts; both require endless vigilance against failures of prediction or inabilities to provide the necessary conceptual base for further inquiry; and both are subject to revision at any time. Thus, in effect, the realist does not offer an alternative because his behavior is indistinguishable from that of the complete provisionalist. There are no observable consequences that could differ; there could be no decisive experiment to differentiate the two. Hence, from our point of view—though not according to the realist, the two positions are equivalent.

ONTOLOGICAL PARITY

The fact that there exists a hierarchical structure implies that there are two basic perspectives regarding the processes of analysis and synthesis. Assume we are considering objects at a given level, say α. The α-level appears to consist of "elementary" objects if we direct our view towards the more complex α + 1st level. This posture leads one to consider conditions for the unitability or combinability of the "atoms" from α to form systemic configurations at $\alpha + 1$. Hence this perspective focuses its attention upon a theory of union or synthesis. Conversely, if we direct our view back toward the more elementary $\alpha - 1$ level, the objects of α appear to be configurations or holons that are decomposable. However, since any complete theory seeks to explain all aspects of the process, both perspectives are needed. Therefore either a theory of union or a theory of decomposition is incomplete by itself and requires the other for its completion. We shall study both versions in order to understand the more general relations that exist.

There must be an equivalence between the description of an object from the two perspectives. A holon viewed as an object in and of itself must be equivalent to the holon viewed as an amalgam resulting from the union of more elementary objects. This equivalence is expressed symbolically by the "equation" $A \cup B = C$. Ontological parity is a

general demand that for $A \cup B$ and C to be equivalent they must have the same general ontology.

Furthermore, within any broad category, there is a hierarchical decomposition. Things are analyzed into more elementary things, until <u>the</u> elementary level is reached. Similarly, things are combined to generate complex systems, which are also combined to generate still more complex systems. In short, there are many different kinds of objects. Everything has a definite ontology or existential character associated with it (see our discussion of the theory of the thing).

Ontological parity deals with the various forms of combination of things; more precisely, it describes which classes of combinations are admissible and which are not. A sentence is a collection of terms (objects) that are related by some general operation, such as addition (union), substraction, and equation. In particular, ontological parity declares that two objects can be equated only if they are of the same ontology—they must be of the same kind in order to be equatable. This disallows equating objects from different hierarchical levels or objects of different genre, e.g., substantive and formal.

Although an equality relates two objects, these objects may themselves be composed of other objects, i.e., may be holons themselves decomposable. For example, in the mathematical equation, $(a + b) = c$, the term $(a + b)$ is itself composed of a and b. In this case the combination of a and b is by simple arithmetic addition and it is fairly clear that the ontology of $(a + b)$ can only be that of both a and b. In other words, arithmetical addition does not alter the ontological status of the united terms. Thus the demand for ontological parity across an equality relation is tantamount to a demand for ontological

parity of each and every term connected by arithmetic addition.

However, there are many other forms of arithmetic operations that can combine individual terms, and many of these do not preserve ontology. For example, multiplication of numbers (that have dimension) drastically alters the ontology of the resultant term. In these more general cases, ontological parity does not necessarily require that the ontologies of the individual terms all be the same.

Furthermore, if one broadens his considerations to general equality and sentencial structure, the possible forms of combination grow astronomically in number. For example, consider the "equations" of nuclear physics, e.g., $p + e = n + \nu$. Clearly this "+" is not simple addition,* but symbolizes a complex interaction process between the electron and proton to form a neutron. The ontologies of p and e^- can be different** and it is the ontology of the resulting complex or system that is significant. Since the number of possible unions is so large, one can only make very general statements about the entire class. In order to make more specific declaration, the character of the particular union must be taken into account.

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*We will elaborate upon the general concept of addition in the next section; hence, this and the next few paragraphs should be viewed as preliminary.

**Depending on the reduction and hierarchical context. In this case p and e^- are of different ontologies if viewed as proton and electron with different charge, etc., but are of the same ontology if considered as elementary particles.

Let us view ontological parity as minimally requiring ontological equivalence across an equality. This will be called the weak form because it only places restrictions upon the macroscopic nature of the two equated quantities. If we further restrict the equality such that the individual terms in the quantities also have the same ontology, then this is the strong form because it restricts both macroscopic and microscopic qualities.

Lets us point out that equality is always with respect to a given set of resources and reduction, etc. Thus A = B if and only if all descriptors of A equal all descriptors of B; the equality is entirely context dependent. We will soon see that this has profound consequences when the "context" refers to the level of resolution of the cognitive agent.

GENERAL THEORY OF THE UNION OF THINGS

In this section we shall consider the general process of bringing two "things" together to form a new thing. This is a generalization of the simple equation A + B = C in which we interpret the "+" and "=" in a broader sense. The "+" shall be conceived of as union, combination fusing, interacting, etc., whereas the "=" is viewed as leads to, results in, etc. Thus, the equation is being recast into a process format that may be symbolized as $A \cup B \rightarrow C$. The precise nature of the union depends upon the specifics of the things being united and the nature of the things is strongly dependent upon the reduction of the discourse, the hierarchical level under consideration, and all the other factors mentioned earlier under the theory of the thing. Hence the union can refer to situations as widely varied as the mutual sharing of an electron by two

atoms to form a molecule to the adoption of a common value system by individuals to form a coherent group. In a real sense, the union entails a shift of perspective from the individual or atomic level to the systemic level. As before, the choice of which is the atomic level in the hierarchy of things is context dependent. The union of elementary particles results in atoms; the union of atoms produces molecules; the union of molecules produces quantities of chemical substances; the union of chemical substances forms human organisms; the union of human organisms forms social structures, etc. The important point to realize is the utter generality of the concept of union; it can encompass any union of "things" from one level to form a new complex at a higher level. The example cited above illustrate the variety of interaction mechanisms that serve to combine the two "atoms" and we shall concentrate upon the broad relational aspects rather than the particulars of interaction.

Let us examine the process of union from a more general perspective. When two things are brought together to form a new system, there is a complicated process of partitioning the holon. When we speak of things as atoms, we are referring to partitions or decompositions of the whole, of the universe of discourse. Remember, things are what they are due to their relation to other things in their universe; and their universe results from a reduction of the universe of experience as suits the cognitive agent and the "facts" of experience. Hence, when we speak of the union of things, it is—is a certain sense—a reuniting of things to reconstitute some whole. More than ever, it is clear that there are two hierarchical levels involved—the atomic and the systemic. Equality can correspond to an equivalence of two partitions. Thus, one could have $A \cup B = C \cup D$ where A, B, C, and D are all atomic and

correspond to two different partitions of the whole represented by $A \cup B$ or $C \cup D$. The equality of two partitions corresponds to the equivalence of two representations. And the representations are of the more general entity, the holon.

We must mention that this section's emphasis upon the two level relationship does not contradict our earlier insistence upon a triadic paradigm for systemic configuration. By concentrating upon the binary structure we are neglecting the substructure and particulars; hence our conclusions are necessary in character, never sufficient. We do not tie down the specifics and are left with necessary constraints, an envelope.

One of the confusing aspects of this entire discussions is the fact that our language and symbols do not indicate the levels from which the objects come; hence, one cannot tell at which level an equality occurs or from which level an object comes. Let us briefly return to the notation introduced in the theory of the thing, viz. let the level of an object be symbolized by a superscript, e.g., x^{α} is from the α level. Then the above discussion translates into

$$A^{\alpha} \cup B^{\alpha} = C^{\alpha} \cup D^{\alpha} \qquad (1)$$

and

$$A^{\alpha} \cup B^{\alpha} = E^{\alpha+1} . \qquad (2)$$

Here the union operation serves to operate upon the ontologies of A^{α} and B^{α} to convert the resultant ontology to that of the α + 1st level. A concrete example is found in the symbolic equation

$$p + e^{-} = H : \qquad (3)$$

here p and e^{-} are from the "atomic" level and H is from the "complex" level. Another example that displays a nonunique partition is

$$2p + 2n + 2e^- = He = 2d + 2e^- \qquad (4)$$

(where p = proton, n = neutron, e^- = electron, and d = deuteron). Note that we have not included the energy in our equation; that has been subsumed in the symbolism, especially the "+."

Despite the awesome generality of the concept of union, it is apparent that it cannot operate (be applied) without restriction; there must be constraints that provide conditions of admissibility. Not <u>any</u> two things can be added; and of those that can be united, there are restrictions upon the acceptable combinations. Therefore, one is led to seek principles or laws that govern the process of union and define the conditions of admissibility.

We have stressed the importance of considering the different levels. Let us use this realization to gain a fuller understanding of the conservation laws of physical science. The conservation laws involve a transformation or mapping between the descriptor values at the atomic level and the descriptor values at the system level. Frequently the relation is one of simple addition, but this is not necessarily so: consider the age of a nation and the ages of its citizens. The standard conservation laws of physics relate the values of "atoms" and "systems" and by demanding equality of the two values, delimit the possible forms of combination. Conservation of energy declares the sum of the energies of the atoms to be numerically equal to the energy of the resulting system. One does not ordinarily see the levels of the descriptors mentioned, but to understand fully the laws, the levels must be considered. The energy of an "atom" is a descriptor at a different level from that of a "system." We could formalize this as follows: Let $E^{\alpha}(x^{\alpha})$ be the α-energy of x^{α}. Then conservation of energy for $x^{\alpha} + y^{\alpha} = z^{\alpha+1}$ becomes

$$E^{\alpha}(x^{\alpha}) + E^{\alpha}(y^{\alpha}) = E^{\alpha+1}(z^{\alpha+1}). \qquad (5)$$

The two descriptors E^{α} and $E^{\alpha+1}$ are different; they operate at different levels and the conservation law relates and constrains the values at the two levels.

There is a conservation law corresponding to each descriptor for each union (partition) but one usually concentrates upon those that are additively related. The conservation laws are general principles that dictate the admissibility of combinations. The very generality of the conservation laws implies that they are independent of the specific realization, i.e., the "objects" of the conservation laws are classes of objects.

It should be noted that our considerations are directed to the formulation of necessary conditions; the sufficiency conditions can only be found by studying the particulars of given problematic situations. The necessary restrictions cover a far wider set of objects; they are more general.

As an example of such a necessary condition that must be satisfied by the objects of an interaction, or union, consider the application of ontological parity. Given two objects A and B, what characteristics must they have in order to be unitable? Our earlier analysis demands the inclusion of a level indicator, so rephase the problem in terms of two objects A^{α} and B^{β}. Ontological parity in its weak form requires the following implication: $A^{\alpha} \cup B^{\beta}$ defined $\Rightarrow \alpha = \beta$ (numerical equality). Now, $\alpha = \beta$ is equivalent to the demand that there exist some context or universe (holon) in which A and B are atoms. In other words, there must be some universe that can be decomposed such that A and B are "objects." This decomposition need not be unique, it need merely be possible.

Let us follow this line of argument somewhat further. We now see the general validity of the expression $A^{\alpha} \cup B^{\alpha} = C^{\alpha+1}$. In other words, the ontology of the entire expression $(A^{\alpha} \cup B^{\alpha})$ is of the $\alpha + 1$ level, $(A^{\alpha} \cup B^{\alpha})^{\alpha+1}$. Hence we have that

$$(A^{\alpha} \cup B^{\alpha}) \cup C^{\alpha+1} = D^{\alpha+2}. \qquad (6)$$

And, so long as there exist appropriate holons and decompositions, one can continue this compounding process. For example, recall our illustration starting with elementary particles and cascading to social structures.

Another important point is that the two unions in equation (6) are generally different in form and character. The first union combines "atoms" of the α-level whereas the second combines "atoms" of the α + 1-level. Thus the first union could be electrochemical while the second is socio-economic. We could enrich our symbolism and complicate the $\cup$ with an indication of the levels but this quickly becomes cumbersome.

This raises still another point, one which we have frequently mentioned. If one sought to specify fully every symbol he would be led to include specification of the entire universe. Thus certain facets must—of practical necessity—be tacitly included and context given.

Thus the thing becomes a thing with respect to a certain context or reduction. The formal modifiers needed to specify unambiguously each word and each sentence would be prohibitively cumbersome. We must rely upon context to achieve unambiguous meaning. Thus, efficient languages, however strenuously we strive for formal rigor, will necessarily depend strongly on conventions regarding context. Conversely, a completely structured context-free language that is notationally practical is too narrow for holistic representation, even as a formal language.

PART V

METATHEORY AND METHOD

Chapter 11

METATHEORETIC INNOVATIONS II—EPISTEMOLOGICAL

From the perspective of conceptual relativism, every act of conceptualization is construed in terms of an interaction system composed of a conceiving subject and a conceptual object. Optimal control of cognitive processes in conceptualization, expectation, and action therefore will be contingent on two characteristic features of subject-object interaction: (1) the indefiniteness of conceptual objects (unconditional decidability unattainable) and (2) the finiteness of cognitive agents (unconditional freedom unattainable). If an idealized notion of "optimal" cognitive control be taken as synonymous with the notion of unqualified rationality, the problem of this section immediately becomes clear. In that broad middle ground of human behavior subject to conditional decidability and conditional freedom—the only ground admitted in actuality by a commitment to conceptual relativism—operational control of cognitive decision processes can be attained only by the institution of a coordinated <u>system</u> of criteria and tests for warrantability of judgments. Such a control system must be designed in consonance with the several distinguishable aims of the cognitive agent in interaction, as well as the several types of subject-object interaction systems arising from involvement with substantive, symbolic, and abstract conceptual objects.

THEORY OF COGNITIVE CONTROLS

Relativism, as the bare denial of absolutism, complicates the task of metatheoretic construction in epistemology inasmuch as it displaces the idealized conception of a singular, immutable, all-purpose regimen for determining the "rational" admissibility of cognitive decisions, substituting in its stead the demand for a multiplex, modifiable system of controls capable of establishing admissibility on the only basis that is actually realizable, i.e., admissibility with respect to criteria specific to distinct objectives of the cognitive enterprise. However, the gain achievable by an epistemology (a metatheory determinative of admissible object-theories) constructed on these lines is a crucial one. A theory of knowledge that is sensitive in detail to the systemic character of subject-object interaction will be capable of rendering explicitly what it means to be rational in important areas of human concerns—particularly the area of valuative judgment—which have heretofore been regarded as necessarily "off-limits" for systematic, warrantable control.

The possibility of epistemological construction of this type originates from a clarification of the cascaded structure of principles required to restore decidability in the face of freedom—and hence ambiguity—engendered by the relativistic character of object-constructs and object-theories. The presence of degrees of freedom admits of corresponding kinds or sources of undecidability. Each kind of undecidability must be resolved by a decision principle of appropriate type. Innovative aspects of an epistemology based

on a commitment to conceptual relativism therefore are most effectively introduced by discussion of the manner in which the criterion "true" has been previously utilized, at the cost of serious confusion, to refer to several levels of what is actually a cascade-structure of distinct admissibility-criteria.

Renovation of the Criterion "True"

To assert that a statement is true is simply to assert that the statement is admissible with respect to a principle that resolves a specific kind of undecidability at a subordinate level of decision. The meta-principles required in order to establish relative decidability are principles which either (a) rule out concepts, sentences, or models which have intrinsic sources of undecidability (ambiguity) as categorically inadmissible, (b) set up a threshold criterion with respect to a norm, (c) enable a unique selection from values distributed in a range of freedom, or (d) select that action which optimizes future freedom. The truth-value with respect to (a) is two-valued, i.e., the norm is either satisfied or not satisfied, (b) is measured on a continuum we refer to as a "warrant." The decision to admit or not in (b) depends on the measure of warrant, i.e., the warrant is deficient with respect to a norm, or is sufficient or better. There is a functional truth-value associated with the case (c), as the measure of freedom is considered an "adjustable parameter" and the cognitive principle is an operator which selects among the degrees of freedom. The last case (d) constitutes the resolution of evolutionary undecidability. There are thus several kinds of truth and measures of truth.

A concomitant characteristic is the requirement that a statement be testable in principle; otherwise a new kind of undecidability is introduced, and is ruled out by the simple and effective device of declaring it inadmissible under an additional cognitive control. Testability involves an interaction—since only through interactions may statements be tested. Thus the principles we seek are in the nature of operators which operate on a function defined in the space of freedom. We seek here a general definition. We may symbolize this as

$$A = O_p[g(f)] = O_p[t] \qquad (1)$$

meaning that the decision A is determined by a decision operator O of particular class p operating on a normative "truth" function measured in a dimension of a "freedom" variable f.

Categorical Truth-Norm. Formal truth is two-valued. The variable of freedom is a measure of ambiguity. The operator admits or rejects depending upon whether the operand has one interpretation or more than one. In other words, a statement or set of statements is rejected categorically if any formal ambiguity exists. The truth-value is "true" if no ambiguity exists, conversely "false." The truth-value must be differentiated from the action operator which may be "admit if true," "reject if false,' or vice versa.

Threshold Truth-Norm. In the case of experimental measures we shall decide the admissibility (i.e., "truth") of an objective theory according to a truth function $t = \omega(f) - C$, $A = H[\omega(f) - C]$ where $\omega(f)$ is a confidence measure such that $0 < C < 1$, and $H(x)$ the Heavyside step-function.

Functional Truth-Norm. At the level of object theories the admissibility may be determined more generally as described. For example, consider a

Bayesian approach to a probability estimate. As an act of policy admit a specific model having two states (heads or tails) and one in which the occurrence of heads is uniformly at random with probability parameter p, $0 < p < 1$. The probability estimates, π, of occurrence of heads in the throw of a two-sided coin may be given by

$$\pi = \int_0^1 (n+1)\binom{n}{k} p^k (1-p)^{n-k} dp, \tag{2}$$

where n previous trials have resulted in heads occurring k times. Here it is to be pointed out that p represents a degree and range of freedom, and the integrand represents an operand of weighted "truth," and the operator $\int dp$ is the decision operator. If follows that $\pi = (k-1)/(n+2)$ is the so-called Laplace-Bayesian weighted "average." This is a meta-decision relative to a magnitude of measure associated with an object variable. The decision whether to bet or not would be a threshold type decision. The decision of what odds constitute a fair bet would be determined by a threshold type decision by the two participants which coincided or overlapped, i.e., the decision to gamble was admissible to each party.

Optimization Truth-Norm. Finally, if the operand is a measure of freedom directly (or probability of a level of freedom), the appropirate decidability operator will be the optimization (i.e., selection of most favorable action). In value-decision theory this operator is that of practical decision; e.g., select a strategy α from a range of possible action A such that

$$Q(x,r) = \operatorname*{Sup}_{\alpha \subset A} \{K(\alpha; y,s|x,r)\ Q(y,s)\}, \tag{3}$$

where x,y are vector states of a system of states W selected from the same set $x \subset W$, $y \subset W$, $r < s$ are time variables, $Q(x,r)$, $Q(y,s)$ are vector sets of values of the respective states at times indicated, and K is a matrix of transition probabilities from states x at r to states y at s.

Ontological Digression. The preceding epistemological concepts are naturally associated with the concept of existence: existence is established by the admissibility of objectifying statements. An "objectifying" sentence is one which literally creates a new concept. An example of such sentences is found in Newton's laws of motion. "Force" and "mass" are conceptualized, and the relations between them described by the laws as the basis for a theory of mechanics. The admissibility of objectifying statements such as these is determined by an elaborate series of tests which comprise the whole integrated structure of cognitive controls.

Since existence is determined by cognitive tests, a kind of existence may be associated with each set of tests, including the appropriate truth function and decidability operator. For example, when a mathematician states a theorem beginning: "There exists a function, f(x), such that ...," he does not refer to a substantive existence, but is declaring that the statement to follow passes a test of nonambiguity with reference to logical consistency (a cognitive control).

We shall recognize three broad classes of existence of objects as determined by sets of tests of admissibility (and by traditional modes of inquiry) to which their objectifying statements are to be submitted. Formal objects (abstractions) are subject to a class of formal controls, but they are non-testable with respect to empirical (extrospective) tests. Substantive

constructs (the objects of the "real" world of the objectivist) are subject to the entire battery of tests—a consideration which will place drastic restrictions on their representation. Finally, value constructs are subject to a holistic set of tests comparable with that for substantive objects but differing in virtue of the duality of objects v. values of objects.

PRINCIPLES OF COGNITIVE CONTROL—THE RATIONAL CANONS

We are now in position to assemble the principles which serve to accomplish or maintain decidability regarding the acceptance of alternative object-constructs, theories, or models. Such principles constitute canons in the sense that they prescribe the course of operations involved in selecting, evaluating, quantifying, and using a cognitive model for practical decision purposes. These canons, which will be referred to as "cognitive controls," are meant to provide a paradigm sufficiently general to encompass the range of behavioral response associated with rational-selective processes in cognition. Consideration of equally significant creative processes is deferred. We have, of course, no means of assuring that the following list is exhaustive. We believe it to be complete in coverage of the sources of cognitive ambiguity now generally recognized. Certainly it includes more tests for ambiguity than have previously been brought together.

We shall classify the kinds of ambiguities that may occur, and hence their respective controls, under two broad classes: the categorical v. non-categorical controls of Table 11.1: Canons of the Rational Process. Subclassification is carried out according to the roles of specific controls

Table 11.1

CANONS OF THE RATIONAL PROCESS

CATEGORICAL CONTROLS	NON-CATEGORICAL CONTROLS
FORMAL	EXTROSPECTIVE (Empirical)
Syntax	Criteria of Fact
Consistency	Extrospective Non-ambiguity
Completeness	INTROSPECTIVE (Pragmatic-Aesthetic)
Superjective Singularity	Problematic Area
Ontological Parity	Risk
Procedural Invariance	Rigidity
Testability	Practicability
	Elegance
	EVOLUTIONARY
	Optimization

in structuring the selection among alternative conceptualizations. Since these controls are interdependent, such a classification is not always clear-cut. In general outline, however, the categorical controls are formal controls which must be satisfied perforce if the objectification in question is not to be irremediably ambiguous and therefore non-meaningful. Non-categorical controls, in the order listed, admit of successively more scope for relaxation, with the higher order criteria of elegance and optimization representing what might be termed "opportunistic" refinements—though these might certainly prove crucial in the long range. The extrospective controls concern the admissibility of raw data by transducers, and the applicability of object-models for prediction of subsequent states. The introspective controls are concerned with the admissibility of object-models with respect to strategy and policy as internal characteristics of the cognitive agent. Finally, the evolutionary controls are associated with the admissibility of a conceptual system—an entire portfolio of programmed responses of the cognitive agent—in terms of its stability and durability, its adaptive range, ultimately its contribution toward the sustained viability of the individual or cultural cognitive agency it serves.

Categorical Controls

The first three formal controls are those of classical logic. Syntactical well-formedness is basic to cooperation among respective parties as to the format of communication, and it is therefore the norm of interpretable expression. Logical consistency, in the elemental sense of the Aristotelian "excluded middle" (an object-statement and its negation are not simultaneously admissible), is similarly the norm of meaningful reference. The conjunction

of an assertion and its negation is unresolvably ambiguous, and such a pseudo-statement is therefore incapable of reference to any existing state of affairs whatever. The criterion completeness demands that the primitive statements comprising an objectification permit, by formal expansion of terms, representation of any of the possible states of the system. The formal character of a complete objectification is therefore that of the tautology, that is, all possible conditions relevant to the model must be representable in disjunctive normal form. This formal-tautological structure of an objectification that is complete, with respect to the context of a given reduction, brings to light the holistic character of an admissible objectification.

The general tendency in the study of formal systems is to limit the consideration of admissibility to these three controls of syntax, consistency, and completeness—with tests for redundancy (independence of primitive statements) sometimes added in the interest of logical elegance. The following additional formal controls, however, prove to be of extreme significance in all non-axiomatic theory construction (which is to say throughout the physical and behavioral sciences). They have appeared separately under various guises in the history of inquiry; our concerted attempt to assemble and to employ them as explicit norms is due to the unusual demands of an organismic concept of "system" as a conceptual format.

Superjective Singularity. This control demands that any particular objectification shall be construed as a member of some more inclusive hierarchical configuration, that it shall derive unique meaning from its relation to some encompassing system (a "superject") of which it is a proper subset. In a given discourse, any object of reference must be taken as

belonging to one and only one systemic context (not itself), else its specification may become ambiguous. An example of such ambiguity is found in the well known paradox concerning the (male) barber of Seville, who shaves every man in Seville who does not shave himself. The question Who shaves the barber? leads to paradox because the act of shaving is simultaneously interpretable in two contexts which are not distinguished. The control supplied by the notion of a singular superject would require that a man be construed as shaving either qua barber or qua private individual.

In the general case, this control rules out such a concept as a set which is a member of itself—a concept which is inherently ambiguous, as shown in another famous paradox of logic. This control will be recognized as similar in effect to Russell's Hierarchy of Types [1], yet it is sometimes overlooked by mathematicians because they are dealing with formal constructs which are empty of substantive content and hence are insensitive to the systemic character of conceptualization. Since the ontology of abstract constructs at every hierarchical level is identical, there is the possibility of undecidability in logic unless additional controls on admissibility are introduced. In dealing with organismic systems, the matter is more straightforward. Our conception of any definite "thing" is the result of decomposition of a universe into compositional and interaction properties. Interaction properties may, of course, be aggregated by reduction in various ways in order to obtain a practicable model for a specific purpose. Alternative reductions thus may result in what is (ostensibly) a "given" thing being associated with quite different collection of properties, hence with different referents. For example, a given man may be the head of a family,

a member of a business corporation, an officer of a social club, a representative of a community of political constituents, and so on. A man who, as President of the United States, has no constituted authority to discipline children will attain to that authority in the role of father. An unambiguous specification of any definite thing must include sufficient interaction properties to establish unique reference in virtue of contextural singularity.

Ontological Parity. This control requires that every term of an objectifying statement be subject to identical tests of admissibility (existence), where "term" refers to symbol strings associated as elements of an analytical equivalence relation (logical, algebraic, linguistic equivalence). Since there are many kinds of truth-measures corresponding to sets of conditions for decidability, it is readily possible to violate this requirement— a fact which is put to effective use under the relaxation of controls in poetic license, e.g., "Happiness is a puppy dog." An example of an significant violation would be: "The probability of heads in the throw of a coin is the limit of the ratio of number of heads to number of throws as the latter goes to infinity." Probability, so defined, is a formal parameter, untestable under the terms of substantive experimental operations associated with throws of coins. It is possible, however, for constructs of different ontological status to appear in a given formulation provided there are operators which reduce each term to the same ontological status.

Ontological parity is an extension of two well known ideas. The first, learned by every student of elementary physics, is that the terms of an equation must be such that each is reducible to the same units. The second

is a generalization of G. E. Moore's concept of a "naturalistic fallacy." If one asserts "this is equal to that," the equality implies that "this" and "that" are testable by the same collection of cognitive controls. That is, each must have identical ontological status.

The great debate surrounding Moore's naturalistic fallacy concerns not the incontestable logic of this principle, but the question of whether there is more than one kind of existence. The logical positivists, for example, viewed entities as having only substantive existence; and since they could devise no substantive tests for values, concluded that value-expressions were merely emotive noises or names assigned to grades. Moore, on the other hand, had earlier maintained that values had a different ontological status from that of substantive entities, that utilitarianism involved the error of equating values to natural entities. The conception that the good is maximized by according the greatest pleasure to the greatest number of people was criticized for the implicit equation of a value (good) to a natural entity (pleasure). Since we shall argue that there are three general ontological categories and that they are subject to distinct combinations of applicable cognitive controls, we attempt to avoid the possibility of ambiguity arising from equating objects of different ontological status. This is essentially an extension of the general effort in contemporary science to avoid such misleading notions as (1) that a mark on a blackboard <u>is</u> a straight line, or (2) that a world globe <u>is</u> a perfect sphere. "Straight line" and "perfect sphere" are names of formal constructs not empirically realizable or testable.

Procedural Invariance. This control is a generalization of Einstein's insistence on invariance of the form of physical laws under a specific class of transformations. Under conditions where procedure is not a part of the objectification, prediction or prescription must be independent of the procedure of analysis, that is, procedure must be conventional in the strict sense. If predictive or prescriptive conclusions resulting from the use of an object-model are not invariant with respect to procedure, the model will be said to be "procedurally ambiguous." Procedural invariance is imperative in any instance of the cognitive manipulation of rational models under conditions where there can be no absolute basis for preference as to type of formal representation or technique of calculation. Ambiguity may be avoided only by demanding that alternative conventions yield equivalent conclusions.

In physical theory, adherence to procedural invariance (the equivalence of conventional reference systems) leads to the introduction of relativistic properties of space-time. In decision theory it leads (in stochastic models) to selection of the Chapman-Kolmogorov equation—which is invariant under translation in time—and subsequently, with the introduction of a practical-decision operator, leads to the principle of optimality of time-dependent programming in order to achieve an invariant form of the value-decision equation. The principle of optimality is therefore but another version of Einstein's principle of invariant transformations. One of the authors has shown elsewhere that finite stochastic models define a space-time metric which is inherently relativistic [2].

Testability. As the last of the categorical controls, we have the formal requirement for testability in principle, where the qualification intended by "in principle" admits of indefinitely extended chains of inference. A concept formulated in such a manner that it is nontestable is literally nonsense, since by lack of this characteristic it has no connectability to the predictive or the prescriptive processes. In short, we are defining "sense" to mean having this connectability. It is admissible to verbalize about nonconnectable concepts—however such concepts can have nothing to do with prediction of substantive events or prescription of practical decisions.

It might be thought that our own primitive concepts fail to meet the requirement of testability. This is not the case, however. It is true that primitives do not permit a priori testing before their initial use. Yet the requirement for stability and competitiveness of the decision-system itself ultimately constitutes a test of its primitives. What is excluded by this control are concepts which by their very nature have no rational connectivity to the general universe of objects, or concepts the testing of which is excluded by their nature.

The requirement of ontological parity combined with the requirement of testability has some interesting consequences with respect to the representation of substantive concepts. Testability rules out infinite procedures in testing. These restrictions, together, eliminate as substantive concepts the notion of continuous space-time. Continuity in either space or time has the status of a formal abstraction since these constructs are intrinsically nontestable by experimental means. "Real" time and "real" space must be discrete [cf. Ref. 2].

The doctrine of substantive determinism (i.e., the doctrine that given all of the initial conditions of a substantive system, and all of the interactions, all subsequent states can be predicted with certainty) is likewise non-testable. Limitation of information storage, retrieval and processing in human brain or computer are such that no test-situation can be realized in terms of all initial conditions for interaction of even a limited complex of elementary particles.

It has become popular in contemporary value-decision theory to use stochastic-definite formulations in modelling. Such models presuppose that even if the world is non-deterministic it is possible to know transition probabilities with certainty. This middle-of-the-road doctrine is also intrinsically non-testable. It is not possible to know that any particular model uniquely represents the events of interest; nor, given a model selected as an act of policy, is it ever possible to establish with certainty the appropriate magnitudes of probability parameters required for quantification of the model.

Non-Categorical Controls

Controls listed on the right of Table 1 are termed "non-categorical" because the degree of admissibility achievable under these controls is determined by compromise among conflicting requirements. With respect to formal controls, a construct, statement, or model is either admissible or not; one may measure admissibility in terms of a two-valued function. In contrast, admissibility with respect to non-categorical controls is measured by a warrant defined on a continuous interval, and no construct or model may be categorically rules out or categorically accepted on the basis of a given outcome of testing.

These controls are further classified in terms of the basic perspectives for judgment on the part of the cognitive agent:

(1) extrospective, i.e., looking outward toward the sources of sensory perturbation and the objectified domain of perceptual experience;

(2) introspective, i.e., looking inward to valuative preferences and requirements of the cognizing subject; and

(3) evolutionary, i.e., looking "globally" in reflection at an individual system (one's self or one's organization) as embedded in the universal context of a selective ecosystem, a physical-social environment in which viability of the self-system must be maintained by adaptive response to a changing context.

Extrospective Controls

The construction of what is normally spoken of as "objective fact" does not depend wholly on information received from an environment via extrospection. Inextricably involved in the construction process are policy commitments and formal relations pre-selected internally by the cognitive agent.

Criteria of Fact. From some set of alternatives (possibly infinite), one must select a particular objectification—as an act of policy—in order to carry out further action, even if this is to be only further inquiry. This initial commitment yields a pre-theory establishing criteria of fact so far forth. Criteria of fact essentially constitute the specifications of a "filter" designed to limit the attention of a decision maker to just that information which qualifies as relevant with respect to the pre-selected range of problematic situations. Thus, even before deliberate observations can be made, the valuative element of policy and the formal element of logical relation have entered into the determination of "fact." This process can lead to the instrumental deficiency we have earlier termed "the predicament

of the perfect filter," i.e., the situation in which the filter is self-reinforcing, admitting only that type of information which would tend to indicate that the prejudgmental bias of the filter is appropriate. In view of the possibility, a nominal indication of the adequacy and relevance of accepted criteria of fact is always indeterminate. Every organization and every individual must constantly test for instrumental ambiguity of this type. Input information is always biased by filtering; hence, instrumental redundancy is essential. The operative control here consists in satisfying this requirement: that initially accepted criteria of fact must prove to be stable with respect to the admission of randomly selected information normally excluded by filtering on the basis of presumed irrelevance.

Extrospective Non-Ambiguity. The particular objectification and filter which have been selected lead, by way of manipulation of the model, to a predictive expectation. Except in the most simplistic of models, complete agreement between expectation and experimental outcome in use of the model is seldom achieved. An inadmissible difference between these two (between conception and perception) signals the presense of extrospective ambiguity. Given a pre-selected problem area, and a pre-selected level of confidence, a measure of extrospective non-ambiguity (admissibility) is determined by a threshold operator on a truth-function comparing the measure of confidence with the pre-selected norm. The determination of the measure of confidence is a problem in the design of experiments and statistical inference; in detail, experimental design must depend on the particular problem area and the formal model chosen as acts of policy.

Selection of the formal model, the problem area, and the policy regarding risk of failure all are non-empirically determined. As in the attempt to assure instrumental non-ambiguity, therefore, the assurance that expectations are consonant with experience (extrospectively non-ambiguous) is no straightforward matter of "looking at the facts." Rather, it involves the complicated interplay of formal, empirical, and valuative issues surrounding the question of the *significance* of disparity between expectation and extrospection. The important points are:

(1) that the relevant truth-measure is distributed over the range $0 < t < 1$, never wholly false, never wholly true, *never without risk*;

(2) that the decidability operator is the threshold operator; and

(3) that stochastic-indefinite models, rather than deterministic or stochastic-definite, are appropriate to the demands of explicit cognitive control.

The theoretician *as* a decision maker in his own right is engaging in two acts simultaneously: evaluating his model and evaluating parameters as quantified in the model. The truth-values associated with the latter comprise a distribution. Because frequently the theoretician discloses only the final outcome of the modeling process, he may be deluded into believing that his last iteration is *the* appropriate description and that all preparatory work, done covertly without conscious programming, was merely false motion. The present emphasis on Bayesian analysis is a promising move in the direction of explicitly programming the procedure involved in assigning numbers to formal parameters for use in practical decisions.

Introspective Controls

Obviously one has at his option the selection of criteria which can make any set of observations either extrospectively admissible or non-admissible, depending on the problematic area selected, the degree of risk one is willing to take with respect to failure in predictive or prescriptive operations, and the personal strategy with respect to rigidity in adherence to previously adopted values and policies. These issues bring up the requirements for sensitivity to introspective cognitive controls.

Problematic Area, Risk, Rigidity, Practicability, Elegance. Selection of the problematic area to be addressed must be derived from a strategy of composition for the entire portfolio of models of a decision maker. This holds because trade-off is involved between practicability and scope: resolution of an immediate problem here and now v. the placement of an investment in analysis which may have strategic payoff at long range. The control at stake is that an admissible objectification must adequately address the precise problem area selected. If it pertains to a practical decision, it must enable a decision to be made in the time permitted for its deliberation. Further, the consumption of resources involved in reaching an optimal decision must be offset by the gain achieved by pursuing a course of analysis rather than habitual, intuitive, or random action—or, indeed, the do-nothing policy of letting nature take its course. Ultimately a strategy of reduction must determine the adequacy of any component of the model with respect to the problem area: Is the conclusion or contemplated action sensitive or insensitive to variation of the particular component?

Risk with respect to possibility of failure to achieve goals is also determined in the context of the entire repertoire of programmed response capabilities of the decision maker. The risk associated with adoption of alternative decision models, risk attaching to alternative courses of practical action (among which resort to modelling or simulation is only one), and the risk attending null action must be coherently related. By intuitive balance among these risks, a responsible decision maker must develop his personal risk-policy. The cognitive control that is relevant to admissibility of a decision model preferred as a directive to practical action is that no strategy or program called out by the model shall violate risk-policy. Admissibility, on this count along, obviously increases with tendency of the preferred model to minimize risk.

Rigidity is associated with adherence of a decision maker to values, policies, strategies--once he is committed--despite a recent history of adverse results. Placement of the compromise between rigidity and pliability in this sense is a fundamental determinant of human personality and character. If a decision maker is completely rigid, he is non-adaptive. He seeks an impossible achievement: to make the world of experience conform to his conception. If he is too pliable, then he is essentially unprincipled and his response is effectively determined purely by immediate rewards or penalties. Because of uncertainty as to the appropriateness of any given cognitive model, and because of the presence of critical stochastic elements in the course of events, some point of balance between these two extremes will be judged as optimal. Wherever that balance is struck, it will inject an additional criterion for the admissibility of cognitive models, since their

adoption as directives to action will depend on their consonance with this basic "attitudinal" commitment.

Practicability is a criterion for admissibility of cognitive models which bestrides the basic point of contact between acting v. thinking for the sake of acting. In order to make contact with action, a model must be interpretable, operational, serviceable in realistic terms. A cryptic way of expressing this is given in the notion of cost-effectiveness of a cognitive model: it must be productive and timely in reduction or of stress, attainment of immediate objectives, or advancement toward long range goals (whether physical, intellectual, aesthetic, or emotive) while requiring expenditures within the constraints of the cognitive agent's available resources. Typical instances of failure with respect to servicability in this sense are: (1) models which entail "exhaustive" collections of multi-attribute alternatives that are defeatingly unwieldy in terms of mental resources; (2) ad hoc models which, in virtue of foreshortened range of attention and fixed reductionist assumptions, are non-adaptive in the sense that any significant modification of valuative basis or dynamic change of situation vitiates the model. The ultimate threat to practicable connection of thought with action occurs if the cognitive agent's total portfolio of models begins to make demands that strike against limitations of cybernetic capacity for deliberation of alternative actions. Practicability, in this event, can be restored by the risky strategy of arbirarily foreclosing an appreciable number of interests and problem areas. A more promising possiblity, though demanding of creativity, is the attempt to regain practicability by reorganization of the overall repertoire of cognitive programs.

Elegance, which is generally thought of as a rather rarefied aesthetic criterion or admissibility, turns out to have surprisingly strong connection with pragmatic considerations. The concept of logical "elegance" incorporates three related desiderata for cognitive models:

(1) simplicity--if possible, the simplicity of immediately understandable primitive concepts--but more important, the sense of simplicity associated with easy accommodation of diverse types of phenomena under a theoretical structure marked by parsimony in its primitive assumptions;

(2) correspondence, the condition that any novel theoretical model whose domain of interpretation overlaps that of a previously warranted model shall--in the area of problems common to their intersection--yield equivalent conclusions; a general principle of correspondence might be stated in this way: the conclusions of any collection of equally warranted alternative models must be coherent in the domain of their intersection;

(3) comprehensiveness, i.e., all other tests being satisfied, the model which is applicable to the broader problem area is to be preferred as contributing to greater theoretical gain with comparable economy of means.

In these terms it becomes immediately clear that increase in elegance serves not only the aesthetic interest of the cognitive agent, but the strategic interest of improved adaptive capability as well. An unwieldy collection of disjointed elemental models is very inefficient in utilization of cybernetic capacity. However, if a number of elemental models can be subsumed under a single model, the resulting increase in elegance permits a net decrease in demand on limited cybernetic capacity. The cognitive agent now has access to available capacity (cybernetic freedom) for development of new programs which may extend his range of adaptive response or, at the least, make it practicable to consider a larger total context of interests contemporaneously.

Pattern formation may be generally described in terms of the minimization of demand on cybernetic capacity. When an appreciable

reorganization results in a sharp decrease in demand for cybernetic capacity, one has created a "pattern." No doubt this is the principal significance of Bertrand Russell's well known maxim: When we know, <u>what</u> we know is structure. Successive increase in the elegance of those patterns or structures in which swarming events are encoded is the goal of a psychological drive that is, properly speaking, aesthetic in character. But such a drive serves much more than <u>merely</u> aesthetic ends. Highly elegant theories, once they have been evolved by aesthetic selection, ultimately prove to be the most "practical" of all accomplishments because they admit of literally innumerable specific implementations. This consideration has been developed in detail by Rukeyser [3] in an account of the cascading practical realizations of Willard Gibb's elegant system-theoretic foundations for physical chemistry. As an endpiece to this work, a remark of Whitehead's is used in summary: "The paradox is now fully established that the utmost abstractions are the true weapons with which to control our thought of concrete fact." Instances exist, particularly in the history of mathematical physics, where pursuit of elegance in the formulation of abstract principles--even in defiance of empirical data--has proved to be a policy superior to that of unquestioning subservience to experimental findings. Belated corrections of supposedly definitive observational data have sometimes disclosed that attention to formally "beautiful" (logically elegant) formulation can be an even more unerring guide to admissibility than empirical confirmation.

<u>Evolutionary Controls</u>

The preceding set of controls consists of reflexive criteria. Failure to pass all of the introspective tests will indicate that a given objectification must ultimately fail. However, admissibility with respect

to these internal controls cannot guarantee that a stable and durable cognitive model will necessarily ensue. A model so warranted may nevertheless still lead to inaccurate predictions and to dangerous decisions over the long run. There remains finally the test associated with evolutionary selection: the question as to what extent a given model contributes to the viability of the cognitive agent under external selective processes of an environment.

Regardless of how logically admissible or how introspectively satisfactory one's cognitive processes may be, they must contribute above all to organizational viability. In specific terms, they must contribute toward <u>optimization</u> over some collection of measure of organizational "effectiveness." Heretofore the identification of such measures has been largely a matter of intuitive artfulness. We maintain that the generalized concept "optimal organization," as a supreme measure of extrinsic value defined on tradeoff between optimal control and maximal freedom, now provides a definitive principle for identification of relevant measures of effectiveness. The essential meaning of the commonsense notion of purpose or goal for any system can be given in technical terms of extremalization of measures associated with extension of either the control capabilities or the range of freedom of the system. Injection of the notion of balanced tradeoff between the antithetical desiderata of control v. freedom permits the conceptualization of a singular extremalization process: overall <u>optimization</u> of the organization of a system subject to simultaneous demands for tactical effectiveness <u>and</u> strategic capability, which is to say subject to the demand for evolutionary viability. The holistic objective functions for any system will therefore incorporate variational measures of control and freedom

(effectiveness measures in general) subject to alternative optimization operators appropriate to specific problem context (max, min, sup, inf, maximin, minimax, etc.). Optimal organization, the generalized extremal measure of effectiveness, is approached when these operators consume immediate freedom (foreclose present alternatives)in such a manner as to maximize future freedom. Test of a cognitive model with regard to its contribution toward optimal organization will therefore not constitute an admissibility test in the proper sense, but rather a test by preferential selection for stability, durability, and adaptive range as disclosed over an appreciable history. Such criteria may be termed "evolutionary" controls because (1) a minimal measure of optimality in this sense in the probability of survival and (2) a general measure of optimality is a measure of viability. The application of evolutionary controls produces a cognitive-selective process which is in every way analogous to the process of evolutionary selection in the biosphere. Predictive theories and value systems alike undergo histories of evolutionary development not unlike those of biological species [4].

In particular the evolutionary test is the primary test of value constructs. A novel value-commitment which institutes a goal, policy, or strategy becomes warranted as a norm of action (vindicated as a "good" directive to action) only if the history of actions determined by it indicate a trend toward increased viability. When first chosen as a basis for decision, a particular policy may have minimal warrant, the sole criterion being that some unambiguous basis for action is required. The policy which increases or maintains the viability of the system will attain to a degree of confidence. The longer it has been successfully used, the greater the confidence--and the greater the resistance to

trial of alternative policies. Untried values are risky, while tried values lead on the average to no worse situation than that of the present. Hence the modification of value systems proceeds at a much slower pace than the modification of substantive constructs.

PROBLEMATIC SITUATION AND DECISION

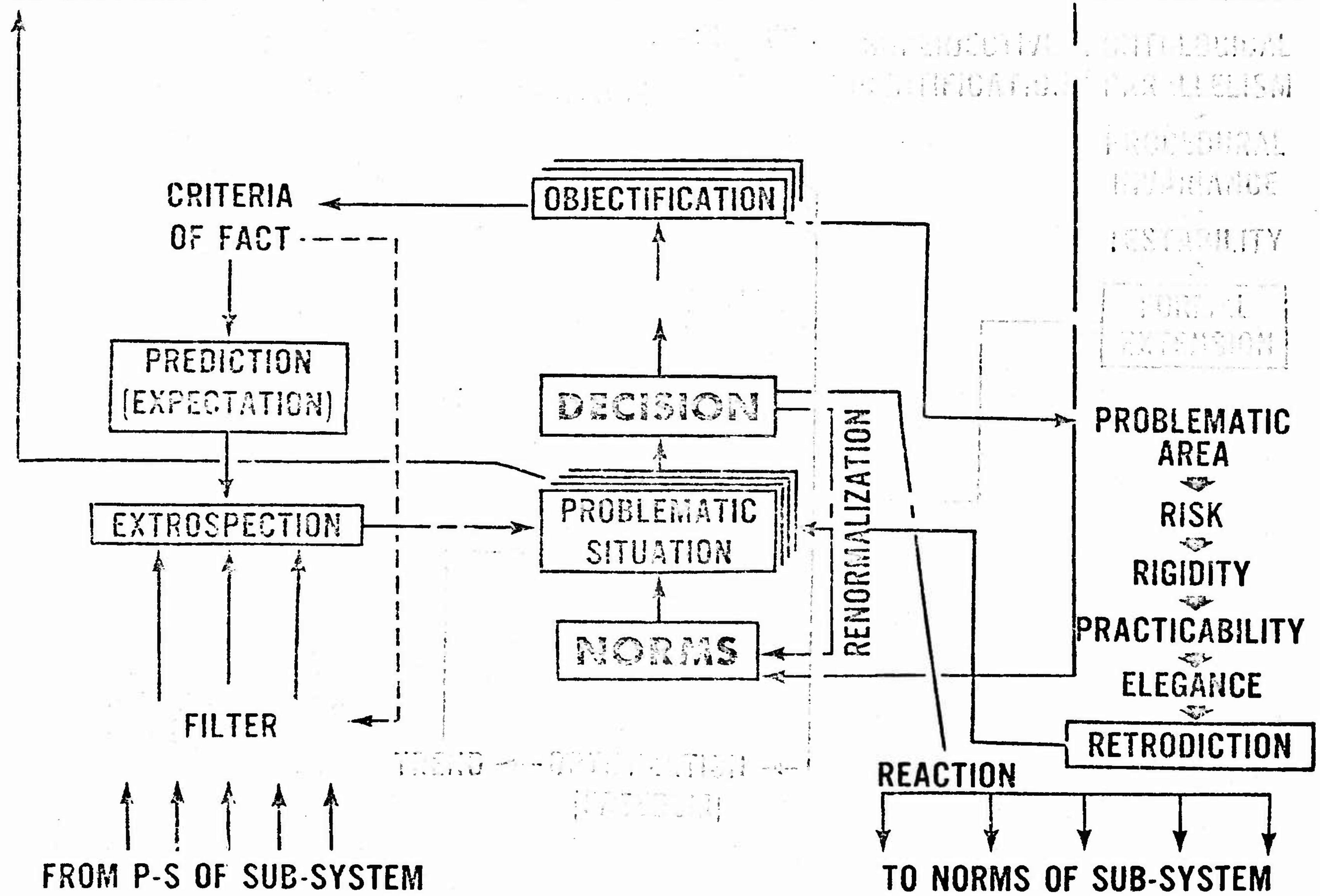
In order to display interaction among the various cognitive controls Fig. 11.1 presents a schematic flow diagram relating all the controls in the previous list of Table 11.1. An emergent objectification, as a creative act of conceptualization, is elaborated initially by formal linguistic extension, that is, by syntactical and logical transformations of primitive statements subject to preservation of formal criteria of admissibility. Modus ponens is the prime example of such a transformation. The results of formal extension are subject to test by the sequence of formal (categorical) controls listed at the top of Fig. 11.1. Any appearance of ambiguity constitutes failure of the objectification as a usable cognitive model; and failure in this sense of blockage to decision is the essential characteristic of a "problematic situation." This concept of a problematic situation is central and crucial. It should be noted that, in general, the whole motivation for engagement of the cognitive process is attributed to undecidability in a problematic situation signalled by encounter with ambiguity--whether formal, factual, or valuative in character.

Under confrontation with failure of a cognitive model so far forth, three classes of strategic decisions are open to the purpose of reconstruction and resolution of problematic situation:

(1) reformalization or reobjectification of the model itself;

Figure 11.1

A COGNITIVE STRATUM



(2) renormalization, i.e., adjustment of the norms operative at the idio-system level of hierarchical organization (the level of an autonomous individual or social organization);

(3) reaction, which is manifested through modification of the norms of subsystems (within limits subject to preservation of the integrity of the total system).

It may seem surprising at first that reaction should be construed as involving modification of subsystem norms, since action and reaction in physical systems have been successfully treated in terms of causal-deterministic relation. In complex adaptive systems, however, while information feedback linkages between mutual-causal processes can give the overall system the appearance of deterministic characteristic response, this holds only as long as all subsystems are operating within their respective ranges of adaptive response.

Subsequent tests for admissibility of "evolving" objectifications appear in a second circuit of extrospective controls (indicated at the left of Fig. 11.1). In this circuit of extrospective controls, instrumental testing of criteria of fact serves to determine the adequacy of the filter which is admitting extrospective input. The cognitive operation concommitant with extrospective testing is that of prediction, i.e., the projection of implications of a model forward in time from an initial state of affairs. This process is productive of the expectations which are to be tested for their correspondence with perceptual judgments. It should be noted that "extrospective input" does not ordinarily refer to signals from an external world but to signals from subsystemic components which indicate by this means their own level-specific problematic situations. At any level of organization above the "atomic" level of information transducers, externality is represented solely by indications of the states of subsystems. Such signals, when transmitted

to the next higher level, constitute constraints which become part of the problematic situation of that level. Only the most elemental subsystems are involved in immediate interaction with the environment. Strictly speaking, human individuals, for example, do not communicate mind-to-mind; they communicate through channels, with the interaction interface between them finally pairing their most elementary subsystems.

Referring to the lower right of Fig. 11.1, the introspective tests of the third circuit of cognitive controls are retrodictive. The objectification is projected backward in time from terminal goal state to present state. Valuative considerations always proceed backward in time since value-decision formulations are adjoint to the objective formulations appropriate to derivation of predictions. More precisely, prediction utilizes a forward projection in the primal version of a formal representation, while evaluation requires a backward projection in the same representation.

It is possible on purely formal considerations to construct an equivalent representation which may be called the anti-system or the dual-system. In such a formulation, the forward projection in the dual is essentially the same (except for certain aspects of normalization) as the backward projection in the primal system, and vice versa. It is possible to address a given problematic situation solely in terms of a primal representation, or solely in terms of a dual representation, or by combination of features of both. Utilization of any of these alternative modes of representation is purely a matter of formal convenience and computational efficiency.

Finally, by the deceptively simple loop at lower center of the diagram of Fig. 11.1, we represent the overriding circuit of evolutionary tests sensitive to the extent to which an objectification contributes toward optimal organization of the cognitive agent(s) it serves. While each of the foregoing regimens of testing includes certain novel considerations in addition to well recognized criteria, the whole circuit of evolutionary control is an innovation. Extension of the very concept of "rationality" is involved in appending to formal, empirical, and pragmatic-aesthetic criteria the additional criterion of cultural-evolutionary viability. The contribution of such a commitment toward acquisition of a more <u>comprehensive</u> version of rational warrantability is no doubt obvious. However, the injection of a novel criterion has its price: difficult methodological issues arise, particularly with respect to legitimate procedures for testing and warranting values. Value-commitments cannot be warranted by means of the objective-scientific test of confirmation by experiment. Consistent agreement between prescriptions (determined by values) and satisfactory outcomes of trial-decisions may only mask the shortsightedness of a course of action that is actually leading ultimately toward disaster. Nor can an adequate test situation, in the sense of a crucial experiment, be conceived in terms of any isolated state of affairs that would irrevocably <u>dis</u>confirm given value commitments. Values, policies, strategies, programs are inherently trend oriented. They are expressly designed to secure benefits accruing overall from trajectories of events that will necessarily encompass incidental failures and losses as well as successes and gains. The procedures for warranting value commitments must therefore feature <u>measures of tendency over durations of time and over distributions of events</u>.

To the conventional battery of test-operations for (1) formal validation, (2) factual confirmation, (3) pragmatic-aesthetic satisfaction, the further operation of (4) evolutionary vindication must be added.

Certainly there is, in concept, nothing at all novel about the "vindication" of commitments--the notion is very well summed up under the Biblical phrase "by their fruits shall ye know them." But, as we shall show in the following chapter on methodological unification, it is no simple or straightforward matter to codify procedures in detail for achieving vindication. It is in the attempt to render the vaguely intuitive notion of vindication explicit in terms of test procedures that an innovation must be advanced and with it a significant extension of the overall criterion-concept of "rational" warrantability for conceptual objectifications in general. The central consideration in this attempt will be the complementarity of objective and normative methods.

Chapter 12

METHODOLOGICAL UNIFICATION

OBJECTIVE VERSUS NORMATIVE METHOD

With the intention of extending the scope of scientific explanation to include biological, psychological, and social systems, behavioral inquiry was initially undertaken from the classical perspective of objective theory. An "objective" theory is a theory constructed under a stringent conception of admissible investigative procedure governing observer-object interaction. Explicitly, the following commitments comprise the essentials of a methodological approach featuring the repudiation of subjectivity, which so vitiated early inquiry, and its replacement by an insistence on objectivity.

Observer-Object Context

With recognition of the inevitability of a subject-object relation at the basis of the experimental method, emphasis is placed on a laudable effort to maintain an unbiased attitude on the part of the subject (observer). The obstructive human tendency of investigators to become involved in the rationalization of a personally satisfying hypothesis is excoriated. As Bernard [1] maintained in his prescriptives for the experimentalist, the preconception of hypothesis and experimental design--which is creative and subjective--must be absolutely severed from the observational phase of inquiry. To experiment is to put a question to nature; when nature answers the observer must be completely submissive. He must see what is <u>there</u>, no

more or less, regardless of his prior commitments and interests. Every question is to be resolved on its merit as the facts determine, and facts shall be construed as only those observations that are open to public scrutiny by at least a coterie of competent and independent investigators.

The creative role of the inquirer as subject, like the origin of his assumptions and hypotheses, has no formal status whatever in this version of scientific method. The resources and procedures of creative insight, being subjective in character, are totally outside the consideration of objective theory. The control of the procedure for confirmation of hypotheses, not the control of the strategy of inquiry, is taken to be the domain of scientific methodology.

This severance of subjective aspects of the observer-object relation clearly presupposes prior commitment to one version of the ontological-epistemological position known as "realism". Owing to the antimetaphysical bent of objectivists in general, it is difficult to obtain a definitive statement of this commitment. Nevertheless it is surely unquestionable that the prescriptions in para 1 can be countenanced only under the assumption that the objects of any inquiry are sufficiently independent of the observer (subject).

Several notions familiar even to common sense are present in this view: that there is some particular, definite "way things are" (equivalent to the conception of things-in-themselves as comprising reality); that "things" are independent of thoughts about things; that facts, peremptory in character for any observer, ultimately constrain the concepts and theories that are warrantable, while being in no way constituted by the preconceived concepts and strategies associated with the creative role of the observer; and that the attainment of theories confirmed by the facts in a given

domain comprises a continuing process of discovery that, in the limit, approaches the truth about nature owing to successive replacement of disconfirmed hypotheses by others sufficient to cover the facts so far set forth.

Format of Inquiry. As a process of discovery the objective-theoretic approach features the following investigative procedure, generally termed the experimental method:

(1) Analysis of an object system, as independent of the observer, to achieve a factorization of measurable properties to which the behavior of the system is sensitive.

(2) Correlation of these measures over some range of states of the system, where this range may be generated in part by perturbing the system.

(3) Formulation of functional relations expressing, as generalized correlations, the characteristic dependency of each defined measure on some collection of "primitive" measures, where such generalization involves the adoption of some formal model furnishing the logical format of relations.

(4) Design of experiments to test the resultant theoretical model for adequacy (primarily precision and comprehensiveness) of prediction with regard to states of the system not previously observed.

(5) Confirmation of the theory on the basis of the correspondence of predicted observations with relevant experimental evidence--a procedure originally construed as verification under the control of binary logic, despite the realization that technically the testing of theories by their consequences might be expected to achieve only probability, not certainty, as a measure of confirmation.

(6) Reiteration of this procedure to increase the scope of the predictive capability of successive modifications of the model or theory.

This pattern of inquiry, presumably originating as early as Galilean physics, had proved spectacularly successful for predicting and explaining a wide range of inorganic systems in the domain of the geosphere. Persistent attempts to extend this success in early behavioral inquiry, however, encountered intractable problems[1] when confronted with the <u>modifiability of characteristic response</u> (literally the adaptivity) of organic systems typical of the biosphere.

<u>New Order of Theoretical Difficulty</u>

Explicitly the crux of the difficulty lay in the general character of the response of organic systems to perturbations induced by an experimenter-observer. For a very numerous class of systems treated under classical mechanics, perturbations are found to initiate reproducible characteristic sequences of subsequent states of the system. In contrast with this unexceptional behavior, repetitive perturbation of an organism typically yields not only a distribution of alternative sequences, but transformations of this distribution as, for example, in the fixation of a habit under conditioning. In view of this situation it was readily appreciated that accurate prediction of the behavior of organic systems involved a new order of theoretical difficulty.

This new order of theoretical difficulty was certainly not restricted to the behavioral sciences alone. In thermodynamics and later theories of

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1. This is not to deny that eminently respectable accomplishments in physiology, physiological psychology, and biophysics have been attained under direct extension of this strategy of inquiry. The point is simply that these attainments have been limited to investigations restricted to consideration of elementary <u>sub-systems</u> of the total organisms or organizations that comprise the ultimate interest of inquiry in these fields.

mechanics the problem of distributions of outcomes, and with it the accompanying requirement for the establishment of statistical criteria for the rejection of hypotheses, had to be faced in phsycis. The point of interest here is that problems of even higher order were encountered from the very inception of behavioral inquiry.

Thus, any expectation that complete descriptions (i.e., adequate factorization of essential measurable properties) for behavioral systems could be accomplished in terms of the familiar primitives adequate for inorganic systems had to be abandoned as hopelessly naive. Organisms clearly required a more complicated objectification, and the general recourse adopted in behavioral inquiry was to attribute to organic system various collections of additional properties, which came to be associated ultimately with the general notion of adaptive control processes.

This imputation of internal control processes under a new objectification had two notable results. First, there appeared in behavioral theories disconcerting numbers of new primitive constructs: reflex, expectation, attention, motivation, appetite, aversion, drive, instinct, habit, consciousness, id, ego, superego, norm, needs, utility, subjective probability, expected value--the full list would finally evoke incredulity. With the introduction of these new primitives the notorious problems associated with "intervening variables" and "hypothetical constructs"--in short, the whole question of unobservables--arose to plague behavioral inquiry and to generate, finally, the well-known reaction of radical positivism.

Without minimizing the importance of considerations regarding criteria of meaningfulness, interpretability, applicability, and practicability that ensued, which remain at the center of controversy concerning admissible constructs and measurable properties, it is the purpose of this section to emphasize a second outcome. In the attempt to accommodate systems typical

of the biosphere, classical objective theory has been utilized in a manner that is indicative of a trend toward modification of its theoretical perspective. A surface indication of this trend is the curiously unnoticed practice of referring to a supposed object-system in certain areas of behavioral investigation as the subject. Behind this apparently innocuous terminology lies an implicit attribution of crucial degrees of freedom to organic systems. When the conceptual commitments[1] involved in this viewpoint are made explicit, it is clear that certain organisms, at least, are being objectified in terms of (a) hierarchical systems involving multiple levels of integrally related control processes that generate (b) characteristic patterns of response to stimuli via stochastic processes (i.e., selection or decision processes) that are motivated by (c) problematic situations involving maintenance or modification or institution of norms at all levels, resulting in (d) behavioral programs that may range, in sophistication of homeostatic response, from selectivity through ultrastability, conditioning, learning, and ultimately to cognition.

With regard to the taxonomy of adaptive systems earlier proposed in this study, what is the significance of this trend in objective theory toward the conceptualization of organisms as subjects? First, it may be noted that this objectification is literally entailed by adaptive systems in general having been constituted as capable of reaction, renormalization, reprogramming, and reorganization--literally, four increasingly complex levels of homeostatic response. Next, an observation almost impossible to miss: That this objectification is patently based on a veiled analogy

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1. Note that the creative role of the theorizer in conceptualizing and selecting this new objectification remains covert. This is characteristic of the objective-theoretic approach.

in which characteristics of human cognitive systems as subjects (available by introspection on the part of the theorizer) are attributed to organisms as objective properties. Finally, from the morass of problems generated by the attempt to employ this objectification in behavioral inquiry, it may be concluded that the trend toward modification of the objective-theoretic approach needs to be developed in a throughgoing reorientation of theoretical perspective. The strategy of inquiry--a metascientific concern from the objectivist viewpoint--insofar as it controls the fundamental process of objectification and selection among objectifications, requires explicit formulation within a systematic methodological structure.

Normative Approach

Such a departure from the traditional objectivist conception of methodology--i.e., the injection of strategies, norms, decision operators, and decision principles at the level of formal theory--would constitute a reconstruction[1] of the basic theoretical enterprise in terms of the addition of a prescriptive or normative activity as a complement to the predictive aspect of inquiry featured in the objective-theoretic perspective. Thus the most significant result that can be drawn from the proposed taxonomy is the realization that, by virtue of its treatment of cognitive selective

1. It seems clear that there is no general tendency among behavioral investigators toward any such reorientation. The typical reaction to the injection of norms, values, and decision processes as properties of adaptive systems is that of the traditional objectivist: norms, values, and decision strategies are taken to be simply additional factual properties of certain classes of objects. Without attention to the consideration that these properties are being imputed under control of inquiry itself by strategies, norms, and values, the so-called "value sciences" are construed as being confronted only by additional problems of measurement. For a critique of this view see Smith [2].

systems typical of the noosphere, a normative perspective for inquiry in general can be envisioned.

We begin with the consideration that the hierarchy of adaptive systems, with its incorporation of conceptual or symbolic systems, provides a category with respect to which we have peculiarly privileged access. For the analysis of these systems as products of the cognitive process, introspective data are available to the theorist. Although the formulation of a theory of cognition is still a decidely open problem, we are able to appreciate--via the self-awareness of introspection--certain implications of the operations we perform in the reasoning process. Whatever we ultimately come to in the way of cognitive theory, it is surely incontrovertible that one particular feature will have to be acceded to: That cognition as an adaptive control process constitutes (a) a decision process operating to resolve (b) problematic situations (c) via the institution of selected policies as norms controlling (d) objectification and selection among objectifications capable of determining (e) an unambiguous line of behavior in the context of (f) terminal objectives (or values) under (g) the constraints of finite resources, subsystem stresses, and modification of norms by an appropriate supersystem.

This is to say, in short, that cognition as an adaptive control process is identifiable as the general paradigm of the gradually emerging concept of heuristic programming.

The significance of this realization lies in its implication concerning the status of object constructs and cognitive models or theories, i.e., the conceptual entities that emerge from the heuristic activity of the objectification process. Such conceptual entities--instituted on a trial basis and objectified under the strategies, values, and norms of a cognitive

agent--comprise, with that agent, a subject-object dual having the character of an emergent adaptive system. The total collection of such systems is precisely what has been earlier designated the domain "noosphere."

With regard to the conceptions of adaptive systems throughout the taxonomic hierarchy, it is now imperative to note that all these constructs appear as elements of emergent conceptual systems in the noosphere. Even the basic conception of the related domains of the geosphere, biosphere, and noosphere is itself an element of the noosphere. As indicated in Figs. 12.1 and 12.2 if the content of the noosphere is detailed, a situation arises in which one element of an initial gestalt consists of a replica of that gestalt. The geosphere, the biosphere, and the noosphere are themselves conceptualized within the noosphere. This is to say that a cognitive system may be imputed to have degrees of semiotic freedom sufficient to allow the entire system and its environment to be modeled within the decision space of one or more of its subsystems (presumably the psycho-neural subsystem).

It is clear, then, that the cognitive process affords a basis for placing any construct whatever in the context of an adaptive system, viz., the subject-object dual composed of a cognizer and his problem of inquiry (or theory) as an object. The basic import of a normative-theoretic approach to inquiry is concerned with the possibility of systematizing or "programming" the heuristic process of trial and error fundamental to objectification and selection. By formalizing the role of the theorizer as subject with regard to the adoption of metatheoretical controls for the selection among object theories, degrees of symbolic freedom may be generated in a hypermodel by means of which whole classes of objectifications become testable, converging onto a common theory as empirical data accumulate.

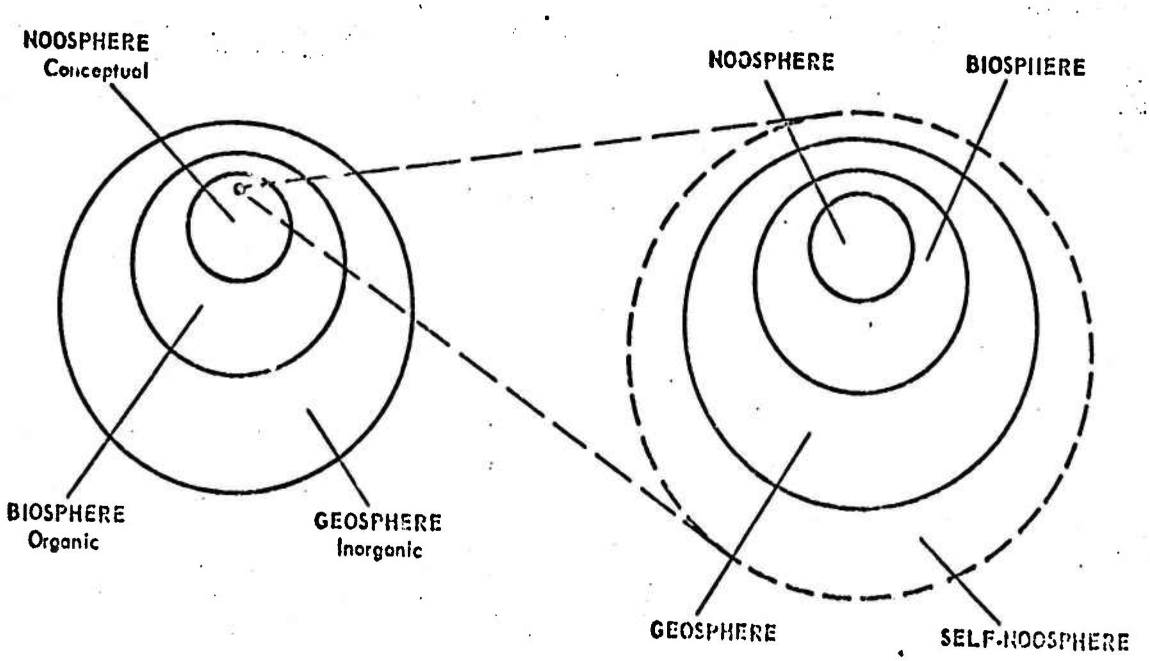

Fig. 12.1 Domains of Systems

Fig. 12.2 Concept of Domains
(Detail of Fig. 12.1)

At this point the normative approach as a methodological option must be sharply distinguished from the imputation of normative character as an ontological option. The fact that any object construct whatever may be considered from the normative perspective of a cognitive decision system must not be construed as suggesting the propriety of anthropomorphizing objects indiscriminately as self-systems (decision systems). To suppose that a thermostat or even an amoeba, for example, literally constitutes a deliberative decision system would be quite as objectionable under normative theory as under the objective approach. The point is that normative theory couched in the format of the mathematics of optimization can be utilized for the explanation of the behavior of even the most elementary mechanistic systems in terms of extremalization of formalized objective functions, with results that are precisely equivalent to those attained under a deterministic approach. It is not a requirement of the normative approach to view such simplistic systems as making decisions that are optional under their own norms; rather the optimization format represents merely a commitment of the theorizer to a general strategy by means of which a class of objectifications may be collectively considered in the interest of attaining optimal decisions for the cognitive enterprise. Subsequent constraints furnished by empirical data will have the inevitable results of reducing the symbolic freedom introduced by the adoption of a hypermodel.

With this development, the range of theoretical approaches open to behavioral inquiry may be summarized by the diagrams in Fig. 12.3 (a,b,c,d). The earliest version (a) based upon the technique of classical mechanics, presupposes the existence of a deterministic object system independent of the observer-subject. Under a reobjectification necessitated by evidence of incomplete factorization the theorizer (b) may covertly attribute to the "object" additional properties suggested by his conception of a subject

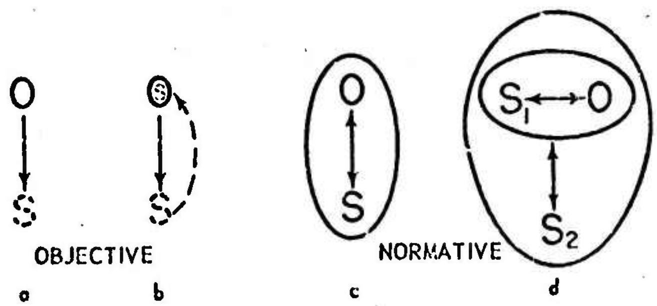

Fig. 12.3 Range of Theoretical Perspectives

or self-system. With the realization that all object systems may be considered in the context of a subject-object dual the prescriptive or normative approach (c) is engendered; and with the objectification of a conformal hierarchy of adaptive systems, normative behavior at a given systemic level (d) may be attributed to the object itself in a subject-object dual.

The problem now is to render this vague intimation operationally meaningful, to vindicate the notion by attaining a rationale of systemic development that possesses predictive and prescriptive significance. That is, we must be able to show, in detail, how the patterns of development and behavior for the specific adaptive systems discriminated by contemporary inquiry are conformal with a unitary format of organization and transformation.

Methodological Option

The new consideration this study puts forward, in addition to the conformality of adaptive systems in general, is the option of a normative-theoretic approach--a methodological commitment to the effect that the following procedures collectively comprise a superior strategy for inquiry with regard to the domain of adaptive systems:

(a) Decision-oriented analysis featuring the inclusions of a decision maker in a subject-object dual for the formulation of a hypermodel.

(b) Formal or theoretic attribution of selectivity to all systems in the context of optimal programming.

(c) Reconstruction of epistemological--if not ontological--commitments to provide for increased complexity of theory addressed to even the most rudimentary systems.

(d) Utilization of sophisticated adaptive systems, i.e., cognitive systems, as paradigms for the identification of primitive concepts.

This normative approach would be complementary[1] to the objective reductionism that has been responsible for extremely fruitful scientific accomplishments but that now appears obstructive to major advance in the behavioral sciences. To do justice, in our primitive notions and in our theoretical format, to the complexity of adaptive systems seems to be the appropriate order of business.

It is our principal contention that a fruitful means of accommodating the complexity of adaptive systems is to be found in the additional degrees of (symbolic) freedom introduced by a hypermodel based on a formalization of the objectification-selection process in terms of (a) a subject-object dual and (b) a programming format incorporating the strategy of the inquirer. In addition to the efficiency of this theoretical approach (it deals, after all, with classes of objectifications rather than singular object-models), the complexity of any object-system is respected inasmuch as that system is allowed to "assert itself" through an experimental history that is relatively unconstrained by conceptual prejudgment of its ranges and degrees of freedom.

COMPLEMENTARITY OF OBJECTIVE AND NORMATIVE INQUIRY

An intuitive basis for a relation of complementarity between objective-scientific and normative-axiological prototypes is to be found in the

1. In the secondary sense in which "objective theory" is used merely for the connotation that a result of rational inquiry must be open to public scrutiny, the normative-theoretic approach itself may be said to be "objective." In a significant extension of this notion, however, the objectivity of a normative theory refers specifically to the requirement that (a) the theory be open to confrontation by input data that, on theory, are extraneous and (b) that the theory be stable and durable under such confrontation.

obvious symmetry which appears in the concluding entries of Table 4.1 (Historical Issues of Dualism, p. 4-41). Prototypes under scientific naturalism are typically reductionist in scope but rigorously warrantable; axiological prototypes tend toward holistic scope but their warrantability is typically questionable.

The reductionism of traditional science has been more than sufficienly belabored. By reference to the column headed Ultimate Control, Table 12.1, we must now emphasize the remarkable accomplishment that has been possible at the price of a sacrifice of holistic scope. Throughout the successive models of scientific thought, one observes the steady accumulation of a battery of cognitive controls--logical, empirical, and aesthetic (in the sense of simplicity, correspondence, coherence, elegance). With the appearance of each additional type of criterion for admissibility, the essential problem of selection among alternative theoretical models becomes more sharply and more unambiguously resolved because the criteria are embedded in a categorical order of priority, logical to aesthetic. In contrast, the corresponding column of Table 12.2 for axiological prototypes transparently discloses a succession of violently contradictory criteria which seem to represent a continuing struggle between rationalist versus antirationalist tendencies of thought. The logical and teleological criteria of the original idealistic model are, in the succeeding stage of a religious orientation, discarded for a theological basis of justification. With the injection of rudimentary intuitional control (purported doubt), rationalism regains the ascendancy only to be again superceded by the anti-intellectualism of an imperative, volitional assertion of "natural" rights as the controlling premises of valuative judgment and social action. Undoubtedly this vascillation is precisely the price that is paid for the

SUMMARY: SCIENTIFIC PROTOTYPES

PROPOSED NAME	TIME PERIOD	PRINCIPAL INNOVATOR	INNOVATIONS	ULTIMATE CONTROL	STATUS OF PREMISES
Axiomatic	5BC-17AD	Aristotle	Valid Syllogisms	Logical	Axioms Self-Evident Necessary A Priori
Empirical	17AD-20AD	Newton	Formal Schema & Induction	Empirical	Abstract Postulates- True of Nature Necessary A Posteriori
Conceptual	20AD-Present	Einstein	Alternative Formal Schema & Induction	Intuitional Aesthetic	Hypothetical Subject to Warrant
Normative	(Current Proposal)		Unified Rational Format	Evolutionary	Objectification Subject to Cognitive Controls

Table 12.1

SUMMARY: AXIOLOGICAL PROTOTYPES

PROPOSED NAME	TIME PERIOD	PRINCIPAL INNOVATOR	INNOVATIONS	ULTIMATE CONTROL	STATUS OF PREMISES
Idealistic	5BC-4AD	Plato	Introspection (Logical)	Logical Teleological	Innate Ideas (Universals= Reality)
(a) Revelatory	4AD-13AD	St. Augustine	Introspection (Psychical)	Theological	Dogmatic
(b) Rationalistic	13AD-18AD	St. Thomas Descartes Kant	Systematic Doubt	Intuitional Rudimentary Control	Synthetic A Priori Categorical
(c) Humanistic	18AD-Present	Rousseau Locke	Voluntarism (Nat. Rights)	Aesthetic Conative	Imperative Volitional
Normative	(Current Proposal)		Relativity Cognitive Control	Evolutionary	A Priori Admissible S/ to Vindication

Table 12.2

general insistence that total dimensions of human values, goals, aspirations, feelings be accorded relevance simultaneously in a holistic treatment of right conduct and wise judgment.

An intimation of the complementarity of the analytical and dialectical modes of thought--with which we began the consideration of rational judgment in Chapter 1--is here almost unavoidable. As we have indicated, no mere intimation that the complementarity of objective versus normative inquiry is a legitimate instance of this larger pattern can have significant force. By such a suggestion we mean simply to indicate the direction that we mean to pursue to philosophical reconstruction. Thus, we give it now as a contention that can be established only by detailed justification: that the two processes of objective and normative inquiry go on side-by-side--not independently--<u>each kind of inquiry serving as a control for the other</u>. Not only do extrospective and introspective controls act on each other, but also in a hierarchical system, the normative processes at one level contribute to the control of those superior and inferior to it. Thus the cognitive structures of intuitive and analytical judgment are interlocked.

From the traditional scientific viewpoint, nature was the one and only control of objective judgment. There are, however, two obvious aspects over which empiricism does not exert control: (1) what constitutes a fact for the test of a theory is not determined by empirical principles alone, and (2) the constructs of science cannot be demonstrated to be other than sufficient with respect to the so-called facts.

The neccssity—that is, the uniqueness—of these constructs can never be established. The realization has gradually emerged that whatever is "true," "valid," or "warrantable," is not to be determined by absolutely singular discoveries produced in flashes of insight, but by selection among alternative creative insights that have been subjected to systematic tests by a process which constitutes reason. The possibility of alternative insights of equal predictive applicability makes necessary the imposition of some principles other than empirical ones to decide among them. These principles are intuitive and categorical. That is, there exists a set of cognitive controls (of which empirical tests are members) which are established for the sole purpose of preventing ambiguity. Some of these principles have appeared in modern and contemporary science, notably the principle of relativistic invariance which can be traced directly to the need of preventing procedural ambiguity introduced by transformations. There are other important introspective controls which delimit acceptable forms for application in the cognitive act. Many of these introspective principles are to be found separately in contemporary theories of value. The long history of successful deterministic science has all but obscured the realization that objective science (empiricism) cannot be construed as self-sufficient in any context where controversial value commitments may enter into inquiry.

On the other hand, normative inquiry, standing alone, can produce

endless variations of logically consistent structure without having any connection to pragmatic purpose, either predictive or prescriptive. The supposition that objective and normative inquiry can operate alone and independently is unacceptable. One cannot pick a format from formal inquiry, and a set of incontrovertible facts independently from empirical inquiry, and simply put them together to form a warrantable theory. Fact is not independent of form (theory); but is structured by formal theory and formulated from information filtered on the basis of formal theory.

Thus, objective and normative inquiry seem to complement—or complete—each other by virtue of closing the feedback systems needed to completely specify a warrantable process; hence, we are led toward the conception of complementary modes.

We may now recognize that the conceptual prototype of scientific inquiry admits of interpretation in terms of a formal-dual format that even more strongly presages a definitive version of complementarity. Consider the nature of a primitive statement which constructs an object-system (or a class of such systems). This primitive statement takes the place of the axioms of the axiomatic model, and the postulates of the empirical model. In the conceptual viewpoint, the primitive statement presents primitive concepts and at the same time relates them in a manner which serves as a prescription. In science this is a prescription for measurement. In axiology it is a prescription for action.

Thus when we make the general statement: "Decisions imply values" we are making a statement constructing a class of object-systems to serve as a basis for the prediction of the behavior of other systems (organisms, organizations, etc.). The statement above prescribes that the decisions of the system of the object of the study are to be inputs from which a set of values will be hypothesized as held by the system under study, i.e., we are attempting to formulate a theory of its value system. Such an inquiry is a value-science.

On the other hand, we may also state "Values imply decisions" as constructing a class of axiologies—inquiry into values or personal norms for purposes of prescribing decision for one's own system. Values are the sources of decisions. The decision is prescribed by some process operating on the values. In value-science decisions of the system studied were primitive inputs; in axiology values of the self-system are the primitive inputs. A value-science is oriented outward—extrospectively; axiology is oriented inward—or introspectively.

Value-science appeals to nature (the behavior of the system as an object) for one requirement leading to establishment of the warrantability of a particular theory of that system's values—i.e., one attempts to predict his future behavior.

On the other hand the personal values in an axiology cannot be warranted through a predictive operation, since self actions are already prescribed. <u>We recognize that the ultimate test of a value system is the evolutionary principle of selection through survival</u>. Thus the

ultimate control of an axiological system leading to warrantability is a history of successful usages. The cultural values which we have acquired have passed such a test. In this respect they are more highly warranted than any new principle of valuation proposed. We are pointing to a way in which value systems can naturally evolve: at the same time we must point to the dangers of irresponsible tinkering with value structures. Any change in policy must be made with due deliberation as to the effects of these changes would have in historical situations. One does not always have to resort to ultimate survival, for in many cases, measures of effect may be invented which indicate a trend toward successful or unsuccessful adaptability to future stresses.

Thus we see that the control of an axiology is in part external—how well is our self-system doing in adapting to external phenomena?

These ideas are shown schematically in Fig. 12-4. Note that an axiological model contains a scientific model formulating and warranting the object-model on which it is based. However, the scientific adequacy of the object-model is to be warranted with respect to the class of decision problems for which the axiology is designed.

The minimal configuration for unambiguous control of decision making, under this concept of interlocking control processes, is clearly a tri-level supersystem-system-subsystem format.

(1) The imposition of controls with respect to the values of the self-system might well be called "executive" control. It represents,

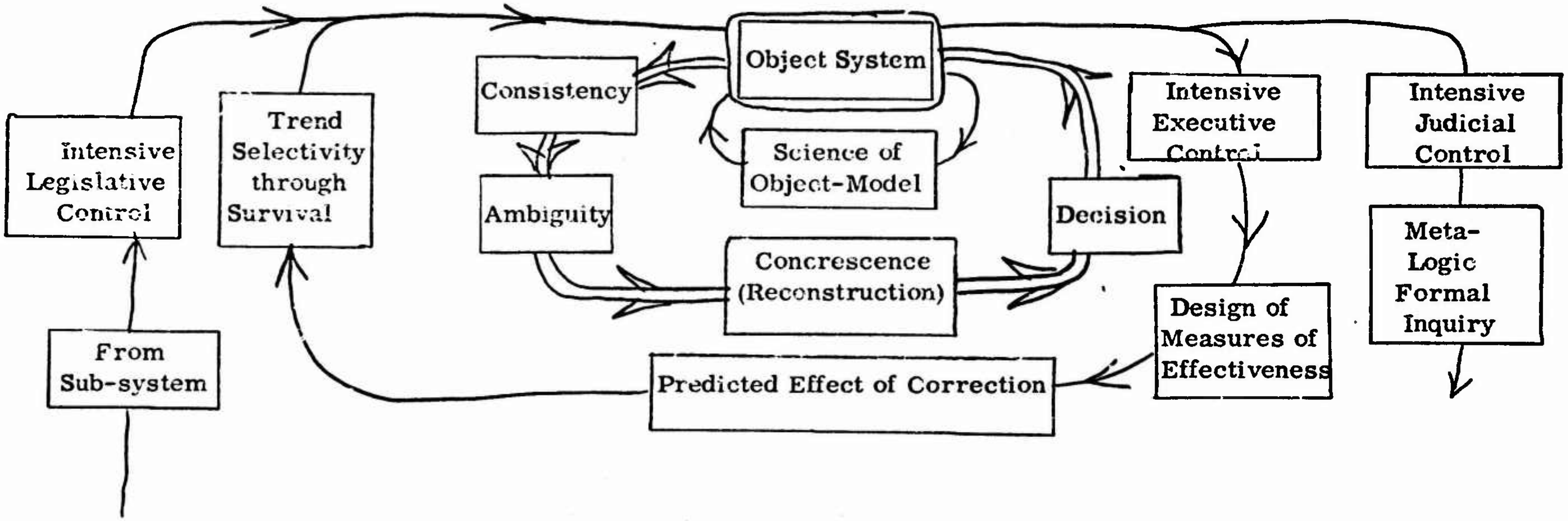

Fig. 12.4 Conceptual Mode of Inquiry: Axiology

in management science, the ideal of total optimization. It is fundamentally concerned with the longevity of the self-system.

(2) The imposition of controls of subsystems within the system we might call "legislative" control. The subsystems have, in effect, delegated to the system certain decision prerogatives. In exchange, the subsystems exert restraints on the decision of the overall system. Some of these restraints have to do with limits of stress on the subsystem, since the survival of the overall system will, in general, depend on the survival of its parts.

(3) The imposition of controls of supersystems of which the overall system is a component may be called "judicial" control. This represents control, limitation, or restraint placed upon the policies of the system by virtue of its membership in a larger community of such systems.

The combined interlocking of normative controls between systems at various levels binds objective and normative inquiry into a stable, warrantable, adaptive cognitive structure. This idea is illustrated in Fig. 12.5. Note that the control loop between a system and subsystem is such that one exerts judicial control over the other (a subsystem), which in turn exerts legislative control over the first.

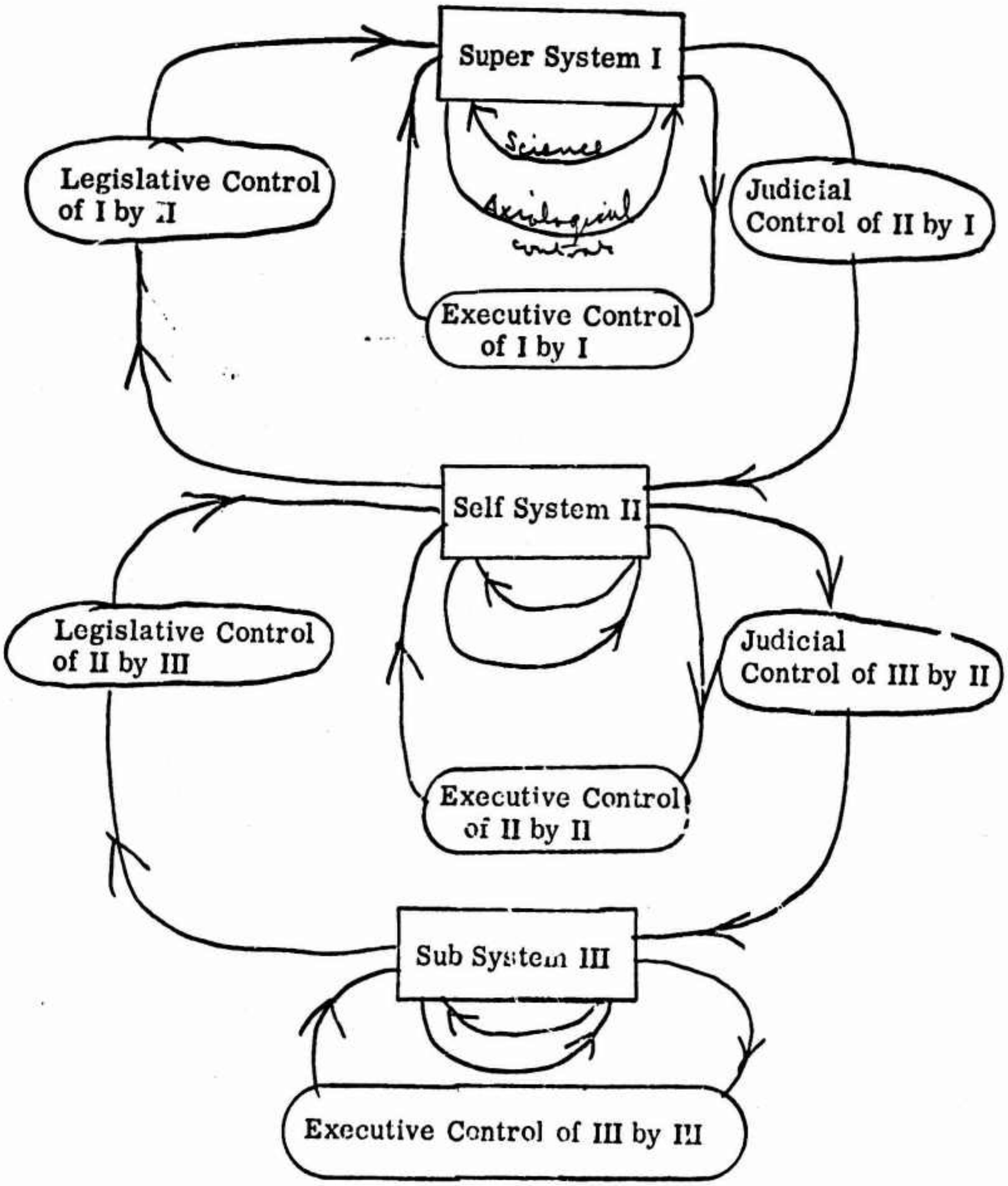

Fig. 12.5 Conceptual Mode of Inquiry--Interlocking Controls

TOWARD METHODOLOGICAL UNIFICATION

While it would not be an innacurate view of normative method to consider it as an extension of conventional scientific method, this would certainly be less than adequate. The suggestion of mere extension of a unitary methodological basis in inquiry is a misleading notion that turns back on the old dream of universal method--a simplistic version of unified science. The complementarity of objective and normative modes now shows this to have been, always, an unrealizable ideal. Methodological unification cannot be attained in any monolithic form. The relations by which objective and normative methods each are required to complete the cognitive control system of the other now make it clear that unified science must be conceived as a "house of many rooms," the sense of unity stemming from connectibility established by a coherent embedding of formal, factual, and valuative orientations in methodology.

The principal impediment to codification and general acceptance of this version of unification via a "continuum of methods" is, of course, the weight of tradition. A major dichotomy of disciplines at present hangs on the fact that the descriptive phrase "normative inquiry" was long ago pre-empted by investigators concerned exclusively with humanistic versions of prescriptive control. Its valuative connotations have been used in an attempt to draw an absolute distinction between inquiry devoted to improved control of cognitive and social behavior versus inquiry devoted to factual discovery. Thus, the traditional normative sciences are the regulative, value-sensitive disciplines of logic, linguistics, jurisprudence, ethics, and aesthetics--as against traditional objective sciences: the analytical and empirical (value-free) disciplines of mathematics and the exact physical sciences.

Under the assumption of fact-value dualism, normative versus objective categories have been viewed as mutually exclusive domains, absolutely distinct in terms of both method and content. (What is the case never implies what should be the case!) It is somewhat difficult, now, to understand how normative theories of optimal control (ethics, for example) could ever have been seriously undertaken in isolation from objective theories of characteristic behavior. Certainly it seems obvious that every control theory, and every decision model purporting to prescribe preferable courses of action, must necessarily incorporate sub-models specifying the logical structure and the factual characteristics of the system one is interested in controlling. No practical significance whatever can be assigned to normative theories which lack this kind of connection with experience. The aims of objective inquiry clearly are embedded in the broader aims of normative inquiry as prerequisites; and all earlier attempts to maintain disjoint divisions of normative and objective science have therefore been misguided.

It must be admitted, however, that acceptance of a "continuum" of descriptive-predictive and regulative-prescriptive activities has been as strongly resisted by contemporary objective scientists (who want no entanglement with subjective aspects of valuative judgment) as by normative scientists (who insist on loftier business than grubbing for facts). In the newer disciplines of operations research and systems analysis, for example, it was apparent from the beginning that a demand would have to be faced that would carry beyond conventional objective inquiry: the necessity of dealing explicitly with purposes, goals, and values in rational analysis. Decision science in practice, however, has continued to work over the familiar terrain of well-formulated, quantitative operations

problems in which it is feasible to use value-parameters that are essentially non-controversial and therefore distinguishable from probabilistic or directly observable empirical measures only in virtue of the fact that they may be even more easily quantified simply by fiat.

In the troublesome area of policy-level decisions, where the underlying difficulty of the decision maker is identified with the question What should be my value-commitments (objectives, policies, strategies, decision criteria)? there has been little readiness to accept responsibility for full scale value-inquiry. The problems are transparently of that type on which the humanistic normative disciplines have frequently come to grief: the problems involved in identifying and warranting value-commitments to be instituted as regulative principles determining decision and action on the part of a complex adaptive system. It is disconcerting to scientific-advisory professionals to hear the practice of systems analysis charged with responsibility, in immediately practical situations, for solution to "holistic" problems of optimal systems design and control which previously have typified the concerns of the humanities. There is no rush to undertake critical research tasks which consist in (1) laboriously eliciting from a decision maker explicit value commitments expressive of previously unrecognized aspects of intuitive judgment; (2) formulating commitments that simply have not been conceived of previously but do appear worthy of experimental adoption; and (3) guiding the process of implementing, testing, warranting values--a process that does not, in principle, admit of any such clear cut situation as a "crucial" experiment. No doubt reluctance is sensible in the face of sharp awareness that the methods of objective science do not legitmately reach to such ends.

The holistic array of cognitive controls which was developed in Chapter 11 provides a rationale for successively embedding the methodologies of analytical, empirical, and valuative inquiry in a way that permits unification. Table 12.2 reinforces the dismissal of monolithic unity. Axiomatics, empirical science, and axiology are presented as metacontrol systems (i.e., methodologies controlling the construction and testing of admissible object theories) for a progressively expanded range of problematic situations. As the aims of inquiry are aggregated (bottom of Table 12.3 upward)--the concern for construction of consistent theories being joined by a new demand for predictive theories, and then by a further demand for practicable cognitive decision models--the restrictions on admissibility accumulate.

It is possible by aesthetic or entrepreneurial commitments (by choice of the range of problems accepted) to operate exclusively under any one of the three distinct cognitive control systems. Yet continuous inclusion of rational canons occurs as one moves from the role of the mathematician (sensitive only to formal criteria of admissibility) to the role of the empirical scientist (sensitive to both formal and factual criteria) to the role of the prescriptive-scientific practitioner (sensitive to formal, factual, and valuative aspects of admissilbe decision.

The background of a cumulative system of rational canons provides connectivity among distinct methods. This allows us to view the whole of inquiry as unified by embedment of methods.

Formal Duality

The property of formal duality is relevant to the topic of unification since it closes gaps in our understanding of essential concepts of

A UNIFIED META-CONTROL SYSTEM

Meta System	Operation	Range of Problematic Situations	Aesthetic Decisions	Scope of Objectifications Practicable
Axiology	Retrodiction	All Practical Problems	All Norms Effective	Most Severely Restrained— Most Reduced
Science	Prediction	Specific Predictive Situations	Action Norms at Indifference	Restrictions Moderate —Richer Range of Objectifications
Axiomatics	Formal Extension	Consistent Axiomatic Systems	Action Norms at Indifference Extrospective Ambiguity Measure at Null	Least Restricted Richest in Formal Content

Table 12-3

substantive things, formal abstractions, values, and the complementary nature of value and fact, subject and object, extrospection and introspection. The objective here it is to give an insight into these conceptual characteristics rather than a precise mathematical formulation.

Consider Euclidian space in n-dimensions and a point denoted by the vector $q = (q_1, \ldots, q_n)$. Consider further a utility function of a specific system $F(q,Tq,t)$, t represents time and T is an operator acting only on q (T may be a constant, or d/dt, etc.).

The utility function F is usually nonlinear, so that a dynamic action problem utilizing this model would depend on the path traversed in object-space. From the viewpoint of cybernetic measures, this is not as simple as desired. Therefore one seeks an enlargement of the model. This is accomplished by the "Legendre transformation" provided F has certain required properties. To each dimension object-space q_i is added a new conjugate dimension p_i such that a model in 2n space (or "phase" space) is achieved which constitutes a "perfect" differential; in effect a path dependent problem in n-dimensional object-space is replaced by a nonpath-dependent problem in 2n-dimensional phase space--in other words, a dynamic problem is reduced to a static problem.

The Legendre Transformation

The Legendre transformation may be written in differential form

$$d(p \cdot Tq) + d(T^+p \cdot q) = dF(q, Tq, t) + dG(p, T^+p, t) = 0$$

where the operator T^+ is "adjoint" to T and where $(x \cdot y)$ represents the scalar product consistent with the operators. T operates only on the primal variable q and T^+ operates only on the canonically conjugate variable p. The function $G(p, T^+p, t)$, complementary to $F(q, Tq, t)$ is

called the "dual" of F (the "primal"). The role of primal and dual are interchangeable.

Designating the partial derivative $\delta F/\delta q$ by F_q etc., and by equating variationals, one has the generalized canonical equation:

$$T^{+}p = F_q, \quad Tq = G_p,$$

$$p = F_{Tq}, \quad q = G_{T^{+}p}.$$

The differential of the dual function, dG, makes up the deficit of the contribution of dF to the perfect differential. For example $T = \frac{1}{2}$, $T^{+} = \frac{1}{2}$ and (replacing $\frac{1}{2}p$ by p) $p = F_q$, $q = G_p$.

The following observations can be made:

(a) In a model where $F = F(q)$ only, the canonical variable p_i conjugate to q_i is interpretable as the holistic value per unit object; i.e., the marginal value of a "thing." Other models can be interpreted appropriately, the dual space is related to concepts of marginal values.

(b) The primal-dual relation is symmetrical; the dual could just as easily represent a value function in a space of some other kind of object.

(c) The differences between the adjoint operators T and T^{+} and the dual functions and conjugate relation determine the essential differences and relation between the concept of value and concept of object. These differences orient values _a priori_, and objects _a posteriori_, so that the rate of warrant is accomplished much faster for objects than for values over some--not all--of the taxonomic scale. Some objects on the taxonomic scale are directly perceived by our senses, i.e., our senses establish a peremptory association with these objects. Values on the other hand, are vindicated at usually a much slower pace, some processes spanning centuries.

However, at the lower (atomic) end of the taxonomic scale, the time scales for vindication of system and anti-system are each much shorter than the interval of practical decision, and furthermore, there is no peremptory association with either--hence both primal and dual systems are interpreted as conjugate objects.

(d) The function F is holistic with respect to a system of objects; the function G is holistic with respect to a related system in conjugate space. This related system may be called the "adjoint" or "anti" system.

(e) For the case $T = d/dt$, $T^+ = -(d/dt)$, it is possible to transform the q variables only, keeping the q variables as parameters. The dual function $H(p, q, t)$ is the classical Hamiltonian for conservative system, $H = (p, q)$; and the conditions for obtaining the perfect differential leads to the classical canonical equations. (Note there are two Hamiltonians. one, $H(p, q)$ associated with the primal, one $H^+(p, q)$ associated with the dual.) One has $H = -H^+$, and $\dot{q} = H_p$, $\dot{p} = H_q$, or in symmetrical form: $\dot{q} = H_p$, $\dot{p} = H^+_q$.

(f) No new information is contained in the dual over that in the primal; however, different forms of the problems may have different difficulties of solution, hence the dual, or the Hamiltonian form may lead to more efficient solutions (obviously, also, they may be more difficult). It is frequently possible to find a form of the Hamiltonian which is time independent, hence dynamic problems in object (q) space become static problems in phase (p,q) space. The introduction of the canonical conjugate variable simplifies the solution of valuative problems (8).

From the viewpoint of decision models the functions F, G, and H lead to stationary variational principles. These principles become major laws in the particular subject covered. For models in which the holistic

utility function is linearly additive and unconstrained, the model already possesses the "perfect" form sought, and the dual variables are degenerate, becoming constants.

Unitary Theoretic Prototype

The property of formal duality leads to stationary variational principles such as the conservation laws in physics (action, momentum, energy). The analogue of these principles may be found in any well formed theory. For example, the relativistic properties of space-time are exhibited by a simple Markov system and arise from the same cognitive requirements. Table 12.4 lists other analogs suggestive of a principle of "analogical conformity," which will be developed at length in the following section.

The conjugate relationship between value and fact is strictly analogous to the relation between momentum and position in physics. Indeed we may regard the concepts "momentum" and "force" to be value concepts in a model reduced to the very simplest properties. Conversely the Heisenberg uncertainty principle applies to the relation between inventory and marginal value.

Where there exists a fairly consistent taxonomic scheme of substantive things, there is no such integrated taxonomy of values. Realizing that at some level a value-concept-space stands in canonical conjugate relation to an object-space, it is always necessary to qualify any value statement by the identification of the object-level at which the primal-dual relation exists. This is a source of confusion in value statements, particularly as between "utility," "value" and "ethics." Yet we have also seen that "force" and "momentum" have a place on the taxonomy of values! However, the latter has a dual relation to object space in a very special subclass of highly reduced models.

PRINCIPLE OF ANALOGICAL CONFORMITY

Hamilton's Principle	Pontryagin Maximal Principle
Stationary Action	Value-Decision Equation
Quantum Mechanics	Stochastic Value Theory
Relativistic Mechanics (Einstein's Principle)	Moving Markov Spaces (Principle of Optimality)
Newtonian Mechanics space momentum	Marginal Utility Theory inventory value of a thing

Table 12.4

ANALOGICAL CONFORMITY

In this section a principle relating to the formalistic aspects of reasoning will be enunciated and arguments presented for its validity. These arguments will be incomplete. A great deal of investigation remains to be done to establish the principle and demonstrate its potential usefulness. The *principle of analogical conformity*, as it is called,[5] is suggested by the formalistic congruency in the evolution of values and the scientific method and in the theories of various sciences, such as physics, economics, psychology, etc.

A. Statement of the principle

The most general statement of this principle is that *there exists a single rational form and that the form of any particular science* or reasoning *is homomorphic to this general form*. This general form is thought to be a more general form of probability theory. A more restrictive definition of this principle can be made. Certain similarities in the form of probability-value theory with the form of quantum mechanics in physics will be observed below. In quantum mechanics certain mathematical operators[6] acting on the scalar product of the wave

[5] This name was suggested to me by Dr. George E. Harmse, formerly of the Operations Research Office.

[6] An "operator" is a symbolic manipulation of mathematical quantities. Multiplication, addition, integration, differentiation, etc., are mathematical operations symbolized by x, $+$, $\int$, $\partial/\partial x$, etc. Operators may be manipulated separately from the operand as, e.g.,

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\phi,$$

where each operator is applied successively to the function ϕ.

function with its adjoint—that is in this particular case, its complex conjugate—produce expected values of measurable observables. In the restricted statement of the principle of analogical conformity, it is stated that the *same operators that produce observables in the physical system when operating on the scalar product of wave functions produce in the general system quantities that behave in that system in an analogous manner.*

For example, the operator,[7] $\partial/\partial x_i$ (the partial derivative with respect to x_i) where x_i is a coordinate of a physical system applied to the scalar product[8] of the wave functions, yields an expected measurement of momentum, $\bar{p}_i$, in the direction of the i^{th} axis:

$$\bar{p}_i = \left(\psi^* \cdot \frac{h}{2\pi i}\frac{\partial}{\partial x_i}\psi\right) \qquad (1)$$

In general, the operator O_i

$$\bar{O}_i = (\psi^* \cdot O_i\psi) \qquad [2]$$

produces an expected measurement of the corresponding observable. The same operator may be applied in a similar manner to the scalar product of the adjoint probability with the forward probability; e.g.,

$$\left(P\dagger \cdot \frac{\partial}{\partial x_i} P\right) \rightarrow (\bar{p}_i), \qquad [3]$$

where the dagger (†) indicates the backward equation. According to the principle of analogical conformity, the quantity so produced will be an observable with respect to the general system, which will have a mathematical form with respect to that general system in congruity with the form of momentum of the quantum mechanical system. The symbol ($\bar{p}_i$) of Equation 3 represents this logical congruency. Should the principle of analogical conformity be established, its potential usefulness will lie in the fact that the theoretical advancement in any one science can serve as a guide for theoretical advancement in any other. This does not mean that the congruity can be applied blindly, since the form of each science is taken to be a special case of a more general form. For example, it can be expected that the laws of physics will have their counterparts in game theory, economics, psychology, and other sciences. The principle of analogical conformity implies that these laws, related as they are to certain operators in the physical theory, will be produced by these same operators in the other sciences.

It is as if the great laws of science such as those of the conservation of momentum, the conservation of energy, the second law of thermodynamics, etc., are the inevitable product of the form of the rational process. The problem of science becomes one of determining the substance of experiment and observation that enables the formation of the constructs that lead to these inevitable formulations.

B. Evidences and plausibilities relating to the principle

The arguments to follow are somewhat sketchy and imprecise. A great amount of time, meticulous work, and moments of insight are required for their rigorous demonstration. It is hoped that at the very least they will be provocative in character. They are published here in the hope that investigation by others may be motivated.

1. Quantum-mechanical analogies. Certain formalistic analogies between probability theory and quantum-mechanical theory are apparent. In quantum-mechanical theory the scalar product of the wave with its complex conjugate is postulated to produce a probability function. The wave function itself is called a probability amplitude function. It is also postulated that the ψ functions are superposable; i.e., if ψ_1 is a wave function appropriate to the description of one system, say an electron in motion through slit 1 and ψ_2 is that function appro-

[7] When this operator is applied to a function defined in a discrete space it will be understood that the difference-operator, $\Delta/\Delta x_i$, is meant.

[8] In the notation used here, the parentheses $(\phi_1 \cdot \phi_2)$ indicates the integral $\int_\tau \phi_1\phi_2 \, d\tau$ where τ is the phase space appropriate to the integral if the functions ϕ are continuous; or indicates the summation

$$\sum_k \phi_1(k)\phi_2(k)$$

if the functions ϕ are defined on the discrete denumerable set k.

priate to the description of an alternative route through a second slit, the wave function describing the combined event is given by the sum of the wave functions of the separate events:

$$\psi_{12} = \psi_1 + \psi_2 . \qquad [4]$$

The probability associated with the combined events is then

$$(\psi^*_{12} \cdot \psi_{12}).$$

For the moment, consider that the wave function, ψ, is in fact a probability function and that its complex conjugate is in fact an importance function. It will be necessary, of course, to assign a meaning to complex probability and value. It can be shown (4) that the forward probability function expressed in Equation 4 in Part I may be rewritten in the form:

$$\frac{\partial P}{\partial t} = PM, \qquad [5]$$

where M is a matrix operator that acts only on the space coordinates of the probability function. The backward or importance equation may also be expressed in this form, as in Equation 4:

$$\frac{\partial P\dagger}{\partial t} = MP\dagger. \qquad [6]$$

The probability function in Equation 5, for example, may be separated into two functions, one a function of space alone, and one of time alone.

$$P = S \cdot T, \qquad [7]$$

where S represents the function of the space coordinates only, and T a function of the time coordinates only. This separation of variables produces an arbitrary eigenvalue ω:

$$\frac{\dot{T}}{T} = \frac{SM}{S} = \omega \qquad [8a]$$

and it is seen that the solution of the time equation is of the form

$$e^{\omega t}, \qquad \omega \leq 0,$$

where ω is restricted to nonpositive magnitudes in attrition systems. Thus it may be concluded that a real state probability is to be associated with a superposition of states that decay exponentially, that is, with transient states. On the other hand, if the probability function had been imaginary, the eigenvalue would be imaginary, $i\kappa$.

$$\frac{\dot{T}}{T} = \frac{SM}{S} = i\kappa. \qquad [8b]$$

The solutions to the time equation in this case would be $e^{i\kappa t}$, and the over-all solution would be a superposition of such oscillating states. Thus, it is indicated that an imaginary probability function, if any exists, would be appropriate to the description of oscillating, or as is said, stationary states. Similarly, it is indicated that imaginary value is to be associated with the net increase of the amplitude of the forward stationary oscillating probability. These statements all apply to conditions under which all the elements of the matrix operator are real. There remains to be discussed an interpretation of an imaginary component of an elemental transition probability. Some indication of such an interpretation will be produced in the discussion on the special theory of relativity to follow.

Nevertheless, assuming for the moment a use for a complex probability notation and for equations of type 5 and 6, it is seen that the solutions would naturally be superposable because of the linearity of the differential equation. In real probability theory this results that the probabilities associated with mutually exclusive events are additive; in quantum mechanics the same rule applies but the imaginary probability has been interpreted as a "probability amplitude." Next, it would also be natural to associate an expected importance with the scalar product of the probability of the complex probability function with its adjoint. Since the quantum mechanical equation is self-adjoint in complex notation, the complex conjugate is the adjoint function of the forward wave equation and may be interpreted as a value function. Thus it appears that the scalar product of probability with value is analogous in value theory to the scalar product of the wave function with its complex conjugate in quantum mechanics, both being *expectations*.

TABLE 1

ANALOGY OF OBSERVABLES PRODUCED BY OPERATORS IN Q MECHANICS AND IN VALUE THEORY

Operator	Expected magnitude of the observable in	
	Q Mechanics	Value Theory
x_i	$\bar{x}_i$ i^{th} coordinate of position	$\bar{x}_i$; useful inventory of i^{th} commodity
$\partial/\partial x_i$ or $\Delta/\Delta x_i$	$\bar{p}_i$ component of momentum in direction of i^{th} axis	$-\bar{v}_i$, avg value of thing of type i

$$(P\dagger \cdot P) \sim (\psi^* \cdot \psi). \qquad [9]$$

What is introduced in quantum mechanics as a basic postulate will then be a natural theorem of probability theory. Now it follows to advance plausible arguments that the quantum mechanical postulate of Equation 1 has an analogue that appears naturally in probability theory. In Table 1, the analogy between the observables produced by two operators in quantum mechanics and in value theory is portrayed. The position operator, x_i, in quantum mechanics produces the observable leading to the expected measurement of position of an atomic particle described by the wave function of quantum mechanics. In value theory, under appropriate conditions, this operation will lead to the expected inventory of an i^{th} type of commodity. The mathematical operator, $\partial/\partial x_i$, in quantum mechanics leads to a mean value of the measurement of momentum in the i^{th} coordinate; in value theory[9] it leads to an average measurement of the value of a thing of the type i; i.e., the i^{th} commodity.

Normally, one might expect that the mean position of a system (that is, the expected state that will be revealed by a random observation on the state) is given simply by

$$\bar{x}_i(w_0, s_0, t) = \sum_{x} x_i P(w_0, s_0; x, t), \quad {}_{t>s_0} \qquad [10]$$

The expected position in the i^{th} coordinate in phase-space can be considered to be the expected inventory of the i^{th} commodity. Equation 10 is appropriate to the situation

[9] See Footnote 8.

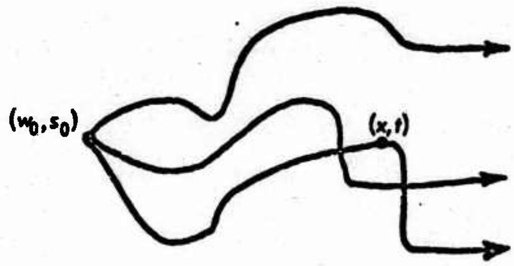

FIG. 2. Illustrating an Average of the First Kind.

Only the initial point (w_0, s_0) is fixed, all possible histories being considered in the average associated with future states of the system.

in which the initial point (w_0, s_0) is fixed *and no further restrictions are considered* (see Figure 2). This is called here an average of the first kind. On the other hand, if both an initial point and a final point are specified, an average of the second kind (see Figure 3) is obtained:

$$\bar{x}_i(w_0, s_0, t, y_0, u_0) = \sum_{x} P\dagger(x, t; y_0, u_0)\, x_i\, P(w_0, s_0; x, t), \quad {}_{u_0>t>s_0} \qquad [11]$$

On the right-hand side of Equation 11 the forward equation gives the probability that the system originating at the point (w_0, s_0) will be in state x at time t. To satisfy the conditions of the average, this is weighted by x_i but it must then be multiplied by the probability that the system moves from state x at time t, to the final fixed point, state y_0, at time u_0. Thus, it is seen that observables associated with operators, as expressed by Equation 1 become a natural consequence in probability theory provided the average is taken at a point intermediate to the two fixed points

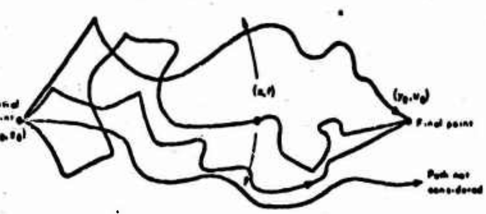

FIG. 3. Illustration of an Average of the Second Kind.

Only those histories that pass through the initial point (w_0, s_0) and the final point (y_0, u_0) are considered. The average is taken with respect to a field point (x, t), which is intermediate; i.e., for which $s_0 \leq t \leq u_0$.

Any physical system described by quantum mechanics fulfills this restriction. The system to which the wave equations apply it is implied in the problem that there is an initiation of the event and an absorption of the event. There are two fixed points in every experiment.

Now consider the space differential operator that in the quantum mechanical system produces an expected measurement of the momentum in the i^{th} direction. The expected value of a thing, $\bar{v}_i$, has been defined as the partial derivative of the value function with respect to the i^{th} commodity:

$$v_i(x, t) = \frac{\Delta Q(x, t)}{\Delta x_i} \qquad [12]$$

If this definition is combined with the fundamental value equation (Equation 11 in Part I), there results Equation 13:

$$\begin{aligned} \bar{v}_i(w_0, s_0, t) &= \sum_x P(w_0, s_0; x, t)\, v_i \\ &= \sum_x P(w_0, s_0; x, t) \frac{\Delta Q(x, t)}{\Delta x_i} \qquad [13] \\ &= -\sum_x Q(x, t) \frac{\Delta}{\Delta x_i} P(w_0, s_0; x, t), \end{aligned}$$

by the nature of the operator adjoint to $\Delta/\Delta x_i$ (see Section VI, D1, Equation 54).[10]

Thus the averages of the type expressed in Equation 1 result naturally in probability value theory. If, indeed, the wave function may be considered a probability function, some of the fundamental relations that appear in quantum mechanics as postulates would result from natural consequences of probability theory.

2. The definition of value and the conservation of momentum. There exists an analogue between the definition of value (Part I, Equation 11) and the law of conservation of momentum. It has been seen above that the negative of the partial derivative with respect to the coordinate as an operator produces analogous quantities in the physical system and the value system—in the first case producing an expected measurement of momentum, and in the second case an expected measure of the value of a thing. For simplicity, assume a simple gambling game. The total money owned by both players is equal to W. The state of the game be represented by the money x owned by the first player. The trapped states, of course, occur when either player wins all the money. Thus the state of the system may be represented as a point on a line segment. It will be assumed that the amount risked in each pot is constant. The value of an added pot to Player 1 is then obtained from the value equation:

$$Q(x) = pQ(x + 1) + qQ(x - 1), \qquad [14]$$

where p represents the probability that Player 1 will win the pot, and q the probability he will lose it. Since $p + q = 1$, the left-hand side of Equation 14 may be multiplied by $p + q$, and rearranged:

$$p[Q(x) - Q(x + 1)] = -q[Q(x) - Q(x - 1)]. \qquad [15]$$

The portion in brackets on the left-hand side represents the difference in state value by an added pot, and therefore by definition, is the value of an increased pot to Player 1. Similarly, the portion in brackets on the right-hand side is the value of a decreased pot to Player 1. This difference taking over a single space interval is, in different terminology, what corresponds to the partial de-

[10] Equation 13 may, of course, be written in terms of the contribution of the k^{th} trapped state (in an absorption system) to the value of a thing:

$$\bar{v}_i(w_0, s_0, t, z_k) = \mu(z_k) \sum_x P(x, t; z_k, \infty) \frac{\Delta}{\Delta x_i} \left\{ P(w_0, s_0; x, t) \right\}$$

obviously,

$$\bar{v}_i(w_0, s_0, t) = \sum_k \bar{v}_i(w_0, s_0, t, z_k). \qquad [13a]$$

In a similar manner the "inventory" average of the second kind may be built up over a set of final states (y_k, u_0):

$$\bar{x}_i(w_0, s_0, t, u_0) = \sum_k \bar{x}_i(w_0, s_0, t, y_k, u_0)$$

If the $\mu(y_k)$ are all unity, then the average of the second kind reduces to that of the first kind. Thus the former summed over only desirable states gives a "useful inventory."

rivative in a continuous space. Thus, Equation 15 is of the form:

$$p\frac{\partial Q}{\partial x}\Big|_{+} = -q\frac{\partial Q}{\partial x}\Big|_{-} \qquad [16]$$

Since the partial derivative operator is analogous to momentum, Equation 16 may be interpreted under conditions where winning and losing are equal; i.e., where $p = q = \frac{1}{2}$ Here, momentum is conserved; momentum in the positive direction is equal and opposite to momentum in the negative direction. This conservation, of course, is true only in a uniform space. When p is not equal to q, Equation 16 gives the weighting appropriate to a more general expression. Thus it appears that the definition of value, in value theory, has an analogous position to a more general form of the law of conservation of momentum in physical theory.

It should be pointed out that to complete the analogy between probability value theory and physical theory, an important postulate is required in value theory that is missing in this discussion.

3. Analogy with the Special Theory of Relativity. A most challenging task in establishing confidence in the analogical conformity principle is that of demonstrating that the form of physical theory is homomorphic to the form of probability value theory. One's interest is naturally directed to the physical sciences. Since they deal with the simplest of perceptual properties, they are the farthest advanced in the conceptual and analytical sense. As an initial step it will be attempted to show that the simplest of stochastic processes—the random walk—when transformations are made on its transition probabilities in such a manner as to produce uniform motion of the first moment of its probability distribution, will exhibit properties similar to those in the Special Theory of Relativity. Indeed, this transformation is the Lorentz-Einstein transformation referred to the space defined by the stochastic process. Generalization to more complex stochastic processes will be indicated. The demonstration will proceed in three parts. The first will produce evidences of relativistic-like properties of the random walk, the second will introduce a notion of *subjective normalization*, which will lead naturally to the transformation equations. It will be necessary to introduce some concepts associated with time transitions purely on the basis of symmetry and completeness. These concepts will lead to an interpretation of an imaginary component of transition probability. All the basic arguments will be included, although the algebraic manipulations will be minimized to conserve space.

a. Indications of relativistic properties of the random walk. Consider a random walk on one dimension. Let k refer to an index of a space point along a line, and n an index of the n^{th} transition. Only discrete points on a line will be occupied. The probability of finding the system in state k after the n^{th} transition, given that it was known to be in state k_0 at the n_0^{th} transition, is designated by $P(k_0, n_0; k, n)$. The initial space point and transition may be defined as the origin in space and transition indices so that it is understood that $P(k, n)$ represents $P(0, 0; k, n)$. Let the probability of a transition to the next adjacent space point in the positive direction be denoted by p, opposite by q, such that $p + q = 1$, then

$$P(k, n) = pP(k-1, n-1) + qP(k+1, n-1), \qquad [16]$$

for

$$P(0, 0) = 1.$$

This has a solution:

$$P(k, n) = \binom{n}{\frac{n-k}{2}} p^{\frac{1}{2}(n+k)} q^{\frac{1}{2}(n-k)}, \qquad [17]$$

$$n \geq k \geq -n,$$

where $\frac{1}{2}(n + k)$ must be an integer, i.

The first moment $\bar{k}(n)$ is defined by

$$\bar{k}(n) = \sum_k kP(k, n), \qquad [18]$$

and the second moment by

$$\bar{k^2}(n) = \sum_k k^2P(k, n). \qquad [19]$$

The standard deviation of the distribution, σ^2, is given by:

$$\sigma^2 = \bar{k^2} - (\bar{k})^2. \qquad [20]$$

There results the average position:

$$\bar{k}(n) = n(p - q), \qquad [21]$$

and the square of the standard deviation

$$\sigma^2(n) = 4npq. \qquad [22]$$

A system that undergoes no change cannot define a space or a time. The actual change itself—the event—is taken to define simultaneously an increment of space and an increment of time. In a system completely isolated from all other systems, time will progress only during the moments of change in that system. Therefore, one is led to the adoption of the index n of the transition number as the measure of the elapsed time. A transition in the system not only defines an ultimate quantum of time, it also defines an ultimate quantum of space. A transition, defining both, can at most move the phase point one unit of space during one unit of time.[11]

The first moment of the distribution gives the "center of probability" of the random walk and therefore its location may be considered the mean position of the stochastic process. Dividing Equation 57 through by the time index n, one gets

$$\frac{\bar{k}(n)}{n} = p - q \equiv \beta. \qquad [23]$$

The ratio of the spatial movement of the center of probability to the time index is equal to the difference between the probabilities of moving to the right, minus the probability of moving to the left. By definitions of time in space, as generated by the system, this ratio, β, becomes the average velocity of the stochastic system. If the probability of moving to the right, p, is greater than that to the left, q, the center of probability will move off to the right with a uniform velocity, β. This velocity will furthermore have a maximum value when $p - q = 1 (q = 0)$. This property is characteristic of the motion of physical systems, where the maximum velocity in physical space is that of the propagation of light *in vacuo*.

[11] "There is no Nature apart from transition, and there is no transition apart from temporal duration" (7, p. 46).

Now consider the first moment and the standard deviation of a random walk that is at rest with respect to the coordinate system. Such a system will be at rest if $p = q = \frac{1}{2}$. Denote the condition at rest by the subscript zero ($_0$), getting:

$$\left.\begin{aligned} \bar{k}_0(n) &= 0, \\ \sigma_0^2(n) &= n. \end{aligned}\right. \qquad [24]$$

The ratio of the size of the random walk in motion ($k \neq 0$) and that of one at rest ($k = 0$) is given by

$$\frac{\sigma(n)}{\sigma_0(n)} = 2\sqrt{pq}. \qquad [25]$$

We have noted that the velocity is given by $\beta = p - q$; and since $p + q = 1$:

$$\left.\begin{aligned} p &= \tfrac{1}{2}(1 + \beta), \\ q &= \tfrac{1}{2}(1 - \beta), \\ \text{and} \quad \frac{\sigma}{\sigma_0} &= \sqrt{1 - \beta^2}. \end{aligned}\right\} \qquad [26]$$

It is observed that the ratio of the standard deviation of the system in motion to the standard deviation when stationary is equal to the square root of $1 - \beta^2$. Now the standard deviation may be taken as a measure of the physical size of the stochastic process. Hence, one derives that the size of the process contracts in the direction of motion, similar to the Lorentz contraction in the Special Theory of Relativity.

Although the random walk as here described has two of the properties of relativity, this description is not relativistic, since it refers to an absolute frame of reference. Is there a transformation that will express the moving system in terms of its own moving coordinate system, and does it have relativistic properties?

b. Random and deterministic components of probability. The word "random" implies uniform probabilities. In this meaning of the term, random walk implies a uniform probability of movement right or left, i.e., $p = q = \frac{1}{2}$. On the other hand, a deterministic sequence of events has been defined as a sequence in which one state follows the other with a probability of unity. The

stochastic process of the preceding section can be either of these two extremes. If $p = q = \frac{1}{2}$, it is its most random condition; if $p = 1$, $q = 0$, the motion of the phase point is to the right in a fully deterministic fashion. Thus a system whose center of probability is at rest is completely random; and a system whose center of probability moves off with maximum velocity is completely deterministic. This suggests that motion intermediate between these two extremes can be broken down into two components—one that it is random and one deterministic.

We therefore seek to break up the transition probabilities in each dimension into two components, as in Figure 4. The excess of probability of motion to the right over that to the left is defined as the *deterministic component*, which is positive to the right. This leaves a *random component*, q to the right, and $-q$, which is therefore directed to the left; these being equal and opposite, are appropriately termed random.

The deterministic component of probability is a vector quantity. The random component, being added both positively and negatively, sums to zero algebraically. The sum of p and q will be defined as the normalized component and depends directly on the metrics of time and space.

It becomes increasingly clear that the concept of probability is inextricably connected with the constructs of time and space. Obviously, then, any operation affecting the metrics of time and space must also affect the constructs of probability. By the term "event" is meant that something has happened; that is, something has happened with a probability of unity. The probabilities associated with an event are usually normalized to unity. This requirement need not necessarily be adhered to. One can associate with an event an expectation not necessarily of certainty so that the sum of the probabilities is different from unity.

Consider now that time is a dimension in addition to the cartesian coordinates. In the description of the foregoing, it appears that a stationary random walk is described as completely random. On the other hand, it has been assumed that every transition

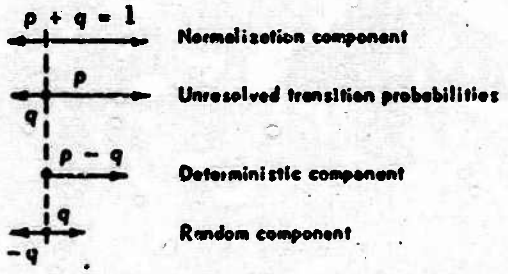

FIG. 4. The Resolution of Transition Probabilities into Deterministic, Random, and Normalization Components.

in space, whether forward or backward, is accompanied in a deterministic fashion (with expectation $p + q$) by transition in time. As long as an absolute frame of reference is adhered to, the transition in space is accompanied by a deterministic transition in time. We now seek to determine what happens to this relation when the coordinate system itself is transformed to move with a velocity equal to the movement of the center of probability of the probability distribution of the stochastic process.

c. Subjective normalization. Consider a universe in which all the denizens themselves, together with what they construe to be materialistic quantities, are all stochastic processes in the same space. Obviously, an observer who is himself a stochastic process, will be subjected to the same distortions to which a local process may be subjected. This is in fact the definition used herein of a *local observer*. He is not only at rest with respect to the center of probability of the process he is observing, but he himself is subjected to the same anisotropies of the probability distributions. Not only is he subjected to the same distortions but he observes the stochastic process as a diffuse probability distribution; he is able to infer the expected movement of the phase point, but he cannot observe discrete transitions to which he could associate a concept of a deterministic event. Hence he is at liberty to adjust the metric of his space and time so that he may reach the normalizations of probability that he desires. In other words, it is useless for him to postulate an absolute discrete framework in which the phase point of his stochastic process is located. Since our observer cannot

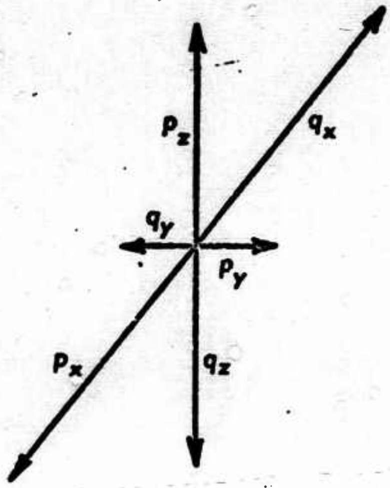

FIG. 5a. Non-isotropic Transition Probabilities in a Stationary Random Walk.

The implications of such a representation made by imposing the observers space metric on the system is that its local time clocks in the three dimensions are asynchronous.

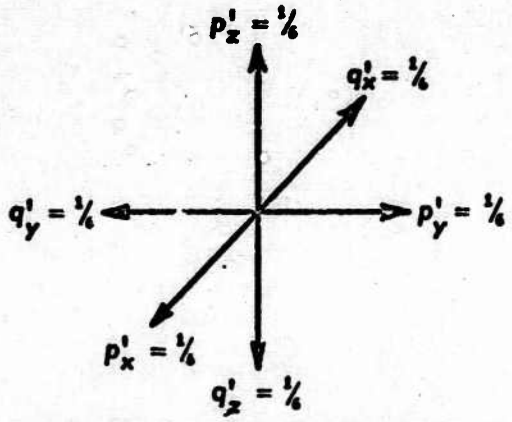

FIG. 5b. The Same Transition Probabilities of Figure 5a as Seen by a Local Observer.

The former observer must conclude that either the systems time clocks are asynchronous and the space metrics of the two systems are identical; or he agrees with the local observer that the time clocks are synchronous (only one time dimension) but then he must conclude that the space metrics of the two systems are different.

conceive of himself as an anisotropic, it would be absurd, he reasons, to believe that his body changes shape merely because it is rotated in space, he sees himself as an isotropic individual. A second observer, presumably ourselves, may see him as flat-headed or pinheaded. We may therefore construe the transition probabilities that define his space to be anisotropic. The distortion is not observable to him, since, to his way of thinking, he is perfectly isotropic. His world may be described in one of three alternative ways: (*a*) we can say that the transition probabilities in the x, y, and z direction are not equal (throughout this part of the discussion we are considering that the process is at rest with respect to ourselves and to the local observer) as in Figure 5a. The local observer, however, considers all of these transition probabilities equal, and equal to $\frac{1}{6}$, as in Fig. 5b.

Now what are the implications of these interpretations? Since time is to be defined in terms of change, the external observer who sees an anisotropic probability distribution *is forced to the conclusion that the clock runs at different rates in different dimensions*, since, for the dimension where the transition probabilities are smaller than the others, transitions will occur less frequently. If the transition in this dimension alone is what determines the movement of its clock, then obviously one is confronted with three dimensions of time—one complementary to each space dimension and all of them occurring at different rates. This interpretation has been brought about by requiring the space metric of the observed system to conform to that of the external observer and leads to the anomalous interpretation of three-dimensional time.

The local observer, on the other hand, subjected as he is to the same anisotropism, cannot observe what we regard to be the distortions of his space. He believes himself to be nicely squareheaded. Not only are his transition probabilities equal in all directions, but the time clocks in all dimensions are synchronized. He can regard time as being a single dimension. There remains a third interpretation, which is the second made by the external observer—ourselves. Instead of imposing our own metric of space on the observed stochastic process and thus arriving at the anomalous description of time, we may wish to preserve the synchronism of the clocks in the three dimensions and infer that the metrics of space of the observed system are different from our own. These metrics of space may be ad-

justed such that the expected motion as observed in the system in the various dimensions agrees with our original interpretation. *This is the meaning of subjective normalization.* An event is interpreted as an expectation. Thus $p + q$, which in the local system is equal to unity, is reinterpreted to mean that $p + q$ multiplied by a space metric is an expectation. It is the expectation that is invariant to these transformations between systems of various isotropism.

In the universe taken as a model for this discussion there is no such thing either as an absolute frame of reference, or an absolutely isotropic system. We have, moreover, inferred there is a separate time dimension concomitant with each space dimension. It is only through the means of subjective normalization that the three time dimensions are synchronized and therefore may be considered as one.

The gist of these remarks is that the normalized component of probabilities in any particular dimension will transform such that the expectation is constant. In other words, that

$$|dx| = (p + q)\,dt = (p' + q')\,dt'. \qquad [27]$$

If the unprimed probabilities are normalized then one gets an equation of the form

$$dt = (p' + q')\,dt'; \quad \text{or } p' + q' = \partial t/\partial t'. \qquad [28]$$

Now consider what happens to the deterministic component of probability. Again assume that the expected motion is invariant to a transformation. The expected motion, dx, is equal to the velocity times dt,

$$dx = \beta\,dt, \qquad [29]$$

but the velocity is a deterministic component. Thus, one gets

$$dx = (p - q)\,dt = (p' - q')\,dt'; \text{ or } \quad p' - q' = \partial x/\partial t' \qquad [30]$$

for the transformation of a deterministic component as implied by the principle of subjective normalization.

d. The probability transformation. We consider two coordinate systems at uniform motion with respect to each other, one designated as the Σ and the other the Σ' system. Consider two random walks, one stationary in the Σ system, one stationary in the Σ' system. Four ways of observing the systems may be considered. The system stationary in the Σ system is observed by an observer local to the Σ system; the same system is observed by local observer in the Σ' system, and vice versa. The following notation will be adopted: (1) The probabilities of transition in Σ as seen by a local observer in Σ, are denoted p_{xx}, q_{xx}. The standard deviation of this system denoted as σ_{xx} and it is measured in terms of the metric of the Σ system. (2) The probabilities of transition in the Σ system as seen by an observer local to the Σ' system, $p_{xx'}$, $q_{xx'}$, $\sigma_{xx'}$. The latter is measured by the local observer in Σ' using the metric of the Σ system. (3) The probabilities of transition in the Σ' system as seen by an observer local to the same system. $p_{x'x'}$, $q_{x'x'}$ and $\sigma_{x'x'}$. The latter refers to the standard deviation seen by the local observer in Σ', and measured by the metrics of Σ'. (4) The probabilities of transition in the Σ' system as seen by an observer local Σ system. $p_{x'x}$, $q_{x'x}$, $\sigma_{x'x}$, the latter being the standard deviation observed by the local observer in Σ', measured using the metrics of the Σ' system. Now any measurement of space using the same metrics of space of the same phenomena should result in the same numerical result. Thus we conclude

$$\left.\begin{aligned} \sigma_{xx'} &= \sigma_{xx} \\ \sigma_{x'x'} &= \sigma_{x'x} \end{aligned}\right\}, \qquad [31]$$

and therefore consider as a fundamental invariant the preservation of the standard deviation, or rather the square of this deviation, when measured by the same space metric:

$$\left.\begin{aligned} 4\,p_{xx}q_{xx} &= 4\,p_{xx'}q_{xx'} \\ 4\,p_{x'x}q_{x'x} &= 4\,p_{x'x'}q_{x'x'} \end{aligned}\right\} \qquad [32]$$

Consider now a stochastic process stationary in the Σ system. The transformation to the Σ' system should result in its uniform motion with respect to the Σ' system, and if the conditions of relativity prevail it will appear to be foreshortened in the Σ' system. Conversely by the fundamental principle of relativity the system that is stationary

in the Σ' system when transformed to the Σ system, will appear again to be foreshortened. The original conditions of the systems stationary in the Σ' system—call it process I—is

$$\left.\begin{aligned} p_{xx} + q_{xx} &= 1, \\ p_{xx} - q_{xx} &= 0. \end{aligned}\right\} \qquad [33]$$

etc. for y and z.

The transformation to the Σ' system will be undertaken in two parts. In the first step the external observer sees the other stochastic system as if it were non-uniform and the probabilities not normalized in an isotropic space. By inference this non-uniformity implies the observed time dimensions are asynchronous. The second step distorts the space in a manner which restores normalization and synchronizes the clocks in the three dimensions; thus permitting time again to be regarded as a single dimension. In the first step, Equation 33 will be transformed in the form that an observer local to the Σ' system would view the same probabilities if he were employing the metrics of space and time appropriate to Σ system. Applying the rules for subjective normalization previously expounded there results, letting r, s, t, be three time coordinates corresponding respectively to x, y, z:

$$\left.\begin{aligned} p_{xx'} + q_{xx'} &= \partial r/\partial r' = e_{rr'} \\ p_{xx'} - q_{xx'} &= \partial x/\partial r' = e_{xr'} \end{aligned}\right\} \qquad [34a]$$

$$\left.\begin{aligned} p_{yy'} + q_{yy'} &= \partial s/\partial s' = e_{ss'} \\ p_{yy'} - q_{yy'} &= \partial y/\partial s' = e_{ys'} \end{aligned}\right\} \qquad [34b]$$

$$\left.\begin{aligned} p_{zz'} + q_{zz'} &= \partial t/\partial t' = e_{tt'} \\ p_{zz'} - q_{zz'} &= \partial z/\partial t' = e_{zt'} \end{aligned}\right\} \qquad [34c]$$

Equations 34a, b, c, together with the invariances of Equations 32 and 31, are almost a complete set of conditions for an orthogonal transformation in the space-time framework, i.e.

$$\begin{aligned} e^2_{rr'} - e^2_{xr'} &= 1, \\ e^2_{ss'} - e^2_{ys'} &= 1, \qquad [35a] \\ e^2_{tt'} - e^2_{zt'} &= 1. \end{aligned}$$

The missing conditions are

$$\begin{aligned} e^2_{xx'} - e^2_{rx'} &= 1, \\ e^2_{yy'} - e^2_{sy'} &= 1, \\ e^2_{zz'} - e^2_{tz'} &= 1, \qquad [35b] \\ e^2_{\alpha\alpha'} &= e^2_{\beta\beta'} \\ e^2_{\alpha\beta'} &= e^2_{\beta\alpha'} \end{aligned}$$

where α and β are commuted through the six dimensions.

We shall assume *ad hoc* that Equations 35b are true[12] and attempt to reconstruct a situation which will produce them. This reconstruction is not unique without the addition of conditions imposed by some deeper insight—which is missing here. Such an insight may be forthcoming if a successful extension of this analogy to the General Theory of Relativity is accomplished. Equations 35b imply that there is a set of probabilities associated with transitions in time, p_{rr}, q_{rr}, etc., for which

$$4\, p_{rr'}q_{rr'} = (\partial x/\partial x')^2 - (\partial r/\partial x')^2\text{, etc.}$$

is an invariant and equal to unity.

Now $p_{xx'} + q_{xx'}$ is the expectation that any space (either positive or negative) transition occurs per *deterministic time transition*. The difference $p_{xx'} - q_{xx'}$ is the expectation that a *deterministic* space transition occurs per *deterministic* time transition. Hence $(p_{xx'} - q_{xx'})/(p_{xx'} + q_{xx'})$ is the velocity of the system. The interpretation of $p_{rr'} - q_{rr'}$ cannot be specifically made on the basis of Equations 35b alone. It seems natural to consider it a deterministic change in time and hence should be equal to unity for a stationary system. Some other interpretation, such as a fully symmetrical one, may well be the better one however. We were led to try the following, since the imaginary notation fits condition 35b and is consistent with the requirement of rotation of space coordinates into an imaginary time dimension; this latter is the result of the invariance of the difference between components in Equations 35b:

$$\left.\begin{aligned} p_{rr'} + q_{rr'} &= i\partial r/\partial x' = ie_{rx'} \\ p_{rr'} - q_{rr'} &= i\partial x/\partial x' = ie_{xx'} \end{aligned}\right\} \qquad [36a]$$

[12] The space-time probabilities producing Equations 35a are insufficient to define a relativistic space-time metric. The adoption of Equations 35b is then the generalization on probability which produces this characteristic.

$$\left.\begin{aligned} p_{ss'} + q_{ss'} &= i\partial s/\partial y' = ie_{sy'} \\ p_{ss'} - q_{ss'} &= i\partial y/\partial y' = ie_{yy'} \end{aligned}\right\} \quad [36b]$$

$$\left.\begin{aligned} p_{tt'} + q_{tt'} &= i\partial t/\partial z' = ie_{tz'} \\ p_{tt'} - q_{tt'} &= i\partial z/\partial z' = ie_{zz'} \end{aligned}\right\} \quad [36c]$$

This results in a six-dimensional transformation.

Conditions 35a, b, now imply an orthogonal transformation which can be characterized as a rotation of space axes into the corresponding imaginary time axes. Aligning the x-axis with the direction of motion simplifies this somewhat. The transformation will involve only one dimension of space and of time. Anticipating that the second part of the transformation will restore the synchronism of the time-dimension clocks, Equations 34 and 36 may be rewritten in a four-dimensional space, assuming only one time dimension. There results, letting $r = s = t$:

$$\left.\begin{aligned} p_{xx'} + q_{xx'} &= \partial t/\partial t' = e_{tt} \\ p_{xx'} - q_{xx'} &= \partial x/\partial t' = e_{xt'} \end{aligned}\right\}, \quad [37a]$$

$$\left.\begin{aligned} p_{yy'} + q_{yy'} &= 1 \\ p_{yy'} - q_{yy'} &= 0 \end{aligned}\right\}, \quad [37b]$$

$$\left.\begin{aligned} p_{zz'} + q_{zz'} &= 1 \\ p_{zz'} - q_{zz'} &= 0 \end{aligned}\right\}, \quad [37c]$$

$$\left.\begin{aligned} p_{tt'} + q_{tt} &= i\,\partial t/\partial x' = ie_{tx'} \\ p_{tt'} - q_{tt'} &= i\,\partial x/\partial x' = ie_{xx'} \end{aligned}\right\}. \quad [37d]$$

Several considerations contribute to the addition of Equations 34 and 36. Before completing this part of the explanation, consider what happens when the second state of the transformation is conducted. The second state of the transformation results when the observer local to the Σ' system imposes normalization, and one gets:

$$p_{x'x'} + q_{x'x'} = \frac{p_{xx'} + q_{xx'}}{e_{tt'}} = 1$$
$$p_{x'x'} - q_{x'x'} = \frac{p_{xx'} - q_{xx'}}{e_{tt'}} = \frac{\partial x}{\partial t} = \beta_x \quad [38]$$

The subjective normalization simultaneously normalizes the probabilities in the appropriate dimension—leaves the deterministic component that of a velocity—and synchronizes the clocks in the various dimensions reducing an apparently six-dimensional space to four dimensions. Now consider what effect these stages of the transformation have upon the standard deviation. The first stage of the transformation, according to Equations 32 and 35, leaves the standard deviation unchanged. Now the imposition of the second stage of the transformation which resulted in the normalization of the probability components results:

$$\frac{4p_{xx'}p_{xx'}}{e_{tt'}} = 4p_{x'x'}q_{xx'}, \quad [39]$$
$$= (1 - \beta_x)^2$$

thus

$$\frac{\sigma_{x'x'}}{\sigma_{xx}} = \sqrt{1 - \beta_x^2}, \quad [40]$$

i.e., the Lorentz foreshortening as previously encountered. These conditions are now those of an orthogonal transformation in the four-dimensional system from the Σ to the Σ' frames of reference. They may be interpreted as a rotation of the axis of motion, say for example, the x axis, into an imaginary time axis. The imaginary is associated with the probabilities relating to the time transition in Equations 34 and 36 in order to preserve the same form of equation in Equation 35. It results, in effect, in a more consistent notation. The $e_{xx'}$ etc. may also be interpreted as a projection of unit vectors in the Σ system on the Σ' axes. The Lorentz-Einstein transformation is implied by the process described. Since the difference of the squares is invariant rather than their sum, the rotation of coordinates must be through an imaginary angle. Assume the rotation to be of the x-axis into the imaginary time axis (uniform motion along the x-axis), this imaginary rotation and the imaginary notation make it appropriate to consider that the unit vector in the time dimension is also an imaginary. The sum of the time transition probabilities $p_{tt'} + q_{tt'}$ has been set equal to $ie_{tx'}$ and the difference to $ie_{tt'}$ in order that the time transition for a stationary system will be completely deterministic. Its projection upon any space axis will be an imaginary. Writing down, now, the conditions for the transformation of the unit vectors in the Σ sys-

tem to the unit vectors in the Σ' system, one gets

$$
\begin{aligned}
e'_x &= c_{xx'}e_x + (ic_{tx'})(ie_t),\\
e'_y &= e_y\,,\\
e'_z &= e_z\,,\\
ie'_t &= (-ic_{xt'})e_x + c_{tt'}(ie_t).
\end{aligned}
\qquad [41]
$$

e. The Lorentz-Einstein Transformation Equations. The transformation equations for the unit vector may now be used to express the transformation for any vector quantity. Suppose the four-vector displacement vector $R = R'$ is considered. This four-vector being an expected motion is invariant to four-dimensional rotation transformation, one has:

$$
\begin{aligned}
R &= \sum_j x_j e_j\\
R' &= \sum_j x_i e_i
\end{aligned}
\qquad [42]
$$

Where e_x, etc. represent the unit vectors and x_j with $j = 1, 2, 3, 4$ replaces x, y, z and it.

One has, now, substituting the transformation for the unit vector:

$$e'_i = \sum_j c_{ji}e_j \qquad [43]$$

getting

$$x_j = \sum_i x'_i c_{ji} \qquad [44]$$

Substituting now into Equation 44 the appropriate coefficient selected from Equation 41, there results

$$
\begin{aligned}
x &= c_{xx'}x' - c_{xt'}t', & y &= y'\\
t &= c_{tx'}x' + c_{tt'}t' & z &= z'
\end{aligned}
\qquad [45]
$$

or, making note of the invariancies of Equation 40, and the relations outlined in Equations 38 and 39, one gets the well-known Lorentz-Einstein transformation equations of the special theory of relativity as referred to the space of the stochastic process:

$$
\begin{aligned}
x &= \frac{x'}{\sqrt{1-\beta^2}} - \frac{\beta t'}{\sqrt{1-\beta^2}}\\
t &= \frac{-\beta x'}{\sqrt{1-\beta^2}} + \frac{t'}{\sqrt{1-\beta^2}},
\end{aligned}
\qquad [46]
$$

Equations 45 and 46 are then a more general statement of the principle of subjective normalization elucidated in Equations 27, 28, 29, 30:

$$
\begin{aligned}
dt &= (p_{xx'} + q_{xx'})dt' + i(p_{tt'} + q_{tt'})dx'\\
dx &= (p_{xx'} - q_{xx'})dt' - i(p_{tt'} - q_{tt'})dx'
\end{aligned}
$$

We have had to resort to arguments of consistency and symmetry to bring in the probabilities associated with the transitions in the time dimension. Since the theory of relativity itself is so firmly established and since an actual stochastic process in real space would be subject to the conditions giving rise to this special theory of relativity, one may regard that theory itself as supporting some of the results here obtained. The equation for the relativistic retardation of the time will be produced by applying the condition of subjective normalization to the second half of Equation 40. It will also result, of course, from the resultant Lorentz-Einstein transformation equation.

The imaginary in Equations 34 and 36 was introduced in order to preserve the form of Equation 40 and to represent $p_{tt'} - q_{tt'}$ as a determinate time transition. This provides a possible explanation for the interpretation of an imaginary transition probability. In the four-dimensional complex the complete description of the stochastic motion requires not three sets of branching probabilities, but four, the fourth one being imaginary. One set of three is associated with space-time transitions, and another set of three which has been reduced to a single entity through subjective normalization and which are associated with the time-space transitions.

f. Non-random walk system. The question naturally rises now, of how to generalize these remarks to apply to stochastic processes not of the simple random walk character, particularly to stochastic processes, the transition probabilities of which vary from point to point in space. Consider three systems, Σ, Σ', and Σ'', moving respectively with velocitic , β_{01}, i.e., the motion of Σ' with respect to Σ), β_{02} i.e., the motion of Σ'' with respect to Σ) and β_{12} (i.e. the motion of Σ'' system with respect to Σ'). These

correspond to rotation of the coordinates by the angle ψ_{01}, ψ_{02} and ψ_{12}, where

$$\psi_{02} = \psi_{01} + \psi_{12}. \qquad [47]$$

or

$$\left.\begin{aligned} \tan\psi_{02} &= i\frac{e_{zt'}}{e_{zz'}} = i\beta_{02}, \\ \tan\psi_{01} &= i\beta_{01}, \\ \tan\psi_{12} &= i\beta_{02}, \end{aligned}\right\} \qquad [48]$$

and since

$$\tan\psi_{02} = \frac{\tan\psi_{01} + \tan\psi_{02}}{1 - \tan\psi_{01}\tan\psi_{12}}, \qquad [49]$$

one obtains

$$\frac{p_{zz''} - q_{zz''}}{p_{zz''} + q_{zz''}} = \frac{\dfrac{p_{zz'} - q_{zz'}}{p_{zz'} + q_{zz'}} + \dfrac{p_{z'z''} - q_{z'z''}}{p_{z'z''} + q_{z'z''}}}{1 + \dfrac{p_{zz'} - q_{zz'}}{p_{zz'} + q_{zz'}}\;\dfrac{p_{z'z''} - q_{z'z''}}{p_{z'z''} + q_{z'z''}}}. \qquad [50]$$

Equation 50 may be used to transform the transition probabilities point by point in such a manner that relativistic invariancy is preserved.

C. The Uncertainty Principle

An important result appearing in quantum mechanics is the Heisenberg Uncertainty Principle (7).[22] The Heisenberg Uncertainty Principle states that the ultimate residual errors associated with the simultaneous measurement of two canonically conjugate variables is greater or equal to $h/4\pi$ where h is Planck's constant. More exactly, the uncertainty principle is stated as follows:

$$(\overline{\Delta p})^2\cdot(\overline{\Delta q})^2 \geq -\frac{1}{4}\left\{\int \phi^*(O\bar{O} - O\bar{O})\phi\, d\tau\right\}, \qquad [51]$$

where p and q are canonically conjugate variables, O is the operator corresponding to p and $\bar{O}$ the operator corresponding to q: ϕ is the state function and ϕ^* its adjoint—or in the case of quantum mechanics, its complex conjugate. Now, the simple operators pertaining to canonically conjugate variables bear a simple relationship to each other: if x_i is a coordinate operator, which on operating on the scalar product produces an expected measure of position, $\partial/\partial x_i$ is the operator that produces a quantity that is canonically conjugate to the first one. In physics, the implication of the uncertainty principle, for example, is that the position and momentum of a particle may not be measured simultaneously with infinite accuracy; as the position is pinned down more and more exactly the uncertainty in the estimate of the momentum increases without bound, and vice versa. Strictly speaking the uncertainty principle in quantum mechanics, which incidentally also appears in sound theory, is applied to wave motion. In its most general aspects this restriction is probably not as necessary as, for example, the appropriate generalization of the reference above. In the terminology of value theory the operator x_i produces an expected measurement of an inventory of a system that is intermediate between two fixed conditions, and the operator $x_i = \partial/\partial x_i$ produces an expected measurement of the value of one thing of type i. Thus *the uncertainty principle in value theory would state that on an empirical basis one cannot simultaneously take an inventory of things of type i and know the value of these things with infinite accuracy.* Empirical values are determined in the market place.

For example, suppose we are among the early Dutch settlers at their first arrival at the island of Manhattan. We have in our possession several trunks full of colored beads that we intend to trade to the savages for their land. How much land will a hatful of the beads buy? In other words what is the value of the beads in this context? We know exactly how many beads we have, but know nothing about their value; so an expedition sets out, trades off a small portion of the beads for land and thus an estimate of their value is obtained. Two things have occurred: one, the market value for beads has been affected; secondly, we have been forced to consume part of the beads and hence did not have as many as we had before. The only way one can deter-

mine the value of all the beads is by trading them all off; but then, of course, there aren't any more. As a second example we ask what is the value of a cake that is placed before us? Obviously a sample bite must be taken to estimate its value. But in the process of eating part of the cake our appetite is affected and the last bite will not be of the same value as the first. The only way in which the value of the entire cake may be measured is by eating it all. But we can't determine the value of the cake by eating it and also have it. This is the uncertainty principle. Applying operators pertaining to value theory but similar to Equation 51 produces:

$$\overline{(\Delta v_i)^2}\cdot\overline{(\Delta x_i)^2} \geq \frac{1}{4}\left[\sum_s P^+\left(\frac{\partial}{\partial x_i}x_i - x_i\frac{\partial}{\partial x_i}\right)P\right]^2 = \tfrac{1}{4}\left[\sum_s P^+P\right]^2 = \tfrac{1}{4}Q(x_0). \qquad [52]$$

Again, these arguments are to be considered plausibilities since the rigorous investigation has not been completed.

D. Analogical conformity in other disciplines

The purpose of this section is to call attention to other disciplines where the principle of analogical conformity may be examined and ultimately exploited.

1. Adjointness in economic models. It has been stated that the adjoint function may always be interpreted as a value function. In most systems the forward function is said to be "real" in that there may be an observable flux associated with it. In the general case of value theory the history of systems is observable and, with a sufficiently large number of observations, the time dependency of state probability may be inferred. In the case of the diffusion of molecules or neutrons, for example, the diffusion may be observed by appropriate detection devices. In such systems the adjoint function has the character of a mathematical construct. In an economic system, however, the situation is different. There is not only a flux of real things diffusing throughout the states of the system, but there is also an adjoint commodity that diffuses in a manner described by the adjoint equation. This commodity is money.

An operator O, which belongs to a class of functions η_i, is said to be adjoint to an operator $O\dagger$, which belongs to a class of functions ζ_j, if:

$$(\zeta_j \cdot O\eta_i) = (O\dagger\zeta_j \cdot \eta_i) \qquad [53]$$

In eigenvalue problems this restriction ensures that the eigenfunctions of O are bi-orthogonal to the eigenfunctions of O^+, a condition that is useful in expanding an arbitrary function into terms of the eigenfunctions. As a result of this condition, a multiplicative operator x, is self-adjoint, that is, $x = x\dagger$. One can show by application of Equation 53 that where ξ is any coordinate including time, the operator partial derivative with respect to ξ is replaced by its negative in order to produce the appropriate adjoint operator:

$$\partial/\partial\xi = -(\partial/\partial\xi)\dagger \qquad [54]$$

Second derivatives are self-adjoint, and it may be shown that a source in the forward system is replaced by a sink in the adjoint system, and vice versa. The replacement of the first derivative by the negative indicates that velocities and other rates of change in the adjoint system are reversed. Hence the adjoint system is often called the backward system. On the other hand, since the second derivatives are self-adjoint, diffusion that takes place in the forward system will take place similarly in the adjoint system. The movement of money obeys all these requirements. Money is always exchanged for commodities, hence its motion is in the opposite direction. It can be expected to diffuse similarly to commodities. And lastly, a source of money is a sink of commodities, and vice versa. This suggests that first-order perturbation theory can be exploited in economic theory.

2. The second law of thermodynamics. The second law of thermodynamics—so important in physical science—can be expected to describe the behavior of the statistical and stochastic formulation of trial coalitions

in many-person game theory. The coalitions so produced gradually move toward an equilibrium configuration subject to the rules of interaction. This equilibrium cannot be reached instantaneously, since every participant of the game does not interact simultaneously with all other participants. Chance meetings occur and trial coalitions are considered. If it is to the mutual advantage of the players involved in the meeting, new coalitions may develop. These trials go on continuously, more often resulting in no change in the coalition structure. The availability of unusual values resulting from shifts of coalitions will gradually disappear or become a minimum as equilibrium is reached. A function—similar to entropy in logical form—i.e., a logarithm of a probability function—will continually increase toward a maximum at equilibrium. The principle of analogical conformity can be expected to give guidance to the application of old well-established formalistic principles to newly formed theories.

Chapter 13

GENERAL VALUE-DECISION THEORY

MATHEMATICAL FORMULATION

In the past half-century ("recent" in terms of history of philosophy) formal mathematical schema have been developed which can be exploited in the development of a specific object system for a value theory. These are: (1) first-order perturbation theory in physics, (2) theory of stochastic processes (motivated as the development of a model for physical phenomena), and (3) statistical decision theory.

Perturbation Theory

Of these, the most suggestive has been perturbation theory. Perturbation theory seeks to predict the effect of specific actions on the stationary states of certain physical systems (quantum mechanical, nuclear reaction, etc.). It is in the sense that it links ultimate outcomes with action that it suggests a format for value theory.

Shorn of mathematical rigor, this view will be presented in the following discussion. Stationary states are represented as solutions of equations of the type

$$\frac{dU}{dt} = LU \qquad (1.a)$$

where U is a function, $U(\underline{r}, t)$, of vector space $\underline{r}$ and time t, and L is an operator which acts only on the special component. By separation of variables set $U(r, t) = u(\underline{r})e^{\lambda t}$, there results

$$\lambda u(\underline{r}) = Lu(\underline{r}) \qquad (1.b)$$

where the constant λ is interpreted as the period. If λ is positive the system is exploding; if negative it is decaying, and if zero it is stationary.

In some boundary value systems Equation (1.b) has non-trivial solutions only for a set of discrete values of λ, given by $\lambda = \lambda_n$. These discrete values of

λ are called "eigenvalues," and the corresponding solutions u_n are called "eigenfunctions." To predict the effect of a specific change $F(S, \underline{r})$ on the system, where S is any set of parameters of the problem, it is necessary to expand F as a sum of eigenfunctions:

$$F = \sum_n a_n u_n \tag{2}$$

Such an expansion is possible provided the constants a_n can be evaluated. This can be done if there exists a set of bi-orthogonal functions $v_n(\underline{r})$ such that

$$\int_V u_n(\underline{r})\, v_m(\underline{r})\, d\underline{r} = \delta_{n,m} \tag{3}$$

where $\delta_{n,m}$ is the Kronecker delta:

$$\delta_{n,m} = \begin{cases} 0, & n \neq m, \\ 1, & n = m, \end{cases}$$

n and m are integers, and where $d\underline{r}$ represents the volume element and the integration is extended over the domain V defined for the system.

One can then expand F by evaluating

$$\int_V v_m F\, d\underline{r} = \sum a_n \int_V v_m u_n\, d\underline{r} = a_m. \tag{4}$$

The bi-orthogonal functions v_n are defined in the same space as u_n and are furthermore assumed to be generated by some other operator, L^+, called the "adjoint" operator, such that

$$\lambda_m^+ v_m = L^+ v_m \tag{5}$$

The relation between L and its adjoint, L^+, can be evaluated by the scalar products:

$$\left.\begin{array}{l} \int_V v_m \lambda_n u_n \, d\underline{r} = \int_V v_m L u_n \, d\underline{r} \\ \text{and} \\ \int_V u_n \lambda_m^{+} v_m \, d\underline{r} = \int_V u_n L^{+} v_m \, d\underline{r} \end{array}\right] \tag{6}$$

$$(\lambda_n - \lambda_m^{+}) \int_V v_m u_n \, d\underline{r} = \int_V v_m L u_n \, d\underline{r} - \int_V u_n L^{+} v_m \, d\underline{r} \tag{7}$$

By the condition of bi-orthogonality, and given that $\lambda_n = \lambda_n^{+}$, one has a means of determination of the adjoint system:

$$\int_V u_n L^{+} v_m \, d\underline{r} = \int_V v_m L u_n \, d\underline{r} \tag{8}$$

On integration by parts one can verify the following sets of adjoint operators:

$$C^{+} = C \tag{9.a}$$

$$\left[\frac{\partial}{\partial t}\right]^{+} = -\frac{\partial}{\partial t} \tag{9.b}$$

or any first derivation

$$\left[\frac{\partial}{\partial x_i}\right]^{+} = -\frac{\partial}{\partial x_i} \tag{9.c}$$

$$\left[\frac{\partial^2}{\partial x_i^2}\right]^{+} = \frac{\partial^2}{\partial x_i^2} \tag{9.d}$$

$$\left|M_{ab}\right|^{+} = \left|M_{ba}^{+}\right| \tag{9.e}$$

Equation (9. a) states that the operation of multiplication by a constant is self-adjoint; Equations (9. b) and (9. c) state that processes described by first derivatives are replaced by their negatives to produce the adjoint, i. e., in the adjoint systems all such processes are reversed.

Equation (9. d) states that second-order derivatives are self-adjoint, and processes described by such operators are the same in both systems (diffusion, for example). Equation (9. e) states that if the operator is a matrix, the adjoint operator is formed by taking its transpose and replacing each element by its appropriate adjoint. One can also show that the role of sources and sinks are reversed in the adjoint system. In the special case in which the matrix is formed of constants the adjoint matrix is the simple transpose. If the matrix is square, shifting from pre-to-post-multiplication effectively produces the adjoint system, i. e.,

$$\lambda u = M u$$

and

$$\lambda^{+} v^{+} = v^{+} M \tag{10}$$

are adjoint systems.

Lastly, it may be demonstrated that the bi-orthogonal function v_0 adjoint to a stationary state u_0 represents the integrated ultimate net change in a stationary system brought about by unit change in the function u_0 at that time and place, i. e., the adjoint is interpretable as a value function. Indeed, in nuclear reactor physics [6] it has been identified as the "importance function."

That this is true is easily seen. Let the eigenvalues be ordered according to increasing absolute magnitude, it being understood that positive values are prohibited, since they represent an explosion of the system. The corresponding eigenfunctions $u_n(\underline{r})$ and adjoint functions v_n can be normalized so that

$$\int_V u_0(\underline{r})\, d\underline{r} = 1,$$

and

$$\int_V u_n(\underline{r})\, v_n(\underline{r})\, d\underline{r} = 1. \tag{11}$$

At any instant the condition of the system is represented by a linear combination of eigenvalues. If stationary this representation is simply given by $\phi = \alpha u_0$, where α is a constant.

Now add a unit quantity of "flux" (i.e., the unit of the eigenfunction) to $\phi(\underline{r})$ at r' as a perturbation, by replacing ϕ by ϕ' where

$$\phi'(\underline{r}, \underline{r}') = \phi(\underline{r}) + \delta(\underline{r} - \underline{r}'), \tag{12}$$

where the Dirac delta function is defined as zero everywhere except where

$$\underline{r} = \underline{r}',$$

and

$$\int_V \delta(\underline{r} - \underline{r}')\, d\underline{r} = 1.$$

Expanding $\phi'(\underline{r}, \underline{r}')$ into a linear combination of eigenvalues, one gets, by integration with the adjoint function:

$$\phi'(\underline{r}, \underline{r}') = \alpha u_0(\underline{r}) + \sum_n v_n(\underline{r}')\, u_n(\underline{r}).$$

The total integrated flux Φ, before the change was

$$\Phi = \int_V \phi(\underline{r})\, d\underline{r} = \alpha$$

has now become

$$\Phi' = \int_V \phi'(\underline{r}, \underline{r}')\, d\underline{r} = \alpha + \sum_n v_n(\underline{r}') \int_V u_n(\underline{r})\, d\underline{r}.$$

Now letting the time, t, go to infinity, all modes u_n decay away except u_0, giving the limiting integrated flux

$$\Phi' = \alpha + v_0(\underline{r}') . \tag{13}$$

Thus, $v_\theta(\underline{r}')$ represents the total net increase of integrated flux brought about by unit change at point $\underline{r}'$. As such it represents the importance to the build-up of the stationary state of such a unit change.

Although in value theory we shall often not be interested in stationary states (we shall deal with systems where the scalar product

$$\int u_\theta(\underline{r})\, v_\theta(\underline{r})\, d\underline{r}$$

is stationary), the structure of perturbation theory is strongly suggestive of a formal schema for a value theory.

Stochastic Processes and Statistical Decisions

In a preceding section it has been observed under the conceptual viewpoint that both science and axiology are decision operations. Our notation and language still bear the imprint of the postulatory approach in which this observation is ignored. Statistical decision theorists, devoting themselves largely to questions of the scientific decision process, have often been blind to the nature of their inquiry, due to their use of outmoded constructs and semantics adequate only for a world in which the "truth" can be known. This is particularly noticeable in the arguments about Bayes theorem currently appearing in the literature.

Every object theory begins with an "object statement" which constructs (in the sense of "forms constructs for") a particular object model. In science we have such object statements as Newton's "laws" of motion, and the Schrödinger wave-equation. These statements simultaneously structure and relate the concepts employed and set up a model for a specific purpose and prescribe a method of measurement. In science this purpose is predictive; in axiology it is prescriptive. The making of the object statement (called "postulates" in the postulatory mode of inquiry) is itself an intensive or prescriptive operation. Thus Newton's laws give a prescription for the measurement of the concepts presented: force and mass. Similarly the statement "decisions imply values" define a general class of object sciences which prescribe a means of measuring a construct called "value." The selection of a particular construction for an object statement is itself an intensive decision, i.e., a policy action. Generally speaking the construction selected must be relevant to the problematic situation faced and adequate to give reasonable warrantability to the action contemplated. There is no means, a priori, of making certain of adequacy and warrantability. This is true of any policy adopted under uncertainty.

Take, for example, an industry making a new kind of product for which there exist no demand-experience factors. How fast should the capital equipment be amortized? If too fast, then the price asked for the product may be too high; if too slow, the demand may be satiated—or a competitive product developed—before the amortization is completed. At either extreme the business may go bankrupt. In the absence of experience, the setting of any amortization rate is a gamble (obviously in such a situation the industry attempts to amortize its equipment as rapidly as possible consistent with maintaining a sound financial balance sheet). Usually there is some information available to narrow the choice, although the history of American business is full of some bad choices (the railroad industry, for example). The selection made by successful business determines precedence, i.e., those businesses making poor policy decisions go bankrupt and the poor policies die with them.

To a greater degree, the selection of a particular object construction is a gamble which must be made as a starting point (i.e., an hypothesis) and is to be revised and improved as experience develops.

This was not realized by Wald when he published Statistical Decision Functions, for he wrote [16]:

" Given (1) the stochastic process $\{X_i\}$ (2) the class Ω of distributions which is known to contain the true distribution F of X as an element. . . ."

The point is, in the view of the conceptual mode of inquiry, that it is not even possible to know that a class Ω contains a given distribution or that a given distribution is the "true" one. The "true" distribution has meaning only in the context of the hypothetical object system. It becomes an abstract parameter. A probability hypothesis is not meaningful, moreover, unless the object construction contains a prescription for supplying numbers for these abstract parameters on the basis of the finite knowledge available. This prescription is also adopted on the basis of policy (an intensive operation), initially unwarranted—a part of the general hypothesis of the object construction—and subsequently either acceptably warranted or rejected in favor of a more warrantable hypothesis.

This is the interpretation of the Bayesian method which makes most sense. It is to be expected that some hypotheses selected heuristically as a basis for the Bayesian method will not be fruitful. That some hypotheses are fruitful is the amazing observation. A proposal to test the Bayesian method [3] by pitting a "non-Bayesian" scientist against a "Bayesian" scientist in a probability estimating contest is just as meaningless as the notion of testing a follower of Berkeley versus a common-sense Ben Jonson by staging a stone-kicking contest.

From the conceptual viewpoint one is testing, by such a contest as that suggested by Bross, merely the relative warrantability of two object constructions, and not their respective metaphysical controls.

The warranting of the metaphysical controls of the process of selection among object systems can be accomplished only at a much more abstract level, and then only on the basis of evolutionary trends.

Construction of Object System

The construction of an object system may begin with bare statements such as Newton's laws of motion: $F = ma$, and the statement of the conservation of momentum. These statements structure the constructs "force" and "mass," prescribe a measurement process for determining their magnitude quantitatively, and thereby lay a foundation for a science of mechanics. However, the Newtonian theory also made use of more elemental constructs—space and time—which were accepted without examination. Usual definitions of a stochastic system similarly introduce constructs which are presumed to be acceptable without examination. These are the construct "state" and the construct "probability of transition from one state to another." The state of a system is defined by a set of numbers $\{x_i\}$ constituting a vector, x. These numbers represent the quantification of a set of qualities. The quantifications are definable in a continuous interval, on a discrete set, or in a yes-no sense representing the presence or absence of the given quality. The qualities making up the collection have been structured (constructured) and selected specifically with the utilization of the object system in mind. The goal of the designer is to include all those qualities "important" to the objective of the analysis. "Important" here means all qualities to which the outcomes of the process are sensitive—either predictive or prescriptive. The structuring of the basic space of the object model is part of the hypothesis of the object system. There exists no a priori means to ascertain its adequacy, relevance, or completeness. These properties may be demonstrated, to a degree only, in the warranting process.

There are, however, ways of indicating adequacy. One is a "sensitivity analysis." If the outcome (prediction or prescription) is not affected by wide variation of a given quality, then that quality could have been omitted. On the other hand the omission of an important quality cannot be demonstrated except through failure of the cognitive model. Intimations of such deficiency sometimes occur. If the outcomes of the process are repugnant "intuitively"—i.e., on the basis of a more general but less well defined cognitive process—without the introduction of "intrinsic" or "intangible" factors, it is usually symptomatic that the object model has failed to include some factor important to its conclusions.

This is merely a symptom, however; its absence is no warrant that an inadequacy does not exist in the object system. The conceptual interpretation leads to an awareness of this constant possibility.

Probability. Next it is common to introduce the notion of a "transition" from one state to another. This transition is a quantitative change in any one or more components of the state vector. We shall alternatively represent the state vector by the lower case letters x, y, z and the time at which these states are occupied by r, s, and t. The vectors x, y, z represent the same set of states, different letters being used to differentiate between particular states. Thus a transition from a state x at time r to a state y at time s is represented by

$$(x, r) \longrightarrow (y, s),$$
$$s > r.$$

Whenever several states (y, s) may occur following the change from state (x, r) it is common to introduce the notion of a conditional probability, $k(y, s | x, r)$, which is read: the conditional probability that the system S is in a particular consequent state (y, s); given that a transition occurred at a particular antecedent state (x, r) (where r>s). The system S is defined as that collection of states connected by non-zero conditional transition probabilities.

This formal schema for an object model therefore assumes that the model is adequate in that (a) the collection of possible states of the model has a one-to-one correspondence with states in the world of experience, the collection therefore being complete insofar as states "important" to the conclusion reached by use of the model in the cognitive operation; and (b) the conditional transition probabilities are known with precision, i. e., that one knows the "true" probabilities. Such a formal model is termed "stochastic definite."

Many investigators have noted the inadequacy of such a form schema for use as the structure for an object model. In particular Reichenbach [[illegible]] has been vocal in depreciating the notion that knowledge will even become "true;" and he has pointed out that the logical consequences of this idea have not been sufficiently realized. Among recent publications, Churchman [4] calls attention to the logical problems involved in setting up a measurement of probability. He offers no solution, but concludes that the science of measurement has not been adequately formulated.

The use of a formal schema involving a concept of known or "true" probability is logically untenable for the same reason that all supposedly "true" constructs must be rejected. One can never demonstrate that he is in possession of the "truth." A construct may be shown to be sufficient, i. e.,

consistent with past observations, but its durability—its sufficiency in the future—is open to doubt. Furthermore the necessity (uniqueness) of any construct or hypothesis cannot in principle be demonstrated. Hence any formal schema utilizing a concept of true probabilities is essentially meaningless, since there exists no prescriptive for measuring these true probabilities.

Baye's decision theory is interpretable from the conceptual viewpoint. The structure of the particular object model, and its incorporation into a general hypothesis, are used to define a "measurement" process by means of which finite information is transformed into quantitative entries into the formal schema. Thus the measurement of a probability depends on the context of use. What, from the postulatory viewpoint, was a "true" probability now becomes merely a parameter of the theory which is dependent on observations for the degree of warrantability associated with the overall object system.

For example, consider a particular situation wherein it is desired to form a predictive object system. The first policy decision is to select a format for a predictive theory. Further assume that this decision resulted in the selection of a simple two-transition model. Call the consequent states A and B, i. e., the system moves from some initial state I to either A or B. Assume that a number μ of such transitions have been observed ($\mu \geq 0$) and of these λ_A were the transitions ($I \rightarrow A$), and λ_B were ($I \rightarrow B$), $\lambda_A + \lambda_B = \mu$. The second decision—on the basis of policy again—to be made concerns the significance of the order of the observation. In cases of μ very small it is generally believed [and sometimes plausible arguments may be advanced] that the safest policy is to assume that the order is random. One posits a parameter $P\{A|I\}$ (represented by p_A) such that the observed sequence, g, would result on a random or uniform basis:

$$P\{g\} = \frac{\mu!}{\lambda_A!\lambda_B!} p_A^{\lambda_A} p_B^{\lambda_B} \quad .$$

This may be interpreted (another policy decision) as a weighting distribution, $F(p_A, \lambda_A, u)$ for an estimate of p_A (or p_B). Normalized, F becomes

$$F(p_A, \lambda_A, u) = \frac{(\mu+1)!}{\lambda_A!\,\lambda_B!}\, p_A^{\lambda_A} p_B^{\lambda_B} \,, \tag{14}$$

where $\begin{cases} \lambda_A + \lambda_B = \mu \,, \\ \\ p_A + p_B = 1 \,. \end{cases}$

and

The action (prediction) produced by the two-state theory should then be such that estimates of k_A from 0 to 1 have been weighted by the function F. In linear cases this will result in the estimate of k_A given by the first moment

$$k_A = \bar{p}_A = \int p_A F(p_a, \lambda_A, \mu)\, dp \,. \tag{15}$$

Under the hypothesis of the present example

$$k_A = \frac{\lambda_A + 1}{\mu + 2} \,. \tag{16}$$

This estimate is of interest for several reasons: (1) it depends on the formal nature of the particular object system chosen, (2) it produces a number for use in the object system for all states of information from minimal ($\mu = 0$) to "certainty," i.e., $\mathrm{Lim}\, \mu \to \infty$, and (3) the estimate of k_A is never zero or unity. This latter is a consequence of the number of possible states posited by the object model. An indefinite run of A outcomes observed would never result in an estimate of k_A of unity and would be only asymptotic to unity—as long as the original two-transition hypothesis is followed. The "induction" that $k_A = 1$ in such a sequence involves the abandonment of the two-transition object-model in favor of a deterministic transition object-model. As data accumulates, other hypotheses may be tested—particularly the hypothesis that the outcomes are random with a constant probability.

There are several trends to be gleaned from this discussion. One is that a statement about the probability of an event begins with a minimum of information and is primarily a policy—or a value. ("Policy" and "value" will sometimes be used synonymously, since the two are essentially transforms.) Such a probability has a minimal component of fact—primarily that rudimentary knowledge which led to a selection of a particular formal model as an appropriate format for a theory. As data accumulates, the factual content increases until, in the limit, it becomes almost all factual. This characteristic is, we believe, shared by all constructs. The old philosophical argument

relative to value and fact is resolved by the complementary approach: looking forward into the uncertain and unpredictable future, all constructs are predominantly of the nature of value; looking backward at the record all constructs are predominantly (but never wholly) factual in nature. At the present both kinds of constructs will be employed. Our current language adequately describes the present, but becomes less applicable as one projects into the future or past. In the stochastic definite model under the postulatory interpretation, a problem projected into the future forces one to use a probability parameter as if it were factual, whereas it may be primarily a policy (value). Conversely, in a value-science, the values an investigator deduces as being held by a subject whose decisions are a matter of record, are primarily "factual" in nature; although we shall see how far from the values held introspectively by the subject these extrospectively deduced values may be.*

Another trend to be observed is the dependence of the "factual" nature of a construct upon the object-hypothesis. The object hypothesis, in effect, determines a prescriptive for the admission of data as "factual," i.e., as pertaining to the object-model under development. It determines, in effect, the characteristic of the filter which admits—or rejects—raw data.

Thus a "fact," as well as a construct; is contextually meaningful (with respect to object-model, state of knowledge, and temporal projection. Abstracted from the context, its meaning is lost or becomes ambiguous. It is this non-contextually anchored usage of the terms "value" and "fact" that leads to the fruitless arguments in the history of value theory [and even in present day literature] as to the nature of value vs. the nature of fact.

Another characteristic of this approach is that the concept "true probability" was replaced by a parameter of the model, p_A, which had a formal status only and which does not appear in the final object-model as one of the quantities which are manipulated.

Modification of the Stochastic Model

We are now in a position to modify the standard approach to the use of stochastic definite models. The conditional probability $k(y, s \mid x, r)$ that the system S is in a particular consequent state (y, s), given that a transition occurred

* I am sure that something like this was in the mind of one of my postulatorily inclined colleagues who, when remonstrated with for leaving a class of facts out of a current research project because he could not intuitively accept the consequent conclusions, rejoined with, "Facts are among the least objective things there are!"

at any antecedent state (x, r), is no longer presented as a true or known probability; it is replaced by a more general construct: Given (1) that in the context of the structuring represented by a system S of states represented by the vector x, a given transition from (x, r) to a (y, s) has occurred $\lambda(x, y)$ times out of a total number, $\mu(x)$, of transitions observed to occur from the state x, and (2) a particular object-structure designed for a specific action purpose—and through it a rule $R(\lambda, \mu, S)$ for evaluating a transition probability from the data λ and μ available and the number or density of states. The transition probability may be expressed by $k[y, s \mid x, r; R(\lambda, \mu, S)]$. We shall shorten this notation to

$$k(y, s \mid x, r; R)$$

where R represents the appropriate rule for converting finite data into estimates of k, and where, of course,

$$\sum_y k(y, s \mid x, r; R) = 1.$$

Since k is an element of a square matrix K we shall, for simplicity of notation, write this as

$$K(s \mid r; R).$$

The Chapman-Kolmogorov Equation

The Chapman-Kolmogorov equation (hereafter referred to as the C-K equation) represents a fundamental restriction on the class of formal schema selected as exploitable for a theory of value. This is a condition which states that the probability of a transition from a state (x, r) to one (z, t) does not depend upon the path taken through some intermediate state (y, s):

$$k(z, t \mid x, r; R) = \sum_y k(y, s \mid x, r; R)\, k(z, t \mid y, s; R), \qquad (17)$$

$$t > s > r,$$

which we may also write, in matrix notation:

$$K(t \mid r; R) = K(s \mid r; R)\, K(t \mid s; R) \qquad (18)$$

There are two ways in which the C-K equation may be utilized: either the initial state (x, r) may be fixed (it is assumed that the system is known to have been in state x, r)—or the final state (z, t) may be fixed. The first usage, allowing the subscript ($_0$) to represent the fixed known state, is then

$$K(t|r_0; R) = K(s|r_0; R)\, K(t|s; R), \qquad (19)$$

$$t > s > r_0,$$

and is called the "forward" equation. The second usage yields

$$K(t_0|r; R) = K(s|r; R)\, K(t_0|s; r), \qquad (20)$$

$$t_0 > s > r,$$

and is called the "backward" equation.

The two equations are adjoints, or duals and the backward equation, yielding the probability of reaching some ultimate state from a field point is always interpretable as a value equation.

The forward equation is often written in the form

$$P(t) = P(s)\, K(t|s; R) \qquad (21)$$

$$P(r_0) = \delta_{x, x_0},$$

$$\text{where } P(t) = K(t|r_0; R), \qquad (21.a)$$

gives the probability of finding the system in a state at time t given the probability distribution of state at a prior time, s. P(t) is a (row) vector and the backward equation

$$P^+(r) = K(s|r; R)\, P^+(s), \qquad (22)$$

$$\text{where } P^+(r) = K(t_0|r; R), \qquad (22.a)$$

gives the probability that a state at time r reaches a given terminal state at t_0; given the probability that it would reach the terminal state at t_0 from a state at a later time s; i.e., $t_0 > s > r$. The forward equation in this form we call the "state equation."

The backward equation is also referred to as the "adjoint equation." It may be constructed into a "value equation" by weighting the terminal states at t_0 by a set of arbitrary weighting functions, $U(t_0)$, which are fundamental posits of value of these states at this time, i.e., we construct a value (vector)* function $Q(r)$:

$$Q(r) \equiv U(t_0)\, K(t_0 \mid r; R). \tag{23}$$

Substituting in the backward equation there results

$$Q(r) = K(s \mid r; R)\, Q(s), \tag{24}$$

where $s > r$.

This equation we refer to as the "value equation."

The Value-Decision Object Statement

There remains one more modification to be made. This is the introduction of a "decision space." The transition probabilities are posited to depend on certain actions, α, taken by the subject. The selection of a particular action has associated with it a particular transition matrix

$$K(s \mid r; \alpha; R).$$

The values of K may be defined for a discrete set of actions, or for one selected from continuous line segments in decision space; or, in the extremely limited case, the action involved may be merely the acceptance or rejection of the object model; or, a more complicated situation is found in the case of a competitive manipulation of the decision space by two players acting independently to optimize their own value systems. Player I selects from a space α, player II from a space β.

We may now rewrite the C-K, state, and value equations as follows:

$$K(t \mid r, \alpha_{12}; R) = K(s \mid r; \alpha_1; R)\, K(t \mid s; \alpha_2; R) \tag{25}$$

$$P(t, \alpha) = P(s)\, K(t \mid s; \alpha; R) \tag{26}$$

$$Q(r, \alpha) = K(s \mid r, \alpha, R)\, Q(s) \tag{27.a}$$

$$t > s > r.$$

* The value vector, a set composed of a scalar value function for each state, should not be confused with vector-values, which are multivalued for each state.

In the equations above the decision space C-K equation has been changed from a one-stage to a two-stage decision space; it being assumed that for a time variable τ,

$$\alpha_{12} = \alpha_1, \; s \geq \tau \geq r$$

$$= \alpha_2, \; t \geq \tau \geq s.$$

The construction is finally completed by the addition of a decision process $\underset{\alpha}{D}$—or algorithm—which prescribes a way to select a particular action from among the set α; yielding the "value-decision equation:"

$$Q(r) = \underset{\alpha}{D}\, K(s|r,\alpha,R)\, Q(s), \tag{27.b}$$

where the decision operator acts on all appropriate selections to its right. This equation represents a class of object constructions for a value theory. It simultaneously structures the material and decision space, introduces a construct "probability" and a means to measure it from limited data, introduces a construct "value," and provides for a decision operator which selects both a unique course of action and associates with it a unique value. The state equation represents an object statement for a purely predictive system—essentially a construction of the model represented by the transition matrix alone. The value equation represents an object statement for either a value-science or a prescriptive value-theory.

Used as a value-science the inputs are the observed actions (decisions) of some external system. The value-scientist attempts to select those decisions for which he believes he can predict the subject's cognitive predictive theory (i.e., the model employed by the subject). A nearly deterministic situation makes this selection more reliable. Next the value-scientist posits a set of ultimate values $Q^*(s)$ which he sets up as hypotheses of the norms which he thinks the subject may be following. The scientist tests his theory of the subject's value-decision process by predicting the subject's future actions. The theory is modified as more decisions made by the subject are available for analysis. The value-scientist constructs a class of object statements by the general statement: "decisions imply values," which we symbolize

$$D \supset V^* .$$

On the other hand, the complementary usage of the value-decision equation is in the inquiry into norms for one's own (i. e, the self-system whether an individual or an organization) decision purposes. A set of values are posited as a policy for action, as well as a decision algorithm $\underset{\alpha}{D}$. Thus the "axiologist," as we call him, has selected a class of object statements constructed by the statement: values imply decision, symbolized

$$V \supset D.$$

The inverse operation to this process is the statement: decisions are presupposed by values, symbolized

$$D | V.$$

The value-scientist has sometimes sought to identify the process

$$D \supset V^* .$$

with that of

$$D | V$$

and assumes that the values posited in his theory of the subject's behavior are those actually followed as norms by the subject's own intensive cognitive process Such an identification cannot be established. The value-scientist cannot be sure that he has assumed the same model as that held by the subject, for the same decision algorithm. He has a small number of decisions to work with as data in a probablistic process. Lastly, he has no means to know, in advance, whether the subject has made a modification of his value system.

Complementarity of Axiology and Value-Science Object Systems

We are now in a position to note more explicitly the complementary nature of axiology and science as illustrated in the object systems. The state-equation (26) looks to the past for its initial or boundary magnitudes; and predicts the probability of a given state in the future— i. e., it is backward looking and forward moving. The value-equation (27. a) or (27. b) conversely looks to the future for boundary or initial conditions and is solved backwards in time to the present—i. e., it is forward looking and backward moving.

The control of the predictive science includes tests for consistency, tests of the design of measurement—i. e., critical scrutiny of the norms or

prescriptives for selection of data and measurement. The control of the primarily extensive activity is then intensive in nature.

The value process to a degree looks to the predictive science to warrant its model. Since the purpose of the intensive value process is prescriptive within a given category of decision problems, the model may be permitted to be poorly predictive in areas not important to the category of decision problems. However, the principle inputs into the value process, the set of boundary values $U(t_0)$ (see Equation 23), are adopted arbitrarily as an act establishing policy. The policies so adopted are, however, warranted empirically. The ultimate test of a value system is the evolutionary principle of selectivity through survival.

The control of extensive inquiry is intensive; that of intensive inquiry is extensive. Thus, extensive and intensive inquiry do not stand apart as independent sources of knowledge, but through the interlocking of their controls provide balance and completeness to the cognitive process.

A value system is not tested wholly by the principle of survival; but there are lesser tests which do not involve survival. These tests are learned by experience to indicate the soundness—i.e., adaptability—of an organization with respect to unpredictable stresses. A business doesn't necessarily have to go bankrupt before its policies show up as poor (although this does sometimes happen). A financial balance sheet will show insolvency and sometimes indicate the risks management is taking. Lastly, the control of (self) survival is not the only empirical control. The value process of every system is controlled by three types of empirical norms. These are those of the self-system (total optimization or executive-control), those imposed by sub-systems which are component parts of the system (sub-optimization or legislative control) and control exerted by the society of systems to which the subject system is a member (social, environmental, or judicial control). These controls serve to interlock the value structure of the respective hierarchy of selective systems which make up a system.

Conflict

Another requirement of a value-decision system is the avoidance of ambiguity. It may sometimes happen that the area of application of two object-decision models overlap. In this area problematic situations may arise in which the actions indicated by the two decision models are not the same. This is a typical conflict. We have, in an earlier publication [12] recognized three processes of resolution of conflict:

1. **dominance—suppression**
2. **schism**
3. **concrescence.**

Dominance—suppression resolves the conflict by repudiating the values in one system in the overlapping area and accepting the norms of the other. This is a special case of the second process: schism. A schism introduces a separation of the overlapping areas of the object models and in one sub-area the values and object-model of one system dominate, in the other sub-area the values of the other object-model dominate. Both of these processes require a higher authority to enforce the dominance of one system over another. Although this process seems essentially destructive, it is however the process by which organizational action takes place. An executive defines areas of responsibility and authority to his subordinates. His overriding authority is used to enforce the schisms built into the organization. Schism is the basis of jurisdictional divisions of the various city, county, state and federal governments.

A third means of resolution is a creative process, given the name of "concrescence" by Whitehead [17], and "dialectic" by Hegel . The presence of conflict produces a tension which motivates a creative reconstruction of the overlapping object-systems, replacing them by a single object-system whose area includes the areas of those systems replaced. New value hypotheses are also introduced such that a scalar value structure is restored. Through concrescence the comprehensiveness of a progression of object-systems is continually increased.

It is natural that extensive inquiry into values—value-science—should be the last to discover the process of concrescence. The demonstration that a subject's value system has undergone a creative reconstruction requires a maximum of knowledge. Such a change is buried in stochastic variation, mixed strategies, and in theories of the subject's values having low warrantability. As a result, the empiricists (Churchman [4], Pepper [9]) among the value-scientists give no attention to this process.

The first great problem faced through intensive inquiry, however, is that of conflict. In searching his own norms for decision the decision-maker becomes acutely aware of ambiguity, and its resolution is to him his most compelling problem. Thus the formal creative process of concrescence is first to be illuminated by intensive inquiry.

The Principle of Invariance *

There are formal controls of intensive inquiry—as we have seen in the avoidance of ambiguity by avoiding conflict. Another formal control which will place restrictions on acceptable decision processes (and is one of the class of controls which avoids ambiguity) is that action indicated in the value-decision process shoud be independent of the procedure of implementation of the decision process. This restriction will further require that <u>the form of the value-decision equation must be invariant to a C-K transformation.</u>

To explain this principle first consider the C-K equation and the value equation without the decision process or decision space:

$$K(t|r;R) = K(s|r;R)\,K(t|s;R) \tag{28}$$

and

$$Q(r) = K(t|r;R)\,Q(t) \tag{29}$$

$$t > s > r.$$

Applying the C-K transformation, Equation 28 to Equation 29, there results

$$Q(r) = K(s|r;R)\,K(t|s;R)\,Q(t),$$

but

$$Q(s) = K(t|s;R)\,Q(t), \tag{30}$$

therefore

$$Q(r) = K(s|r;R)\,Q(r).$$

The form of the value-equation has remained unchanged when undergoing a transformation in time (the C-K transformation).

Now taking the C-K equation (25) and the value-equation (27.b) and going through the same process one gets

$$\left.\begin{aligned} Q(r) &= \underset{\alpha}{D}\,K(t|r;\alpha;R)\,Q(t) \\ K(t|r,\alpha;R) &= K(s|r;\alpha_1,R)\,K(t|s;\alpha_2,R) \\ Q(r) &= \underset{\alpha_{12}}{D}\,K(s|r;\alpha_1,R)\,K(t|s;\alpha_2,R)\,Q(t) \end{aligned}\right\} \tag{31}$$

---

* The material in this section was presented 15 November 60 to an informal seminar calling itself the Joint Research Group on Military Resource Allocation Methodology [6].

The transform no longer leads necessarily to an invariant form of the value-decision equation. A change of form opens the possibility that the value Q(r) and the action α_2 or α_{12} may depend on the staging of the decision analysis— i.e., the procedure of application of the decision process.

Let us require that the form of the value-decision equations be invariant [we desire action to be based solely on the substantive and not procedural factors in the problematic situation]. This is equivalent to the requirement that the decision operator $\underset{\alpha_{12}}{D}$ be distributive: $\underset{\alpha_{12}}{D}$ is restricted to those decision processes for which

$$Q(r) = \underset{\alpha_1}{D}\, K(s|r; \alpha_1, R)\, \underset{\alpha_2}{D}\, (K(t|s; \alpha_2, R)\, Q(t) \tag{32}$$

it being understood that each decision operator acts on all components to the right of it.

Now we may define

$$Q(s) = \underset{\alpha_2}{D}\, K(t|s; \alpha_2, R)\, Q(t)$$

and there results

$$Q(r) = \underset{\alpha_1}{D}\, K(s|r; \alpha_1, R)\, Q(s) \tag{33}$$

and invariance is preserved.

Since each decision operator acts on all components to the right, it is necessary to perform the (temporally) last operation first, etc. This also has the effect of reducing the range of decision space examined from that of the α_{12} space of Equation (31) to that of an $\alpha_1 + \alpha_2$ space. This (Equation 31) is the principle of optimality of dynamic programming.

It is necessary to prove that a given decision algorithm permits the distribution according to Equation (32). R. Bellman [1] and others have assumed this distribution as intuitively obvious, and through use has demonstrated its application to simple optimization and minimax principle of game theory. But this property is not a necessary property of any decision algorithm. We have proven it for simple maximization. D. Blackwell [12] has proven it for the minimax algorithm, but it is conceivable that some of the decision processes in vogue do not permit this distribution.

Boundary Values

In the foregoing optimization procedures it is to be noted that there is no longer a concept of "the optimum," inasmuch as warrantability of the predictive model is constantly being modified by additional data operating through the rule, R, for estimation of probabilities. "The optimum" is an idealized concept like the "true" probabilities.

The foregoing treatment of an object-theory of values leaves some problems dangling. Present time values in an object-model are referred to values over future states. These, in turn, are referred to more remote future states, etc. Can this process be terminated? The value-decision equation successfully refers the internal values (oddly enough called "extrinsic values" by philosophers] of a system to a set of external values [called "intrinsic values"—to the particular object-system—by philosophers] embedded in the larger domain of object-systems. These are the final set of boundary values at which the value referencing terminates.

Optimization theory, which includes among other techniques fixed time programming (linear and nonlinear)—and time dependent (dynamic) programming, is concerned with the methodology of problem solving. In this section we shall be concerned with some aspects of the less tangible side of the problem involved in setting the boundary values for the value-decision equation. Although this equation refers present values to future values, it is solved by calculating backwards. This same procedure is involved with the application of the principle of optimality. Thus a dynamic program involves the same problem as does value theory—how far into the future is it necessary to project one's analysis?

The present discussion can be clarified by expanding the value-decision equation into an n-stage form

$$\left.\begin{aligned} Q(r) = {} & \underset{\alpha_1}{D} K(s_1|r;\alpha_1;R)\, \underset{\alpha_2}{D} K(s_2|s_1;\alpha_2;R)\ldots \\ & \ldots\ \underset{\alpha_n}{D} K(t|s_{n-1};\alpha_n;R)\, Q(t) \\ \text{where} \qquad & r < s_{i-1} < s_i < t \quad \text{, and} \end{aligned}\right\} \tag{34}$$

where the time interval r to t $(t > r)$ has been divided into n stages by the interpolation of $n-1$ intermediate time divisions s_i. In Equation (34) the

decision space of each stage is assumed to be independent, i. e., the decision is made independently on the appropriate strategy for each stage. The decision operators represent a sequence of decisions made relative to future strategies—except D_{a_1} which is the decision relative to the immediate strategy. This decision, D_{a_1}, shall be called the "current decision"—those relative to future strategies shall be called "projected decisions."

Let us note, first of all, that only the current decision is irrevocable—the projected decisions are included only so that the current decision will be made with due regard for its consequences on future actions. It is not the intent in dynamic programming to fix the entire set of future strategies, but only to fix the immediate current strategy in such a fashion that long term benefit will not be revoked purely for the expediently maximum present benefit. It is the intent to make only those decisions which must be made as an optimal balance between timliness for implementation, and warrantability of the object-model. As a decision is delayed, the object-model relating to it becomes more warrantable—on the other hand, the degree of success of the implementation of the decision goes down, and its cost goes up as the decision is delayed.

The information applicable to the current decision is more detailed, in greater quantity, and more reliable than for projected decisions, It is observed that the projection into the future degrades the warrantability of the decision process in several ways:

(a) the estimate of the transition probabilities contains less factual content, and hence has lower warrantability. In the limit of minimal factual content the rule suggested above would consider the transitions of uniform probability. Thus the model degenerates into a random walk as projection is made into the remote, unpredictable future.

(b) the warrantability of the formal properties of the object-model (its construction, dimensions, number of states, possible (non-zero) transition, etc.) degenerates. Innovation, technological (extensive) or ideological (intensive) may change the nature of the object model in either construct space or decision space before the time period it represents becomes current.

The degeneration of the cognitive process through factual degradation as projection is made into the future is an inherent characteristic. As the applicable model takes on more and more of the characteristics of a random walk the projected action has less and less effect on the outcome—until, in the limit, action (decision) becomes totally decoupled from the valuation process. The importance of this observation is that the onset of degradation provides a natural termination of the continued forward referencing of the evaluation process.

PART VI

NORMATIVE SYSTEMS THEORY AND ANALYSIS

Chapter 14

OPTIMAL ORGANIZATION

The task of this chapter is to search for an adequate way of identifying and defining a completely general organizational norm. The term "optimal" implies the existence of some normative measure--a degree of overall goal attainment or of avoidance--such that this measure is extremalized by the behavior of the organization[1] as a whole. In strongly polarized organizations, the identification of such a norm appears to be comparatively easy, at least in a superficial sense. A business corporation seeks to maximize profits; an athletic team seeks to win the game; a military task force seeks to attain its assigned objective. Under analysis, however, these notions turn out to give only a simplistic account of overall goal-seeking behavior. A corporation must remain viable--it must stay in the "black"--as a condition of being able to strive for maximal profit. Further, the corporation's strategists must decide to what degree they shall opt for future profits by deferring immediate gains, even by expanding present resources.

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1. "Organization" is meant here in reference to a system of functionally inter-related elements, whose behavior with respect to a changing environment is controlled by internal norms associated with the viability of the system as a whole.

The athletic team will have objectives other than winning: to teach sportsmanship and teamwork, to develop the determination and physical stamina of players--or if a professional team, to maximize profits. The military unit may seek to win under conditions in which it inflicts minimal injury to itself, or minimal damage to the enemy, or even to convert the enemy to nonbelligerent behavior.

Even the most elemental of organizational norms--the probability of staying alive--when looked at in temporal sequence, is not a simple one. It is the nature of the valuative process that it is oriented toward the future. One makes those decisions today that maximize the chances for survival tomorrow. The surviving states of the organization tomorrow are not all of the same valuation; some may lead to ruin during one of the immediately following time periods, others to a long history.

Although a nation may enter a war solely valuing the winning of it, as victory becomes probable it begins to be sensitive to the difference between winning states. It is no longer indifferent to the level of casualties or damage to itself. Hence the decision is influenced by the threats which may be encountered in the future temporal sequence--ad infinitum.

If there is anywhere in the future the certain death of the organization (organism), values projected only on survival will collapse. The capability of decision becomes trivial, since all actions lead to the same death state. All actions become evaluated as equal under the realization of mortality. Survival as a sole organizational norm is equivalent to the presumption of immortality (i.e., a finite probability of indefinite survival). An individual--or an organization--committed

to survival as a sole norm may undergo a traumatic loss of the power of decision when confronted with the sudden realization that its survival is not indefinitely probable.

Ultimately, then, an organization whose norm is that of survival may in fact act counter to its own norm in the face of imminent danger to itself. It may act "irrationally," i.e., inconsistent with respect to values it has professed to follow. In the face of danger it may lose its value system and decisions may be made haphazardly. Propagandizing the invincibility of a military force among its men is not conducive to its stability under fire. It may be subject to the catastrophic loss of morale under a moderately severe test. The harsh reality is that such a force must accept (as a whole and as individual men) the concept of its own expendability before it can become a truly effective and dependable force in battle.

An individual so oriented in his values will seek to perpetuate the relationships in his personal world. When faced with a loss by death of any of his friends or family, or the loss of a personal relationship by separation, or the maturation of his children, or the threat of the loss of a relationship by the presence of an unfamiliar third party, his reaction is one of anger--the world that is rightfully his is being diminished. The more mature individual would have accepted the reality of the inevitable termination of his relationships. His reaction in the face of disaster is one of acceptance. He reorients his life in a changed situation.

As suggested by these examples, the concept of a generalized organizational norm must successfully cover many complicated particulars.

In support of the thesis that the concept "optimal organization" can serve this function as the most general of organizational norms, we must accept the burden of proposing a constructive specification of the concept in terms of a <u>measure</u> of optimal organization.

ORGANIZATIONAL PARADIGM

These reflection are made in the context of the paradigm presented in Figure 14-1. The characteristic structure of organization is a three-level hierarchical system. An organization is conceived as a <u>holon</u>[1] which is partitioned into interreleated elements, such element of which is itself a holon when viewed from sublevels of organization. Furthermore, the total organization exists in a context (environment) in which it may have a subordinate role, i.e., it is itself related to other holons. Particular collections of such holons constitute a superior level, or "superordinate." We refer to the self level of the organization as the "idio-ordinate," to its partitioned elements as "sub-ordinates," and to a particular larger community as a "super-ordinate." The organization is characterized by the presence of norms unique to each holon. Furthermore, an action (or decision) made at the idio-level is always determined by more than the norms appropriate to that level alone (the idio-norms).

Three kinds of decision can be taken. <u>Reaction</u> consists in the modification of the norms of sub-ordinates. As examples, a military

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1. So far as we are aware, Arthur Koestler initially developed this use of the term "holon" (National Institute of Health, Seminar on Organization Theory, 1967). Whatever the facts as to its origin, we use the term in the sense he then proposed: a sub-assembly of an individual-organizational whole (i.e., a sub-whole) which, relative to its own components, is capable of at least quasi-autonomous behavior.

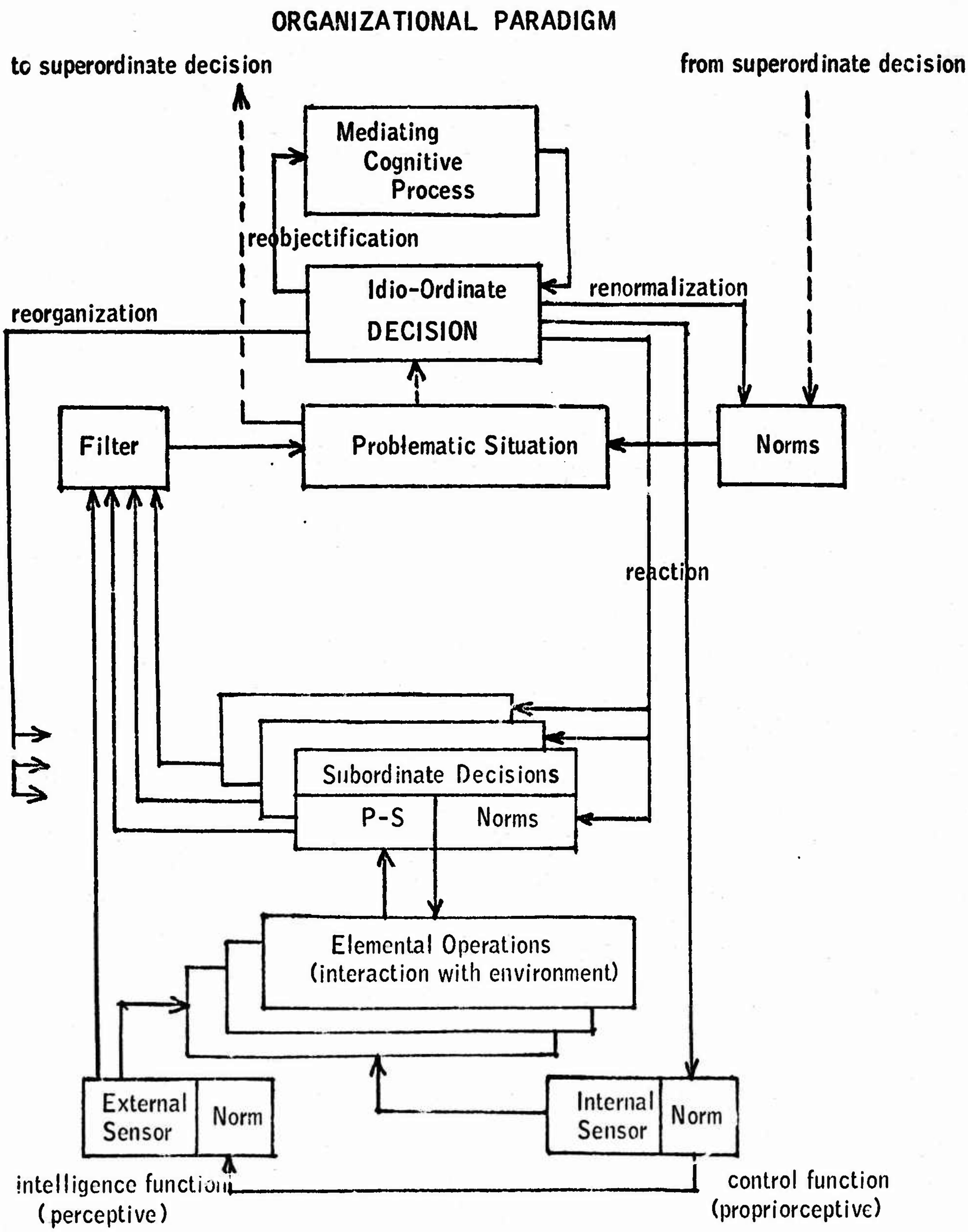

Figure 14.1 Schematic: An Organization as a Normative-Adaptive Decision System

unit defines the "objectives" of its elements and they act under their own drive to accomplish the goal set by higher authority; we turn up a thermostat and the furnace system operates under a new norm to raise the temperature of the room. These subordinate elements may serve to effect interactions with the environment (operating or "motor" function) or be sensitive to a level of intensity of inter-action with the environment (sensory or intelligence-gathering function). Interaction upon--or sensitivity to--the environment is always accomplished at some elemental level.

The norms of sub-ordinates determine the realizable stresses which the idio-ordinate can superimpose on them by modifying the sub-ordinate norm. The military element may run into unacceptable casualties and force a modification of its defined objective. Our arm may encounter stress leading to such pain that we reduce our demand upon it; the furnace may overheat because of our demand for warmth and cut itself off (a feature designed to avoid damaging the furnace). These stress-limited overriding controls imposed by sub-ordinates may be supplemented by an internal system controlling the internal stresses or rates of utilization (e.g., heartbeat, respiration, body temperature--or in a business corporation, the control of rates of expenditure is delegated to a comptroller).

In a second type of decision, an idio system may neutralize the self-motivating drive by a modification of the self-norm, i.e., by <u>renormalization</u>. The objective may simply be changed in the face of continuing failure to achieve it. In a similar manner the norms of the idio-ordinate may be modified by influence of the super-ordinate. These may be cultural, legal, social, professional, supervisory or

biological super-ordinate norms, depending on the system under discussion.

The decision process itself is a "mediated cogitive" process; that is, the idio system constructs--or "objectifies"--a model, a reduction of its universe of experience of the situation. It then projects the consequences of its alternatives and selects a unique action from among these alternatives. This mediated process may fail to find a feasible alternative, i.e., an alternative meeting all of the imposed constraints. A third kind of decision may now be required: a decision to reorganize. Reorganization may be made in the elemental partitioning of the organization itself, but only if the model of the decision process is itself reorganized or, as we shall say, "reobjectified." We regard any functional organization of people in a social or business institution as representing essentially the theory or model (of the higher managers) of the organization's decision process itself.

The states, or conditions, of the sub-ordinate elements are either of external conditions (reason) or of internal conditions (proprioceptive) and are communicated to the idio level through a filter. The characteristics of the filter are determined in part by the structure of the organization (what kind of thing it is) and, in part, is under the control of the idio system. To a degree the idio system determines the signals to which it will be responsive.

A significant difference between the filtered indication of the internal and external state of the organization and its norms constitutes a "problematic situation." The determination of significance is itself a norm or strategy of the idio-system. The existence of a problematic situation is then the motivation for decision.

Motivating Problem: Possible Organizational Norms

In this chapter we are focusing attention on the idio ordinate norm(s). What does an organization seek to develop most fully? What norms result in "optimal" behavior?

Let us now identify and analyze some of the norms which have been used or proposed as appropriate values of an organization. These have been listed in Table 14-1 in a rough ordering that might be associated with their sequential acquisition by an organization.

The minimal or lowest ordered values is that of survival. Obviously, before any other goal can be achieved, the organization must be in existence. Yet we have seen that indefinite survival, in itself, is not a usable value in the long run. However, let us confine our attention to a time span short with respect to the normally expected lifetime of the organization. A corporation, a human individual, a nation (alas, even a committee) seeks to perpetuate itself. Whenever the probability of survival is significantly less than certainty (again the measure of "significant" is a strategy or norm itself), survival may dominate the value structure. A nation at war acts as if any survival state is equal to any other as long as non-survival is significantly probable. Whenever the probability of survival is judged (significantly) near certainty, then that nation acts to win the war while optimizing some other value--say minimizing personnel casualties, or minimizing economic damage, or costs. That is, its primary decision norm shifts to a next higher level. We have termed this level "affluence." It is measured by the quantity of elements: lives saved, dollars saved, stockpiles of materiel or of raw resources. Presumably the quantitative level of these measures

POSSIBLE ORGANIZATIONAL NORMS

Maximal Self-Realization

Altruism (the "truly good" life)

Freedom Seeking

Truth Seeking

Maximal Benefit to Constituents

Minimal Regret for Constituents

Growth in Size

Power

Prestige and Peer Status (group-belonging)

Affluence

Survival

Table 14.1 Values Successively Instituted as Terminal Goals

is proportional to the probability of successfully meeting (surviving) some future threat.

It is possible to attain a level of abundance such that its marginal contribution to future survival is diminished (or even becomes negative). Too many military forces drain the economy, too much food saved requires too much effort to store and preserve it. The organization seeks uses for its affluence beyond that of mere acquisition: in order, it may institute the values of belonging, prestige and power. An individual seeks to enlarge his peer group--he joins social and professional clubs, he seeks to be recognized as one of the group until he is assured of being one of them. Then he seeks to be elevated within the group as a recognized superior or leader; he seeks status through superiority or prestige. Assured of this, he may seek to influence the action of others through power.

The values of size and/or flux of activity are in fact kinds of affluence. Business corporate entities seek to grow, to have a high volume of business. As an organization gets established as sufficiently large in its operational area, it may turn its values toward improvement of its sub-ordinate elements. It may maximize some measure of good (at the sub-ordinate level) for all or a sub-set of its elements, or it may seek to minimize the greatest stress placed upon its sub-ordinates (minimum regret).

The system may forsake competitive values (survival through power); it may place itself in a noncompetitive, prestige-assured status and seek more abstract goals. Its objectives may be to contribute to knowledge (truth seeking), or to seek through abstract channels the expansion of freedom of action for individuals, or to maximize the assistance it contributes to other organizations or individuals (altruism). It may seek an

abstract, self-defined existence termed the "good life," and finally, it may seek to strengthen the sense of self by self-polarization.

These values, in one form or another, appear as norms controlling organizational decision. Is there some common element in all of them, or do they stand as a multivalued structure? We do know that under stress the organization professing to having one of the higher norms may progressively shift its norms downward toward more basic values. We may gain some insight into the common properties of these values by analyzing the decision process itself, first observing the factors that serve to defeat or obstruct optimal behavior under any set of idio-norms.

OBSTRUCTIONS TO OPTIMAL BEHAVIOR

In seeking to identify obstructions to optimal behavior, we shall be led to the recognition of some additional properties of optimal behavior. We shall then see better how to close on an acceptable definition. Non-optimal decisions due primarily to the obstructions listed in Table 14-2 will be treated first.

Formal Obstructions

In discussing the test of invariance or stability of decision subject to formal changes in the model, we shall be testing our decision format against ambiguity--in particular, ambiguity associated with injection of additional information not passed originally by the "filter" (as part of the model). This particular form of ambiguity is encountered when one utilizes different reductions (reduced models) presumed to apply to the same problematic situation. Note that our test, to be definitive, will require that the indicated decision be stable with respect to any additional

OBSTRUCTIONS TO DECISION

FORMAL OBSTRUCTIONS

Ambiguity

- Logical
- Procedural
- Ontological

OBSTRUCTIONS DUE TO FINITE CHARACTER

- Uncertainty
- Finite Amount of Data
- Impracticability (finite resources)

REDUCTIONISTIC DECISION MODELS

- Open Set of Models (sometimes disjoint)
- Alternative Sets of Norms
- Multi-system Norms (value conflict)

CONSTRAINTS

- Infeasibility
- No Higher Principle
- Institutionalization of Values

Table 14-2

consideration. The fact that two different reductions lead to the same choice does not constitute proof that choice is necessarily the best--only that the reductions are sufficiently comprehensive with respect to their differences. A proof of necessity is impossible (except in a closed finite universe in which <u>any</u> and <u>all</u> elements can be considered). One must depend upon the intuition of the reviewers who impose tests of invariance against any consideration they deem may possibly be relevant--or which other critics are judged to believe may be relevant.

There are, however, certain formal properties of models which are certain to introduce ambiguity. These are properties identified and discussed elsewhere under the designation of <u>canons of rational decision</u>. The canons are categorical--i.e., their violation necessarily results in ambiguity and hence inadmissibility. The reader should review our detailed discussion of these cognitive controls in Chapter 11. It will be sufficient here to note the following considerations.

<u>Procedural Invariance</u>. The control of procedural invariance is a generalization of the Einstein principle of relativity; it is the control utilized to obtain a measure of sufficiency of the models used in the decision process, particularly with respect to the adequacy of the <u>reduction</u> process. Since reduction is a procedure process acting on the universal model (finite and constantly under change imposed by new experience), and since the number of different reductions is combinatorily large, it is beyond the bounds of consideration by a finite cognitive agency (decision maker or decision making organization) to consider all of the possible reductions upon the universal model as constituted so-far-forth. Indeed the universal model itself is an idealization since no cognitive agent has the capacity to store and manipulate the "universal model." We mean

by this term the collection of all there has-been-to-know. The point here is that there exists no finitely attainable algorithm for the systematic and exhaustive test for procedural invariance.

Invariance of Choice. The test of invariance of indicated choice becomes a means of testing the appropriate scope of application of a specific decision model--particularly when the problem has been partitioned to such an extent that such scope is severely constrained. In such an instance, the model constitutes a marginal perturbation upon some set of operating conditions. For example, consider a marginal change in the characteristics of one of a family of tactical weapons system. The analyst frequently imposes the perturbation only--keeping all other factors the same. He may compare several such marginal changes and then rank the choices with respect to their effectiveness-to-cost ratio. The analysis has assumed that the environment of comparison does not change (or is the same for each perturbation). Actually the environment does react to the perturbation--and not necessarily in the same way for each perturbation. Were the effectiveness-to-cost ratio determined with due regard to the reaction of the environment, it is possible that the rank ordering of the choices would not be the same as before--nor would be preferred choice be necessarily the same. In such a condition the model would be determined to be non-warranted for the purpose used.

The Need-to-Know. In every decision making body it is sometimes assumed that the ideal is that each decision making element should be supplied with exactly that information he needs-to-know--and none other--since the consideration of irrelevant information is wasteful. However, someone must decide in advance as to the relevance of this information. Such a perfect filter presupposes a full knowledge of the decision problem.

It may at best be appropriate to the case of an automaton where a specific stimulus produces a fixed and determined response.

In a viable organizational decision body the decisions relative to the need-to-know are made iteratively with the decision as to the optimal choice. One involves again the test of invariance: is the choice indicated by the admitted information invariant with respect to the admission of additional information (prejudged to be irrelevant). If the same choice is indicated then the added information is not relevant. On the other hand, if another choice is now indicated as optimal, then the previous choice had been conducted with inadequate (over-filtered) information. It is an irony that the test for relevance, i.e., the "need-to-know," requires that one have access to any information in question before he can determine whether he needs to know it.

Because of the changing and unpredictable nature of the environment, the viable organization must always protect itself from over-filtering of information (in the guise of decision efficiency). In such a case it is possible for one to make rational decisions which nevertheless are in fact very poor decisions.

It is necessary that any decision be tested against the random admission of presumed irrelevant considerations. The optimal allocation of decision-making resources between formulating and testing is a matter of higher decision strategy. The allocation of too little resources devoted to testing will inevitably lead, sooner or later, to poor or even disastorous decisions. Too much resources devoted to testing will make the decision process costly and sometimes non-competitive.

Ontological Parity. We have recognized two fundamental sources of obstructions to decisions: the existence of inherent ambiguity and lack

of coherence associated with the characteristics of decision models utilized by a finite cognitive agent.

In the first class we have drawn attention to ambiguity introduced by the violation of the "law" of the excluded middle, and by violation of procedural invariance. Another very important source occurs with violation of a requirement for <u>ontological parity</u>. By this term we mean, in brief, that the parts of a set of statements about equivalence should be testable by the same existential tests. This canon is a generalization of the physical requirement that each of the terms of an equation should reduce to the same units of measure. In the case of a mathematical equation, the partition denoted by the +'s and -'s are numerical: hence each individual term to be combinable must have the same existential status. In other cases, for example multiplication, the partition is not numerical but operational. The partion of a hyper-space into sub-spaces is another instance. In such a case the ontology of the system and that of its combined partitioned parts must be the same.

The requirement for ontological parity is categorical. It is a necessary formal property of meaningful discourse. The underlying philosophical issue concerning the meaningfulness of the naturalistic fallacy concerns the debate over the acceptance of alternate modes of existence. Since we have equated the meaning of "exist" to the admissibility under test of existential statements, and since we recognize several such tests, we can accept the concept of multiple modes of existence. By "multiple modes" of existence we do not refer to anything mysterious or occult--merely to the simple observation that the expression "exist" conotes an appropriate test. For example, the mathematical statement of a theorem may begin: "There exists a function f(x) defined on the interval . . . ."

The word "exists" here refers to the formal test of consistency with respect to the initial axiomatic statements. The statement of an empirical existence such as "It is raining" is not testable by formal tests, but by substantive (extrospective) tests.

In general the determination of substantive existence involves the use of confidence measures and is never resolved categorically. In the case of "It is raining" the question of truth or falsity is never demonstratable with certainty.

This property of empirical existence is attributable to the finite character of cognition. The information applicable to any given question is always finite. In the case of "It is raining" one might define the state of "raining" by the appearance of "more than one drop of rain per square meter per second." There is always some situation in which the confidence level of determination is small. Suppose we devise an algorithm presumably leading to a determined action (decision)--say the decision to carry a parasol. We observe a square meter of exposed horizontal surface for one second. If any drops of rain fall we carry the parasol, otherwise we leave it off. Even this apparently determined response to a probabilistic experience has conditions in which the attainment of a decision is defeated: Suppose a drop of rain falls just tangent to the border of the square meter--or on the square meter precisely (i.e., unresolvably) at the moment the clock ticks. In either case there are conditions which lead to an undecidably response. Decision has been defeated by the finiteness of the data, or by the finite level of resolution of observation. Note that one can continue to refine his algorithm and specify a decision when such undeterminable situations are encountered; but in every instance the decision gets more decoupled from the observables of the case--in the

last resort one may "toss a coin" in order to decide the issues. (Even this algorithm has its undecidable condition!) Or one may simply leave the decision up to circumstance (the "null-decision").

The point here is that an admissible model <u>must</u> provide for a decision as the situation gets progressively devoid of information; but that such algorithms degrade toward actions that are less and less rational in character, until ultimately the algorithm is "do nothing (rational)." As models become more complex, they require successively more information before an algorithm can be formulated at all in the more complicated format. Whenever information is insufficient to determine a response, the decision maker invariably discards the complex model for a simpler model such that existing data will lead to a determinate response. Thus, the term "optimal response" must have relative reference in our usage: a decision may be said to constitute an optimal response only <u>with respect to</u> the model and data used for decision.

Whenever we attempt to design a notation that <u>completely</u> specifies the ontology of the concept represented by a symbol, we face a dilemma: The set of ontological identities for each symbolic representation contains all that there is to know in the context of some closed, finite universe. If a technical language is to say everything specifically, it must say everything redundantly. It is the marvel of ordinary language that the ontological identies are for the most part implied by the context of discussion and by the background of the discussants. As a rule, however, one is trying to convey only a simple idea when he asserts that A=B, namely, that A is equal to B not with respect to all ontological measures, but merely with respect to a few. The principle of ontological parity requires parity with respect to <u>some</u> measure. In a sequence of sentences it is

further required that the ontology of each symbol remain the same over a span sufficiently broad to convey the idea intended. Otherwise, we generate sequences that are quite literally nonsensical, like that in the example "Why Fire Engines are Red," (Figure 14-3). There must be some thread of common ontological sense from the beginning through to the end in order for the discourse to have meaning. Its meaning is then contained within whatever ontological sense is common throughout the discussion. The effect of the imposition of ontological parity upon cognitive operations is more profound than we have described here. This principle may be regarded as the basic conservation principle. The laws of conservation of energy and of momentum are consequences of the principle of ontological conformity.

Obstructions Associated with Finiteness

In contrast with obstructions imposed by formal ambiguity, we deal now with obstructions to optimal decision imposed by the finite nature of the cognitive act. The fundamental source of this finite nature is the cognitive agent himself. He can assimilate information only at a finite rate, process it at a finite rate with recourse to finite storage capacity, and communicate ideas to others only in finite strings at finite rates.

One is therefore always uncertain whether the best model is being used for decision, and uncertain of the numerical value of the variables and parameters used. A decision as to the appropriateness of his model is made implicitly between each decision in a sequence. That is, any decision at the level of direct action connotes a cascade of decisions at more abstract levels. The model parameters are estimated by finite experiments. These experiments and observations are made prior to the decision--estimation of antecedent state is made on the basis of information which had aged.

WHY FIRE ENGINES ARE RED[1]

1. A fire engine has 6 road wheels and two steering wheels
2. 6 x 2 = 12
3. Twelve inches = one foot
4. A foot is a ruler
5. Queen Elizabeth is a ruler
6. Queen Elizabeth is also a ship
7. A ship sails the seas
8. In the seas there are fish
9. Fishes have fins
10. The Finns fought the Russians
11. Russians are known as "Reds"
12. Fire engines are always rushing
13. Hence fire engines are red

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[1] Invented many years ago by a lady of my acquaintance, a logician to be sure.

Figure 14-3. Illustration of Violation of Ontological Parity

Hence, there is always an uncertainty as to the extent the situation has changed since observations were made. Finally, when one has chosen a model and estimated the magnitude of the parameters, the operations indicated by his "optimal" decision algorithm must be carried out. In the meantime the problematic situation motivating the action in the first place has changed in a not-altogether predictable manner. (This is an illustration of the classic Heisenberg uncertaintly principle translated into general decision terms.)

The search for the "best" or "optimal" decision cannot be made in a context of well-behaved, fixed models and have a determinate relevance to the actual problematic situation.

A decision which is optimal with respect to the information which can be immediately assimulated by one individual is not, in general, the optimal decision if one were able to use a score of professional analysts and scientific observers with unlimited resources over a period of years. "Optimal" is relative to the time and resource limitations constraining that decision process. There is no universally "best" decision which can ever be identified (even with hindsight).

The implication of the normative approach is that whenever a model is projected into use for future decisions it becomes progressively empty of factual data and the appropriate probabilities of outcomes approach that of a random walk--i.e., all outcomes or allowed transitions are equally probable, and the expected outcome in this future problematic situation, being random, is the same (from the perspective of the present) for every alternative action available at that time. Decision becomes decoupled from outcome. Indeed if this onset of unpredictability did not occur, decision would be impossible. Decisions made in the present are determined

by tomorrow's expected values associated with various outcomes. These expected values are in turn referred to further future values and so on. A deterministic model having "complete" knowledge would require infinite forward staging to enable a choice to be made in the present. The onset of uncertainty limits this process; otherwise rational decision could not be accomplished by a finite cognitive agent.

The point here is that the data for all decisions are always finite. Hence, all decisions are made under uncertainty. This uncertainty carries over to apply also to the determination of an optimal decision.

Impracticability. The term "impracticability" here refers to the relation between the cost of resources needed to improve the decision and the increased effectiveness associated with such an improvement. In the words of our profession, the action must be cost effective--i.e., it must result in a net gain when measured against the costs.

Certain models and decision algorithms are inherently impracticable. Particularly impracticable are those directives which demand an exhaustive search among all the possible outcomes of a projection. It has been estimated that there are 10^{47} different chess games. The number of different histories in a battalion sized simulation might well involve the order of 10^{500} elemental calculations. Since the number of atomic-nuclear events in the entire universe is estimated to be about 10^{100}, the universe as a computer would be incapable of making an exhaustive search among all possible outcomes.

It is more common, however, for a search for an optimal decision to be impractical for purely economic reasons; the search for improvement costs more than the benefit.

Reduction and Embedding

We have recognized that any decision model is a "reduction," i.e., that it is utilizable within a limited scope of decision. The organizational norms which we seek have been implicitly determined by the device of embedding one model within another. For example, a model pertaining to a nation at war may be concerned only with the probability of survival as a basic value. As such it values equally all of the winning states, regardless of the losses sustained. But as the winning of the war becomes probable, the values shift because the decision model becomes embedded in another one which includes conditions following resolution of the conflict. Now, the nation no longer values its winning states equally. It values those states more having more surviving persons, economic production base resources, and military resources--presumably because it must be ready to defend itself against some future threat. Thus it is lead to adopt measures of abundance of surviving resources as newly pertinent value measures.

The analysis of the progression of values in the sequential embedding is further complicated by the recognition of (at least) three categories or dimensions of embedding: (1) embedding in object-space, (2) embedding in temporal sequence, and (3) embedding in levels of successively more abstract decision principles.

The following example suggests primarily the first category of embedding--but, to a degree, embedding in time also. Indeed, we shall discover that embedding in space and time are interchangeable.

In a sense a model pertaining to the scope of a particular decision faced here-and-now, finds its boundary values as determined by models

overlapping these boundaries. (In a general sense "embedding" is a form of overlapping.)

In Figure 14-2 we have depicted "the" decision model (the here/now model) embedded in more remote models A, B, C, etc., and these again embedded in a set of models, α, β, γ, etc., representing a fixed environment.

In assessing the response of the environment to contemplated actions in the here/now model, the more remote models may be constrained to slow, linear change in their variables, while boundary values in the fixed-environment models are held constant. That is, it is considered that actions taken in the set of models A, B, C, etc., affect the outcomes in the here/now model only marginally; whereas the actions taken here/now are so little affected by actions in the boundary models α, β, γ, etc., that the precedence relation of values associated with action is not affected.

Spatial Embedding. Now consider the question of convergence for this embedding sequence. By "convergence" we mean that the sequential embedding can be terminated inasmuch as further embedding would not alter the decisions made in the here/now model. This termination is a condition to be achieved in the selection of a portfolio of models as part of the strategy of reduction. Convergence does not necessarily follow, but its demonstration is a desired control-condition to be imposed on the rational decision process.

If the decision maker in the here/now situation had unlimited current information, the process could never converge, since the data processing capability of the cognitive agent is finite. However, the information acted upon in the here/now situation is finite; and, furthermore, it has aged to some degree associated with the remoteness of its origin.

We know of no a priori procedure for proof of the necessity of convergence. One may demonstrate the sufficiency of convergence--i.e., that the process converges for all conceived situations--by the testing of the

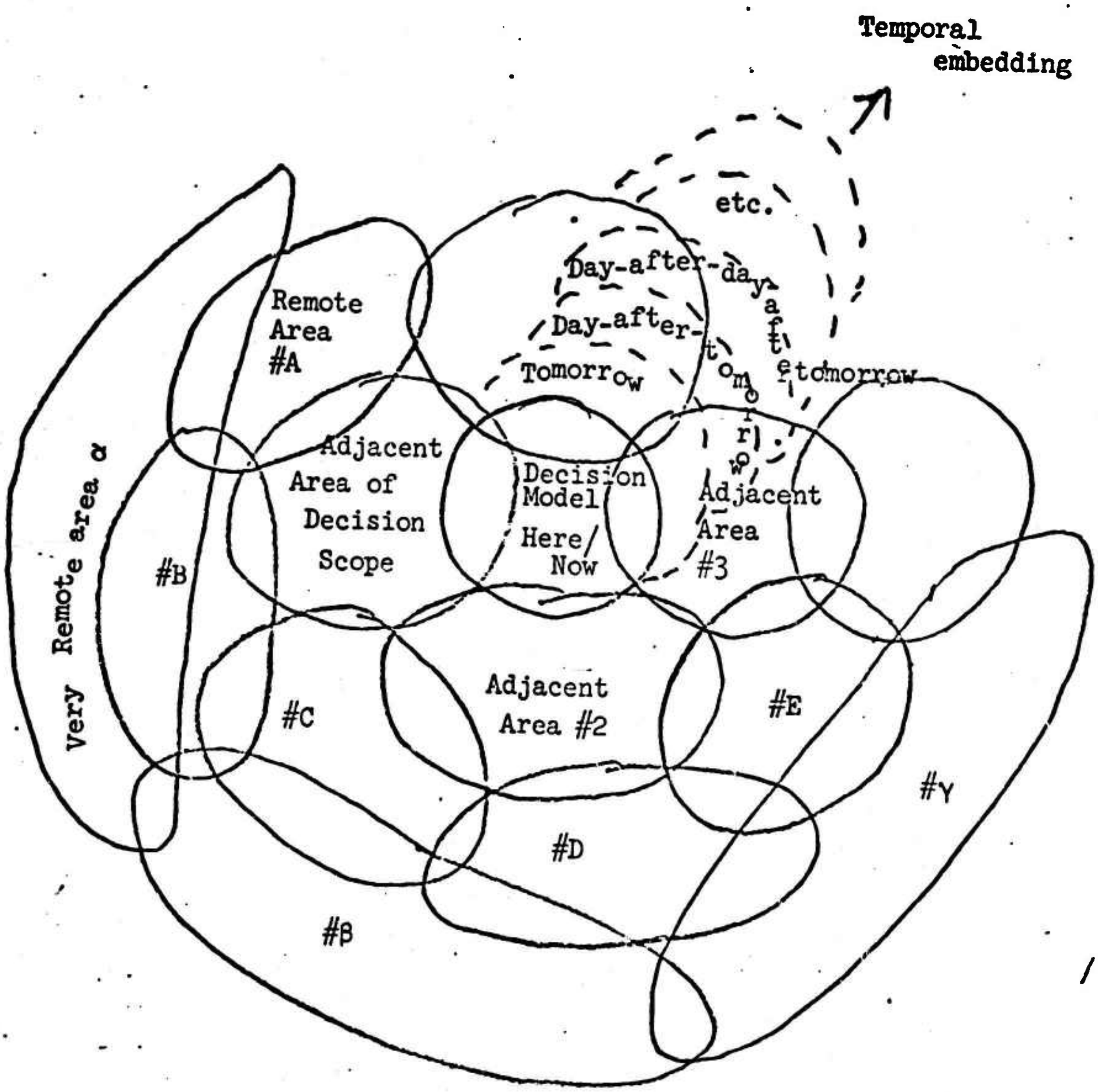

Figure 14-2. Illustrating Two Dimensions of Embedding

stability of the indicated decision against foreseeable and conceivable variations.

Essentially, the achievement of convergence may be construed as the attainment of optimal organization. In virtue of the normative and relativistic character of "optimality," no version of logical or empirical test procedure can adequately confirm such attainment. As with any value-measure whatever, the problem of warranting convergence involves test procedures appropriate to <u>vindication</u> rather than confirmation or validation of the set of decision models in question. The distinction to be kept in mind concerns state-measures v. trend-measures. Programs, strategies, policies, values--and, in the present instance, optimal organization--expressly admit of sequences of outcomes that necessarily include incidental failures and losses as well as successes and gains. The justification of a claim that a given portfolio of decision models converges (or, equivalently, that a given decision system is optimally organized) therefore rests on demonstration that output decisions are stable under confrontation with additional information input, that they satisfy the <u>sufficient</u> conditions for viably adaptive response to perturbations of the decision context (i.e., change of environmental measures).

Were it not for the effect of convergence, any decision whatsoever would entail re-analysis of all the decision there are. The efficiency of a decision model and its embedded structure can be measured by the degree to which ancillary problems in adjacent models must be re-examined.

<u>Temporal Embedding</u>. Any decision process implies a temporal sequence. Today's values are determinable in so far as today's actions affect tomorrow's situations. The values adopted in an analysis of tomorrow's decisions in turn depend on the outcomes day-after-tomorrow apparently

ad infinitum. If a here/now model could be believed to be indefinitely applicable to future situations, then rational decision could not be reached because this ever forward-in-time referencing would never terminate. In practice, however, the relevance of the here/now model to the future diminishes because the factual content of the model diminishes as one projects it into the future. Decision becomes decoupled from expectation; when uncertainty holds over the whole range of variation, there is no differentiation with respect to actions taken at that projected future time.

As a model is projected into more remote future time frames, observations relevant to hard data decrease and ultimately become negligible. At this time the appropriate model is a random walk; outcomes are identical for all actions possible. That is, action and expectation are decoupled.

It is at this onset of ignorance that further embedding of decision models can be terminated, because any further embedding will not change the here/now decision (the only one to which the decision maker is irrevocably committed). Hence, by our test the process has converged in a finite sequence of models.

Should the system terminate because it became trapped in a state which it cannot transform, the process of embedding may terminate for that reason alone.

Spatial and temporal embedding are to a degree interchangeable. Referring to Figure 14-2, note that the decision maker using the here/now model must embed it in yesterday's "adjacent" models--and these in turn are embedded in day-before-yesterday's remote models. In particular, there is a temporal delay between the source of information and its consideration by the decision-maker in the here/now situation. He must

make his decision on the basis of information generated in advance of his action, and the more remote the source the older the information.

This consideration is in analogical conformity with those leading to the relativistic theory of space-time and to the concept of advance potentials in electromagnetic theory.

Hierarchical Embedding. By the term "hierarchical" embedding we refer to the concrescence of groups of predictive or prescriptive principles into a more general principle. This eventually becomes a hierarchy of abstractions. Since such embedding results in more generality, it necessarily results in fewer principles as applied to a finite store of experience. Here, our commitment to the development of a conceptual system which is operable by a finite cognitive agent leads to a natural termination of the hierarchical embedding process whenever a "universal" principle is achieved. Such a principle may be universal only so-far-forth--since finite man is exposed to an open-ended source of experience. However, there is reason to believe that any particular cognitive agent--or any species of cognitive agents--may be limited in its span of hierarchical generality by the finite nature of the cognitive mechanism. One measure of the level of cognitive evolution of a species may be found in the nature of generalities it achieves. The nature of ultimate cognitive generality is probably species-specific, dependent upon the psycho-physiology of that species.

Reductionistic Decision Models

In preceding sections of this chapter we have discussed at some length the role of the process of reduction in the attainment of manageable models for decision purposes. This process turns out to be crucial

to the attainment of optimal behavior. The progression of development of a portfolio of reduced models for decision purposes contains elements of pure strategy. By this we mean that choices of models are made on the basis of policy commitments prior to knowledge of their claim to viability--a matter which can be demonstrated only by evolutionary test.

The evolution of a portfolio of decision models will start with many disjoint, simplistic models--"ad hoc" models or "rules-of-thumb." They apply, at best, to highly specific and limited situations, and they may be totally inconsistent when interpreted outside of this context. The attempt to apply such models outside the original context will usually result in complete failure.

For example, consider the so-called "side-wise" effect in the propulsion of a motor boat by a single-screw inboard engine. If the propeller rotates clockwise (when looking in the direction of motion and thrust), the boat will experience a turning force to port. An ad hoc model is often used to "explain" this side-wise force to the novice boatman: The water pressure is greater the deeper one goes below its surface. The bottom of the propeller is rotating into denser water than its top and thus tends to drag the stern to the right turning the boat toward the left(!).

Now this "model" is perfectly satisfactory for predicting the direction of turn and for making the decision as to which way to turn the rudder to offset the size-wise force. However the model breaks down completely when one must predict the magnitude of the force; and of course it is also inconsistnet with everything one learned in

freshman physics.[1] **Decision on such a basis becomes impossible due to the disjoint character of the ad hoc model. More important ad hoc models, which were perfectly good in their limited context, are the valence theory of chemical bonding and the Bohr planetary model of the atom.**

Conflict

The existence of many models of limited scope in a portfolio of decision models will result, sooner or later, in confrontation with a problematic situation which is believed to lie in the domain of more than one model. Furthermore, the different models will usually prescribe different decisions as leading to optimal behavior. The decision maker thus has to choose first among what appear to be equally eligible alternative models.

This . ate of conflict can be resolved in one of two ways: (1) find a iple which enables him to choose one of the pre-existing models (including value systems); or (2) engage in a creative reconstruction. The first method uses a principle as an authoritative edict (or strategy) to maintain a schism; the second way (which, following Whitehead, we call "concrescence") encompasses the domains of problematic situations of the conflicting models and creates a new model which is satisfactory in the distinct parts of the domains of the original models, and which also provides a unique prescription in the overlapping

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1. For the interested reader, a more acceptable model which can be made quantitative, and which does not do violence to other knowledge of the properties of water, is based on the observation that the line of thrust of the propeller is set at a downward angle with respect to the motion of the boat and the flow of water through it. Hence the down-moving side of the propeller takes a heavier "bite" of water than the upward moving side.

parts of their domains. In the first instance the decision maker is using a principle to maintain a schism in which the original conflicting models are partitioned. In the second he is using a principle to create an entirely new holon in which the identify (or applicability) of the original conflicting models is lost. Both methods entail the conceptualization of a more comprehensive holon. The progressive growth of any portfolio of models therefore generates a hierarchy of models.

A central problem in the developing of a portfolio of models for optimal behavior concerns the question of convergence of this escalade of hierarchical constructs. Do higher principles entail even higher and more abstract principles? Does this escalade terminate? These questions constitute a central problem in any attempt to identify a general organizational norm in terms of which one might measure optimal behavior. Indeed, the establishment of convergence--of termination--of these processes is essential to the development of a rational decision process.

Constraints

Every problematic situation entails elements constraining the admissible decision alternatives. The whole field of mathematical programming concerns the solution of problems expressable analytically in terms of extermalization of an objective function (utility) subject to constraints. We are concerned here with a larger class of constrained problematic situations, particularly those for which there exists no feasible solution, no decision which satisfies all of the constraints.

Non-feasibility is all too common in the complex decisions of government. The primary question is not what is the best alternative

but whether one can determine any admissible decision whatever.

The decision maker must not only seek higher principles (higher hierarchical holons in which feasible alternatives do exist), he must also persuade others to accept those values implied by the higher principles, or to modify their own values systems, or both.

It is the nature of a policy or values system that the older values are decreasingly considered as candidates for modification. Confidence in their viability has been built up over a history of successful behavior. Occasionally such older policies become regarded as sacrosanct and are not submitted to consideration for modification. This feature of institutionalization can lead to "ossification" of value systems. In an infeasible situation some policies and values clearly must be modified. Institutionalization of value systems makes them inflexible and non-adaptive.

Behavior and Decision

"Behavior" as used here refers to the characteristics response of a system. A cognitive agent, in order to be recognized at all as a distict entity, must have some enduring properties of responsiveness to similar problematic situations.[1] Unless a correspondence relation can be established, identifying a characteristic response over some span of context and of time, it is impossible to attribute to the cognitive agent a singular "personality."

The empirical problem of establishing the existence of a characteristic response is not an easy or simple one. An individual cognitive

1. This is implied by the origin of the word "ethics."

agent is constantly modifying his strategy on the basis of his experience, and the environment itself cannot in the strict sense ever be controlled to the extent that it can be returned to exactly the conditions of an earlier state. Furthermore the empirical observer is only forming, at best, his theory of the behavioral pattern of the observed cognitive agent. This theory is necessarily based upon a limited and finite amount of observation on a cognitive agent whose behavioral pattern is constantly subject to modification in an environment that is never reproducible.

If decision is to be an element of behavior then there must exist some set of alternative behavioral actions which are free for selection by the cognitive agent. Decision then implies more than mechanistic determinism--it implies the freedom to choose a unique action.

One must distinguish between the free choices of a cognitive agent and any stochastic character of the outcomes associated with choice. The presence of stochastic outcomes does not imply freedom of choice. Only if the action taken modifies the stochastic probabilities of outcome can there exist a range of freedom associated with choice. Another point to bear in mind is that a cognitive agent will present totally different aspects when viewed extrospectively (as an object) as against being viewed introspectively (as a subject).

If the cognitive agent has adopted a policy or developed some algorithm for the selection of a unique action from among alternatives, he will regard his act as a free choice subject to some selection criteria which he is free to adopt or reject. To the extent that the cognitive agent is consistent in the application of his principle of

choice, an observer will see him as behaving in a deterministic manner. It is necessary to adopt a "normative" perspective, a view of the cognitive agent as a <u>subject</u>, in order to be aware that there is a decision process involved at all.

This projection of the theorizer into the situation of the agent under observation is the essence of the nomative perspective as distinct from the "objective" perspective. Whenever a range of outcomes is left as a free parameter, with unique choice made by some higher principle--an optimizing or extremalizing principle--the observed entity can be described in a normative setting, even though the selection or choice may in fact be in the cognizing process of the observer.

For example, one can predict the motion of an object in a gravitational field by the deterministic equations of Newtonian mechanics; or one can recast the problem in normative form by attributing to the object: (a) the freedom to move anywhere, and (b) a characteristic tendency to extremalize some variational measure--in this case, to minimize action--subject to conservation laws and any physical constraints present.

In the normative view one might be tempted to say the inanimate object is making the decision to minimize action. However a more reasonable view is obtained by acknowledging that the observer is part of the act. He, as a subject, is predicting the path of the object. Hence in the case of the inanimate object decision rests with the observer who predicts the trend of the motion. Any rational theory of objects presupposes this relationship. The existence of any object is manifested <u>with respect to</u> the observer by its interaction with him.

From the perspective of the cognitive agent then, decision is a freedom-of-action-consuming operation, whatever the algorithm for its accomplishment may be. Decision will be regarded as goal-seeking whenever: (a) there is a utility function expressing some higher principle, and (b) the algorithm for decision involves a choice which extremalizes the utility function. The hyperspace of choice constitutes a crude measure of the span of (free) control over which the optimization process will be exercised. The central point of these discussions is that decision as an overt act is a freedom-consuming operation (at the idio-level of that decision). In the act of decision here-and-now, the range of "free" choice (free at the idio-level of the cognitive agent) is narrowed down to just one action.

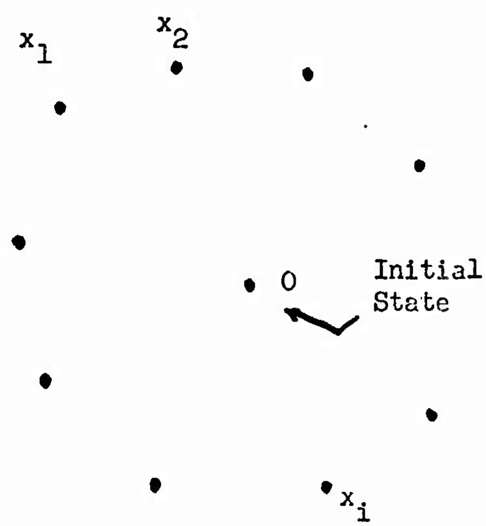
Any discussion of freedom vs determinism is sensitive to the hierarchical level of organization under consideration, to the adoption of objective vs subjective perspectives, and to the temporal basis. Therefore any statements made in generalization of these topics must be placed in the appropriate context.

Time itself, in the passage of future into past, consumes all freedom. If the "present" is that instant in which all freedoms are revoked, whether one takes an overt action or lets events transpire uncontrolled, then obviously an act taken in the present is taken to affect the future. Otherwise there would be no difference in the value of outcomes brought about by the act of decision as against that resulting from null-decision. Either one does nothing and nature takes it course and freedom is lost by default, or one chooses a specific act and thereby consumes such freedom of action as existed. In either case freedom is totally consumed.

Consider, for example, a sleigh sliding down a hill. If the sled is not steered it might end up at the base of the hill with terminal points given by the figurative probability distribution of Fig. 14.3. When controlled the sled might be steered to end its path at or near A. We are concerned with two ideas here: How does the controlled outcome represent the expression of freedom of choice? How is the choice of an end state, say A, to be expressed within out concept of freedom?

In the interest of more precise language, we define the following terms: "Expected control" is the maximum change that can be effected (by overt action) in the expectation of achieving a given state, summed over all states, valuing all goal-states equally positive and all aversive states equally negative.

In order to clarify the definition we shall simplify the situation by use of the following discrete model. Let the initial state be denoted by 0 and



the possible adjacent states achieved in the next move by X_i. The probability of moving from 0 to a particular X_i under action C_j we denote by:

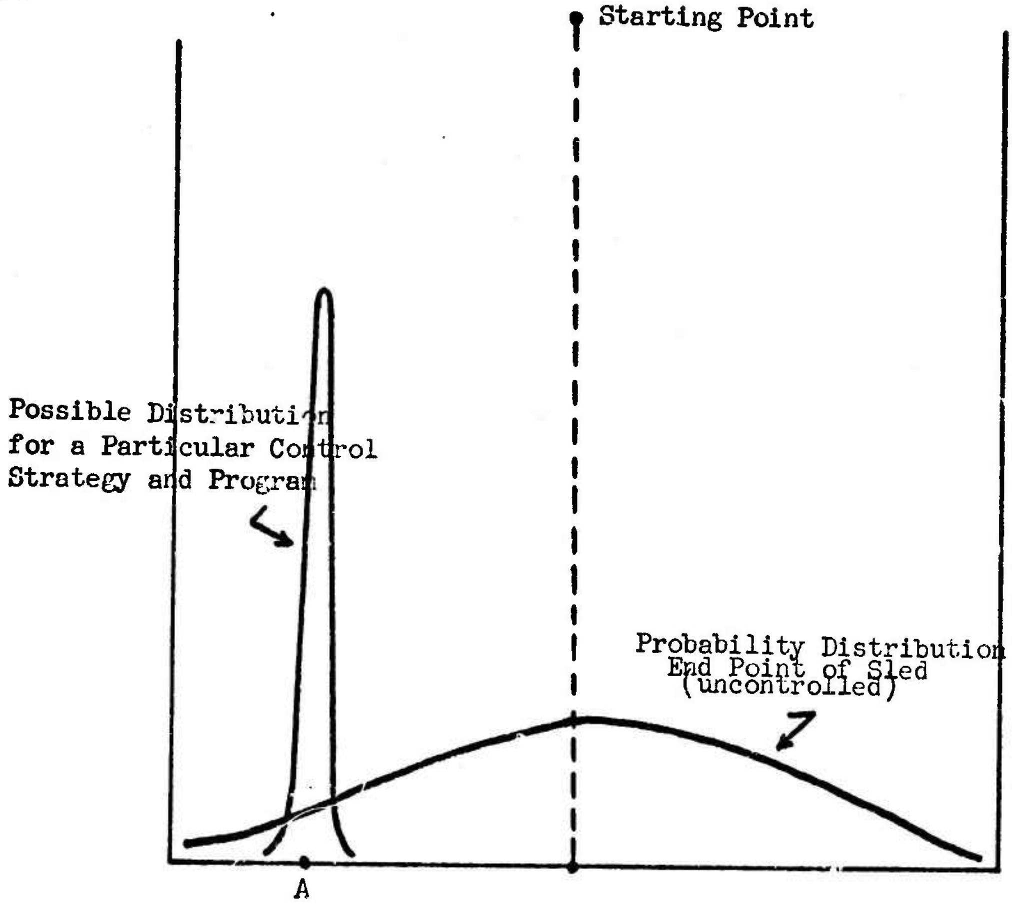


Figure 14.3. Figurative Effect of Control

$$P(X_i, C_j).$$

There exists one action we call the "null action" denoted by C_o in which we do nothing and let nature take its course. The conative control is to be measured for each state by:

$$\sup_j P(X_i, C_j) = P_c(X_i)$$

and the aversive control by

$$\inf_j P(X_i, C_j) = P_v(X_i)$$

(if conative and aversive actions are independent of each other). The expected control in this case is given by:

$$\sum_i P_c(X_i) - \sum_i P(X_i, C_o)$$

$$- \left\{ \sum_i P(X_i, C_o) - \sum_i P_v\ (X_i) \right\}$$

$$E = \sum_i [(P_c(X_i) - P_v(X_i)].$$

Suppose that $P(X_i, C_o)$ is uniformly random, i.e., that if there are N states that:

$$P\ (X_i, C_o) = 1/N \text{ for all } i.$$

The maximum expected control would occur if there exists a control choice for each i such that:

$$P_c\ (X_i) = 1 \text{ for all } i$$

$$\text{and} \quad P_v\ (X_i) = 0 \text{ for all } c.$$

That is, that one can seek or avoid any state with certainty. The maximum expected control is then given by:

$$\sum_{i=1}^{n} (1-0) = N$$

Were the maximum probability to be less than certainty, say $1/M$, $M < N$ and the minimum probability greater than zero, say $1/R$, $R > N$, then the expected control would be:

$$N\ (1/M - 1/R),$$

for example, if $M = N/4$ and $R = 4N$, then expected control = $15/4$.

Implicit in this measure there is another quantity we may call the "span of control" which we define as the number of states whose probability of outcome can be modified in a measurable amount by acts of willful control.

The above definition assumes that conative and aversive acts are independent, and therefore ignores the resolution required to differentiate between these outcomes.

For example, consider in the clock model above that the desirable and undesirable states alternate around the perimeter of next moves. Consider that there exists a control strategy which concentrates the outcomes to a random outcome among two adjacent states (it cannot resolve the difference between desirable and undesirable states).

$$\sup_j P\ (X_i,\ C_j) = 1/2$$

$$\inf_j P\ (X_i,\ C_j) = 1/N$$

and, under the assumption of independence, the expected control would be $N/2 - 1$.

However, since the acts are not independent, one is maximizing his goals seeking while minimizing his aversive outcomes <u>by the same action</u>. Therefore the non-independent control can be measured only in terms of the expected value. That is, the probability of outcomes weighted by the value of that outcome and expected control may be defined as:

$$\sup_j \sum_i \Big[P(X_i, C_j)\, Q(X_i) - P(X_i, C_o)\, Q(X_i)\Big]$$

When $Q(X_i)$ is the value of the ith state, then the above is simply the expected value. When we merely distinguish among desirable and undesirable slates by setting $Q(X_i) = 1$ for the former and -1 for the latter, then we obtain a measure that is sensitive to the resolution required by dependence on the special configuration of desirable and undesirable states.

In our attempts to discuss control purely on the basis of a modification of the probability of outcomes, we cannot escape the necessity of including at least an elemental valuative property of the outcomes--at least to the point of differentiating between desirable, undesirable, and indifferently valued states.

These observations boil down to a fundamental observation about the basic relation among the concepts "value," "freedom," and "level of reduction" of a model:

1. A reduction has not reached its ultimately useful level of hierarchical partitioning until all states can be lumped into at most three categories:

   A. States which the cognitive agent seeks to reach as ends in themselves.

   B. States to which the cognitive agent is indifferent except as they are connected with other states.

   C. States which the cognitive agent seeks to avoid as ends in themselves.

2. If the model above is replaced by one made simpler by aggregatic::, a differentiation of value among goal-states and among aversive states will result and the equality of valuation will be lost. The objectification has been simplified at the expense of complicating its canonical conjugate: the valuation.

3. If the model in #1 is replaced by one of more detail, or one having more hierarchical levels, no increase in the resolution of control will be gained, but merely the introduction of uncontrollable random transactions. One simply has added chaotic elements by specifying beyor l the resolution of his data.

We have shown elsewhere that the value concept is canonically conjugate to an appropriate objective concept. The value of an object is cononically conjugate to the measure of that object (count) in a static system. These relations result from the invention of the conjugate variable in such a manner as to satisfy the Legendre perfect differential. For example, consider an objectification $x_i = (x_1, x_2, \ldots x_n)$ and a total system value function $F_c\ (x)$ (a scalar); when there exists a set of actions $c_i = (c_1\ c_2, \ldots c_j)$. In order to relate the total function with outcome states it would be necessary to evaluate the $F_c\ (x_r)$ for all possible histories from the state x_r--computationally impractical procedure for all but the simplest of decision models.

One can introduce a conjugate variable $p_j = (p_1, p_2, \ldots p_n)$ such that :e scalar product of x with p_j

$$\delta(p_j \cdot x) = \delta F_j\ (x) + \delta G_j\ (p) = 0$$

is a perfect differential. This produces the canonical differential equations.

$$p_{ij} = \frac{\delta F_j}{x_i} \; ; \; x_{ij} = \frac{\delta G_j}{p_i}$$

Here p_{ij} is the change of total system value per unit object of type i and action j--i.e., the value of a thing of type i in relation to this particular whole system as evaluated in the vicinity of some antecedent state. The cybernetic usefulness of substitution of a 2n-dimensional (p, x) space for the n-dimensional x space is afforded by the nature of the perfect differential: Problems which were path dependent in $E^n(x)$ space are path-independent in $E^{2n}(x, p)$ space. One need know only the antecedent and consequent states to solve his problem. The computational effort is reduced drastically and a solution becomes attainable and practical, where before it most probably was unattainable for practical reasons.

What we are observing is that if $F_j(x)$ is measured by the density of conative/aversive states (equivalently valued respectively), the p_{ij} are all valued equally for all i--for the same c_j.

The uncertainty principle of Heisenberg was derived from a closed physical system where F is a function of time and the canonical differial equation takes the form:

$$p_i = \frac{\delta F(x, t)}{\delta x_i} \; , \; x_i = \frac{\delta G^*(p, t)}{\delta p_i}$$

and $G^* (p, t) = G (p, -t)$.

The error in measurements of p_i, Δp_i, and simultaneously in x_i, Δx_i were such that Δp_i and Δx_i could never be zero for the same measurement, that is:

$$\Delta p_i \cdot \Delta x_i \approx \text{a constant.}$$

Note again: the Heisenberg principle of quantum mechanics is propounded in the context of a pre-objectified space-time, with the concepts of force, mass, momentum, already well developed, and with respect to a "closed" set of objectives. In this work we take the position that the process of optimal response involves iterative operations--one set optimizing in object space a well defined utility, the other involving operations <u>within</u> the cognitive agent in the choice among alternative objectifications. Cybernetic space, the range of states within the cybernetic organ of the cognitive agent, is closed by the fact that it occurs within a single finite cognitive being. From the objective viewpoint we were concerned with manipulation within a fixed objective system; in the subjective view we are concerned not only with the manipulation (modification or successive replacement of) a single objectification as a programmed response, but also with the entire repertoire of programmed responses. Part of the cybernetic capacity of the cognitive agent is filled with this file of programs, part with active operations upon a single program, and part is free of commitment and available for the development of new objectification as candidates for inclusion in the file of working programs.

The working file or programs could be composed of a large number of programs of very limited application. In this case the chief cybernetic problem consists in selection, as to which working program to apply. Occasionally a new program--in itself perhaps more complicated than any of the working programs--is devised which can replace a large number of small working programs. It has greater comprehensiveness, with the result that the total repertoire of working programs is significantly reduced, thereby leaving a much larger portion of cybernetic

capacity free. This act, we shall maintain, constitutes the essence of the creative process--and produces the euphoria suggested by the comic stereotype-report, "Eureka! I have it!" One implication is apparent in the mysterious manner in which this result occurs: there is no propriorceptive awareness of the creative process in action. There is only the happy consequence of the process.

OPTIMAL ORGANIZATION

We turn now to the central topic of this chapter: the concept of "optimal" organization. We have been endeavoring to identify a measure of value which is common to all value measures, and have been developing the thesis that there is such a measure which we can associate with an appropriately defined concept of "freedom." In developing this thesis we have identified objective freedom with capacity to control (or determine) outcomes irrespective of the value of these outcomes--except for the necessary differentiation between desirable and undesirable outcomes.

This path has lead to expansion of the objective detail of the models employed for decision purposes. The notion of one single model of super detail has been rejected as cybernetically inefficient and has been replaced by a nested hierarchical set of reduced models.

We have further observed that there exists a level of detail beyond which there is no corresponding gain in the span of control. The only cognitive utility in increasing detail beyond that justified by the current state of observations is in the methodological guidance of future experiment and observation.

The act of valuation has been simplified at the expense of complicating the act of objectification. We are seeking a justification of the premise that there exists some level of objective detail which permits the equal valuation of all ultimately desirable states. Thus the problem of valuation has been made easier while the problem of model structure has been made more difficult. On the other hand the cognitive agent may elect to simplify his objective model at the expense of maximal complication of his valuative problem by taking the present situation and looking at the value to be associated with all possible immediate-future moves with respect to all possible systems of which the present situation is a member.

With respect to a particular cognitive agent, with finite experience and cybernetic resources, there is this fundamental trade-off. He can utilize his finite cybernetic resources for maximization of his control over the situation by an increase in the detail of either his objective consideration or his valuative consideration. His control will be increased to the point of detail permitted by his experience. Adding objective detail without substantive data simply complicates the problem by adding random transitions without permitting increased control. Indeed, one measure of the adequacy of detail occurs whenever the increase of detail does not increase control.

We have been examining the method of increasing objective detail by hierarchical embedding because this permits a large amount of historical and culturally acquired valuation to be explicitly utilized in the decision problem. Most single state objective models are deficient in that they lack consideration of the many alternate systems to which the situation is common. We can now move toward a delineation

of subjective principles with respect to a finite cognitive agent with finite cybernetic resources and with finite experience. Maximal control can be exercised by a nested set of models in which a trade-off is achieved between objective detail per hierarchical layer and the number of such layers ultimately achieving a simplified valuative situation: seek or avoid.

Consider the contrast between the resolution of valuative analysis and the achieving of a boundary of uniform values (contrasted only by difference between avoidance and goal seeking). One can, in principle construct a single micro-model having sufficient detail to admit of differentiation between all significant alternatives bounded by uniform values. Such a model would require a maximum of detail; its use would be prohibitively costly or, more likely, impossible.

Consider next a cybernetic alternative: embed one model within another. The model covering a more comprehensive range actually will have correspondingly less resolution of detail. It is used to get a rougher measure of the boundary values of the primary model (the model in which the immediate action alternatives are simulated). One may now explore the primary alternatives with, say, one-tenth the cybernetic labor but at the price of a greatly reduced resolution.

This embedding could continue, the extreme example being a set of models each having only two alternatives but requiring a large cascade of embedding. As this embedding increases (under constant total cybernetic resources) the cybernetic labor decreases while the uncertainty entailed by decreased resolution increases. The minimum labor is associated with almost no resolution. The highest resolution is associated with prohibitively large cybernetic labor. (The number of elemental

cybernetic operations associated with an exhaustive exercise of moderately large micro-model involves numbers of the staggering magnitude 10^{500}. Somewhere between these extremes, given constant cybernetic resources, there is a combination of detail and embedding which would represent optimal organization.

Let us now review the constraints we identify in rational problem solving:

1. The sequence of antecedent-to-consequent transitions must be sufficient to link the present state to outcomes which can be valued uniformly as attractive or unattractive. Otherwise the chain of analysis will, at best, rest on poorly structured strategies--cultural behavior patterns, or just mere open strategies.

2. With fixed cybernetic resources, the embedding of models first connects as in (1) above, and then fleshes out the objective detail at each level in such a manner that the product of variation in objective outcome-estimate with that of the valuative (value of the immediate future states) is minimized.

The decision maker then looks at the entire set of problematic situations he faces and allocates his cybernetic resources with the goal of optimizing his total behavior. This presupposes that he has enough cybernetic resources to apply to all his problems. Usually he is apportioning deficits and must rely on cultural or habituated patterns of response everywhere he does not recognize obvious exceptions to standard situations.

Additional sections of Chapter 14 are in progress.

Chapter 15

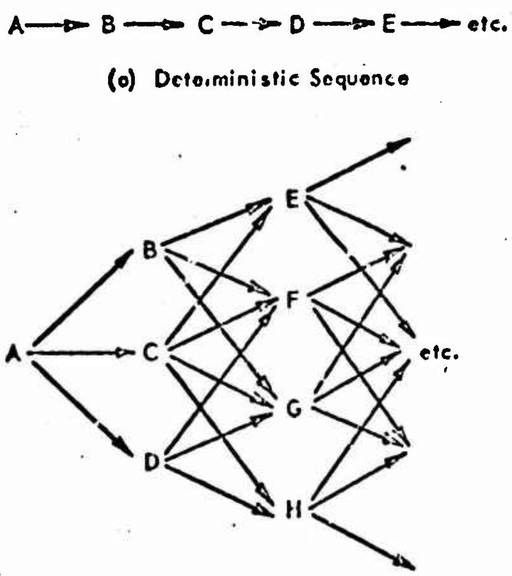
STOCHASTIC-NORMATIVE ANALYSIS

Value analysis is undertaken in this chapter on the basis of a stochastic system model. This format of analysis has been chosen with the intention of addressing primarily the formal aspects of values in their most general interpretation, i.e., in association with the concept of a state value function.

STOCHASTIC MODEL--STATE FUNCTIONAL APPROACH

A. Definitions

It is necessary to define a technical language. For this technical language we shall draw upon the terminology of physics and mathematics as it pertains to stochastic processes (1, 4, 7, 8, 28). A stochastic process is to be distinguished from a deterministic process. If event A is always followed by event B, which is always followed by C, etc. (see Figure 1a), each change occurring with a probability of unity, then once the



(b) Stochastic Sequence

FIG. 1. Illustration of Deterministic and Stochastic Series of Events.

condition A is reached the complete chain of events is fully determined. On the other hand (see Figure 1b), if event A may be followed by either B or C, then the event is said to be "stochastic." A series of such events, each followed by many alternatives, is described as a stochastic process. If the branching ratios are known, the system is *stochastic definite*. The real world can be more nearly described in terms of a stochastic model than in terms of the deterministic model.

1. State. The condition of a system at any moment in which it is not undergoing change is described as a "state" of the system. A state may be defined as a collection of numbers:

$$x = (x_1, x_2, x_3, x_4, \cdots x_i) \qquad [1]$$

where the state x of a system is described by the set of numbers, x_1, x_2, etc., and a number of components are included *sufficient adequately to describe the system with respect to the particular problems faced*. For example, in a system comprised of two opposing military forces, the state of the system might consist of a listing of the numbers of separate men and weapons on both sides. Such a collection of numbers, together with the transition probabilities, comprises a model. This collection will be represented here by any of the lower case letters in the latter part of the alphabet.

2. Transition and transition probability. Any change in the system is considered to occur discretely. For example, in the array of numbers x_1 to x_i, x_3 might possibly represent the numbers of friendly tanks in the opposing military forces. One of these tanks might be destroyed by the enemy. The system is said to have undergone a transition:

$$(x_1, x_2, x_3, x_4, \cdots x_i) \rightarrow (x_1, x_2, x_3 - 1, x_4, \cdots x_i). \qquad [2]$$

That is, the change that occurs may be in only one of the array of numbers (i.e., the "coordinates") that describe the state. The word "event" may be used synonymously to express such a transition. The probability that any particular transition occurs is defined as a "transition probability" and is represented by the symbol $P(x, s; y, t)$, $t > s$. Special consideration must be given as $t \rightarrow s$ or as $s \rightarrow t$. (See, for example, Arley, Ref. 1, p. 25.) This notation is to be read as a probability that the system initially at state x at time t will be found in state y at time s.

3. System. A "system" may be defined as a set of all states that are connected by non-zero transition probabilities. This definition, although adequate for the first part of this discussion, will have to be modified later. In the first part of this discussion only attrition systems will be explored. An attrition system is one in which the vector components always decrease—as would occur, for example, in a battle that is isolated from the sources of supply of the participants.

4. Special states. There are special kinds of states, or sequence of states, to which reference may occasionally be made. Mention already has been made of the "deterministic sequence" where one state follows another with probability of unity—that is, there is no branching. There may be cyclic states in the system; i.e., a deterministic sequence that folds back on itself to produce a continuously recycling path through the same set of states. In simple attrition systems primary interest occurs in the "trapped" states. A trapped state may be defined as a state for which there exists no chance to escape; i.e., there are finite transition probabilities to the state from others, but one for which the transition probabilities away from the state are zero. A system once arriving in the condition of a trapped state undergoes no more change. It becomes "trapped" in that particular condition. For example, such a system could be a gambling game. The state of this system could be expressed by the amount of money held by one player. The system will become trapped if either player wins all the money. When a system becomes trapped—there being no further change—the game ends.

B. State probability

Consider a system initially in a state w at time s, and at some later time u in a state y. It may have arrived at the state y along many different paths. Consider a path that

traverses state x at an intermediate time t. The probability that the system went from w to y along the particular path that led to x is the product of the probability that it first moves from w to x and then from x to y in the proper temporal sequence. If now these conditional probabilities are summed for all possible paths there results the well-known Chapman-Kolmogorov equation (12, 13):

$$P(w,s;y,u) = \sum_x P(w,s;x,t)P(x,t;y,u),$$

$$u > t > s.$$

where [3]

$$\sum_y (P(w,s;y,u) = 1,$$

and $$0 \leq P(w,s;y,u) \leq 1.$$

1. Forward state equation. There are two distinct ways in which the Chapman-Kolmogorov equation may be regarded. First, it may be considered that the system is known to have been initially in state w at time s. Equation 3 is then employed to determine the probability that the system may be found at state y at some later time u. That is, the equation so interpreted is given a known condit.on in the present to predict the probability of the future.

In this usage the equation is known as the "forward" state equation, or more simply as the forward equation. The subscript zero ($_0$) will be employed to designate a fixed known state, the forward equation being written as

$$P(w_0, s_0; y, u)$$
$$= \sum_x P(w_0, s_0; x, t)P(x, t; y, u), \quad [4]$$

$$u > t > s_0.$$

2. Backward state equation. On the other hand the final condition of a system may be assumed to be known. That is, it may be known that at time u the system was in state y. The Chapman-Kolmogorov Equation 3 then may be employed to determine that the probability that the system initially in the field point (x, t) traverses the fixed point y_0 at time u_0. This is the so-called "backward" or "adjoint" equation and is designated here with the zero subscripts as follows:

$$P(w, s; y_0, u_0)$$
$$= \sum_x P(w, s; x, t)P(x, t; y_0, u_0), \quad [5]$$

$$u_0 > t > s.$$

The adjoint equation can always be interpreted as a value function. This interpretation arises as Equation 5 represents the contribution that state w makes toward the achievement of the final state y_0. That is, the backward probability gives the *importance* of state w to this final condition.[2]

3. The value equation. Although Equation 5 is fundamentally an *importance* equation it is not in the form in which the value equation appears. Let the value associated with the state w at time s be denoted by $Q(w, s)$. The fractional contribution to the value function $q(w, s, y_0)$ by a particular trapped state y_0 is to be obtained in an ultimate sense. This contribution is defined as proportional to the limit of the adjoint probability function (Equation 5) as u_0, the time to reach the final state, moves out to infinity. Or, in other words, this contribution is defined as the probability for any time that the system will pass from state w at time s ultimately to state y_0. The contribution to the state value is defined to be proportional to this limit:

$$q(w, s, y_0) = \lim_{u_0 \to \infty} P(w, s; y_0, u_0). \quad [6]$$

For the time being, attention will be limited to those systems that are certain ultimately to come to rest in a trapped state. This may be regarded as an assumption, or conversely as a constraint placed upon the present considerations. Simple attrition systems, if left to themselves, would have such an ultimate fate. If in the gambling game previously mentioned the players gamble long enough, one or the other will surely win all of the money. In a battle fought to the last man where other factors do not intervene to stop

[2] This is related to the equation of retrodiction. See Ref. 28, Part III. To get retrodiction the transition probabilities must be a probability measure over the antecedent states, not the consequent states as expressed in Equation 3.

the bat.le, the fight would continue until there were no survivors on one or the other of the participating sides. Since, by hypothesis, only attrition systems are considered for the moment, there need be no concern about the system's avoidance of its ultimate fate.

Having assumed that y_0 is a trapped state, that is that

$$\sum_{y_0} \lim_{u_0 \to \infty} P(w, s; y_0, u_0) = 1, \qquad [7]$$

enough time has elapsed so that the system is in one or the other of the possible trapped states. Then, an *arbitrary* weighting of the value of the trapped state, $\mu(y_0)$, leads to the value definition:

$$\begin{aligned} Q(w, s) &= \sum_{y_0} q(w, s, y_0) \\ &= \sum_{y_0} \mu(y_0)\{\lim_{u_0 \to \infty} P(w, s; y_0, u_0)\}. \end{aligned} \qquad [8]$$

Equation 8 says that the value of the state w at time s is the sum over all trapped states of the product of an arbitrary weighting of each trapped state multiplied by the probability that the system ultimately reaches it. For example, if there are two kinds of trapped states, those desirable y_d and those undesirable y_u, and if furthermore the desirable trapped states are given a weight of unity and the undesirable trapped states a weight of zero, then the value $Q(w, s)$ becomes identically equal to the probability that the system ultimately comes to rest in any one of the desirable states:

if $\mu(y_d) = 1,$

$\mu(y_u) = 0,$

then

$$Q(w, s) = \sum_{y_0 = y_d} \{\lim_{u \to \infty} P(w, s; y_0, u_0)\}. \qquad [9]$$

The value function, as it appears in Equation 8, is more often expressed as the relationship between the value of one state at the reference time to the values of the states that may be reached at some later time. For example, referring to Equation 5, let us ask what the value is of the state x at time t. In conformity with the definition of Equation 8 this becomes:

$$Q(x, t) = \sum_{y_0} \mu(y_0) \lim_{u \to \infty} P(x, t; y_0, u_0). \qquad [10]$$

Referring back to the adjoint Equation 5, if the time u_0 is allowed to go to infinity and the definitions of Equations 8 and 10 are applied, one obtains:

$$Q(w, s) = \sum_{x} P(w, s; x, t)Q(x, t), \quad t > s. \qquad [11]$$

Equation 11 states that the value of the present state w is the average (or expected) value of a future state x, or in other words, is equal to the product of the probability of transition from state w at time s to state x at time t, with the value of the future state x at time t. Equation 11 is the value equation.

C. The meaning of value

From the preceding discussions of the definition of value and its derivation from the adjoint probability, its fundamental meaning becomes clear. In Equation 9 it is seen that the value of a present state w is linked to the probability of ultimate success, that is, of ultimately reaching desired goals. In general, since the arbitrary weighting functions $\mu(y_0)$ have been introduced, the value to be associated with a particular state at a particular time becomes an arbitrary linear combination of the probability of ultimate arrival of the trapped states. It is a usual convention to consider that the more desirable states are valued more positively. Since this weighting is arbitrary, at least in this state of the description of value one may choose the weightii , assumed in Equation 9, or, if he prefers symmetry, may assign the undesirable value states a value of -1. It is said that the value placed upon the trapped states is arbitrary. It is arbitrary in the sense that any set of values $\mu(y_0)$ permit a solution of Equation 11. Equation 11 also calls attention to the continued future reference or estimate of values. The values of the states today are to be determined by the values of the states reached tomorrow. Those in turn are the values of the states that may be reached the day after tomorrow, and so on apparently ad infinitum. In order to obtain a solution to Equation 11 it is necessary to define some boundary conditions under

which the values are assumed to be known. In the present instance, this boundary was assumed to exist on a set of trapped states.

In contradistinction, the forward probability function (Equation 4) has a unique solution if it is known that the state is initially at state w_0 at time s_0. One is essentially looking backward to determine the probability for the future in the forward equation. Conversely, in the value equation, one looks to the future to determine the value of the present and the past.

1. Values in nonattrition systems. It is not necessary that the definition of value be restricted solely to attrition systems, nor to systems to which the ultimate attainment of a trapped condition is certain. Value may be defined in terms of specific endpoints that may occur at time u_0. That is, for Equation 8 one replaces Equation 12:

$$Q(w, s; y_0, u_0) = \sum_{y_0} \mu(y_0, u_0)P(w, s; y_0, u_0), \quad u_0 > s. \qquad [12]$$

In Equation 12 the summation is taken over all the possible states that could be reached at time u_0. Equation 11 again follows from this more general definition. In either case an arbitrary constant or arbitrary function has been introduced to the value equation. Throughout this discussion this arbitrariness will be emphasized. Value determinations continually refer to future states, and those in turn to states further into the future, and so on, until either an endpoint is reached or until the state is so far into the future that prediction becomes impossible. At either of these points rational operation ceases and one must resort to an intuitive designation. These are essentially postulates of the value structure. Later in the discussion it will be suggested that there are more fundamental and more absolute principles that constrain the intuitive approach to the assignment of ultimate values; the application of these principles, however, does not lead to a unique set of values—that is, to a unique solution to the value equations. There is a natural repugnance felt by some readers that ultimate values are to be considered arbitrary and intuitive and they prefer to consider them otherwise. This possibility requires further development of the theory in its present rigid framework and will be discussed in a section Part II on the epistemological problem in value theory. After showing the relationship between decision criteria and value theory, a more detailed discussion of value postulates will be made.

D. Values and decision procedures

At this point, value theory may be linked with decision theory. Decisions rest not only on the probabilities of outcomes, but upon the values of these outcomes, or in terms of the present state w at the present time s there is some requirement to maximize the expected value—all other factors being considered. Consider the simplest situation first.

1. The elementary criterion of decision. Consider now that an individual has some control of a situation existing in a stochastic system. That is, he may take certain actions i that affect the transition probability. The transition probabilities thus depend upon the action taken and are denoted

$$P^{(i)}(w, s; x, t),$$

where the superscript i indicates a particular action taken. The value to be associated with the state w at the time s thus becomes Equation 13:

$$Q^{(i)}(w, s) = \sum_x P^{(i)}(w, s; x, t)Q(x, t), \quad t > s. \qquad [13]$$

In terms, then, of a set of values $Q(x, t)$ over the future states x, there now exists not a single value associated with state w at the present time s, but a set of values—one associated with each of the possible courses of action i. One may now introduce an elementary criterion of decision (sometimes referred to as a game against nature), namely, one selects that course of action i for which its associated value is a maximum:

$$Q(w, s) = \operatorname*{Max}_i \{Q^{(i)}(w, s)\}. \qquad [14]$$

This decision criterion simultaneously produces a unique course of action and at the same time selects a single value from among a set of values associated with each course of action. Equation 14 may be written as follows:

$$Q(w, s) = \underset{i}{D} \{Q^{(i)}(w, s)\}, \qquad [15]$$

where the particular instructions to select the Q having the maximum values are replaced by the more general instruction symbolized by D_i; namely, to apply a particular decision criterion. The relationship between a single value $Q(w, s)$ on the left of Equation 15 and decision criteria that may be applied to the right of Equation 15 constitutes the subject of the theory of decision processes, and is not the subject of this discussion. Equation 15 may be generalized to cover the two-or-more-person game situations.

2. Statistical criterion in n-person games. It frequently occurs that not one, but two or more players control the magnitude of the transition probabilities. If two players have such independent control, the situation is called a two-person game and Equation 15 becomes Equation 16:

$$\left.\begin{aligned} Q(w, s) &= \underset{i,j}{D} \{Q^{(i,j)}(w, s)\}, \\ Q^{(i,j)}(w, s) &= \sum_{x} P^{(i,j)}(w, s; x, t) Q(x, t) \\ &\quad t > s. \end{aligned}\right\} \qquad [16]$$

The second part of Equation 16 indicates that not a single set of values is associated with the transition probabilities in the future state x, but a matrix of values—one value for each combination of actions (i, j) taken by the two players. The decision criterion D determines the course of action i for one player, j for the second player, and at the same time a unique value is associated with the matrix of values. A decision criterion (3, 24, 26) commonly applied to the two-person game is the so-called minimax principle. Another might be the so-called principle of minimum regret. In the remainder of the discussion of value theory it will be assumed that an appropriate decision criterion is imposed to determine courses of action that permit the replacement of the matrix of values with a unique value, and considerations of application of Equation 16 will be taken for granted and not explicitly mentioned. The generalization of Equation 16 for high-ordered games is obvious.

3. A fundamental invariancy. The above definitions, equations, and conditions result in a fundamental invariancy that is of interest in applications to game theory. Consider a system that passed through a fixed initial point, that is state w_0 at time s_0, and then at a later time u_0 passes through a fixed endpoint y_0. Both the forward and the backward probability functions are defined for the time interval between s_0 and u_0; that is, for a state x that is reached in a time t in this interval. The state x at time t may be considered a variable field point (see Figure 2).

The forward function described the probability that the system will be in state x at time t having started from the fixed initial point. The backward probability expresses the importance of the variable field point to the fixed endpoint. The product of the forward and the backward probabilities is the weighted importance of the variable field point. The sum of this product over all possible intermediate variable field points constitutes what is called the inner scalar product of the forward and backward probability functions—or as we shall refer to it, simply as the scalar product. For conditions where the Chapman-Kolmogorov Equation 3 holds, it is very simple to demonstrate that the scalar product defined in the interval

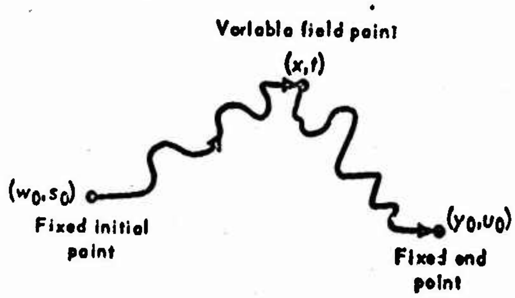


FIG. 2. Stochastic Process with Fixed Initial and End Points.

$u_0 - s_0$ is independent of the time. That is, in this interval it is an invariant:

$$\left.\begin{aligned}&\sum_x P(w_0, s_0; x, t)P(x, t; y_0, u_0)\\ &\quad = \sum_{x'} P(w_0, s_0, x', t')P(x', t'; y_0, u_0),\\ &\quad = P(w_0, s_0; y_0, u_0),\\ &\text{where}\\ &\qquad u_0 > t > s_0,\\ &\qquad u_0 > t' > s_0.\end{aligned}\right\} \quad [17]$$

The proof is a simple one and is omitted. A simple extension of this theorem leads to the invariancy of the scalar product of the state forward probability with the state value function:

$$\left.\begin{aligned}&\sum_x P(w_0, s_0; x, t)Q(x, t; y_0, u_0)\\ &\quad = \sum_{x'} P(w_0, s_0; x', t')Q(x', t'; y_0, u_0),\\ &\qquad u_0 > t > s_0,\\ &\qquad u_0 > t' > s_0,\\ &\text{or, for attrition systems}\\ &\sum_x P(w_0, s_0; x, t)Q(x, t)\\ &\quad = \sum_{x'} P(w_0, s_0; x', t')Q(x', t'),\\ &\quad = Q(w_0, s_0),\ t > s.\end{aligned}\right\} \quad [18]$$

The significance of this theorem is that values so defined, coupled with a particular decision criterion described above, permit even attrition games to be treated as zero-sum games. A nonzero-sum game is to be defined in terms of the inapplicability of the Chapman-Kolmogorov equation. Such a situation would arise if values are associated with the particular paths that the system takes in going from the initial to the final point rather than upon these final points alone. For example, in a gambling game where the house takes a percentage of each pot, the value of reaching the winning state certainly depends upon the path taken. In such a system the Chapman-Kolmogorov equation does not apply; it is a nonzero-sum system.

III. FUNDAMENTAL VALUE POSTULATES

The definition of the value function as expressed in Equation 8 or as generalized in Equation 12 has incorporated into it an arbitrary function. The insertion of this arbitrary function is essential since the adjoint probability function alone is only the probability with which a given state will lead to a certain outcome. In the adjoint function there is no means of differentiating between desirable and undesirable outcomes. There is furthermore no means of differentiating between degrees of desirability, as for example, in Equation 9, where desirable goals are assigned a value of plus unity and undesirable goals a value of zero. As has been stated, such assignment is completely arbitrary and may be considered a postulate of the value system. The arbitrary function introduced into the definition of value permits an intuitive weighting of the final states of the system. If many of us can agree on our basic value postulates and if furthermore the decisions are made consistently with respect to values determined from these value postulates then there can be a source of common or of uniform action. Or the viewpoint of the cultural anthropologist may be taken and the behavioral patterns of a culture observed in terms of decisions made. Next a search is made to find a simple set of value postulates to which these actions appear to be consistent. Obviously, varying degrees of success will be encountered in testing these postulates with which a given culture appears to adhere; they must be advanced tentatively, and tested against direct observation. It is in this spirit then that the subject of the assignment of fundamental value postulates is here developed.

A. Fundamental theorem

It is illuminating to explore values in attrition systems as imposed by some extreme value postulates, and it is desirable to add one further restriction on the characteristics of the attrition system. For the time being, at least, the discussion will be confined to time-wise linear systems. A stochastic system is said to be time-linear if the transition probabilities are not functions of the absolute time, but only of the time difference.

That is:

$$P(w, s; x, t) \to P(w, x; t - s), \quad t > s. \qquad [19]$$

If this condition is applied to the limit expressed in Equation 7 it is seen at once that the resultant function is independent of the time. Thus the value associated in such a linear attrition system depends only on the state of the system and not on the time coordinate, and the value Equation 11 may be written simply as follows:

$$\left. \begin{array}{l} Q(w) = \sum_x P(w, x; t - s)Q(x), \\ t > s. \end{array} \right\} \qquad [20]$$

As already indicated in Equation 18 the summation on the right-hand side of Equation 20 is independent of the duration of time $t - s$ during which the system might evolve from state w to state x. A fundamental theorem may now be stated: *There is a unique value function $Q(w, s)$, satisfying Equation 11 above, which agrees with an arbitrary assignment of values $\mu(y_0)$ assigned to the trapped states, and that furthermore this value is independent of time when the transition probability $P(w, s; x, t)$ depends only on $t - s$.* The uniqueness proof is not included herein although it evolves from a straightforward procedure (23). The fundamental theorem may be generalized to nonattrition systems of the type for which the value definition of Equation 12 applies, provided again that the transition probability is time independent and furthermore provided that the postulate for value of any boundary state is a constant in time.

1. An example of the solution of the value equation in an attrition system. Solutions of the value equation in simple attrition systems have been studied for a variety of conditions. One of these will be described here. Consider that task forces of two opposing nations—RED and BLUE—are roaming around in a large area. Occasionally two opposing forces will meet and engage in a battle of annihilation. For simplicity assume that the elements of the task forces are identical. In a battle between a particular RED and BLUE force, the number of BLUE

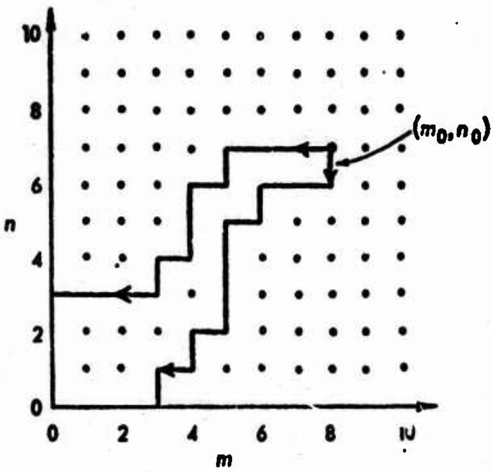


FIG. 3. Two-Dimensional Representation of Battles of Attrition.

elements will be designated by the letter m, and the RED forces by n. The pair of numbers, (m, n), $0 \leq m \leq m_0$, $0 \leq n \leq n_0$, describe the state of the system comprising the battle between these two specific forces. The history of the battle may be represented on a phase plot in two dimensions with the m axis the abscissa and the n axis the ordinate (see Figure 3). Each permitted discrete combination of nonnegative numbers m and n represent a possible state of the system. The state (m_0, n_0) is the initial state. It shall be assumed furthermore that the nature of the engagement is such that Lanchester's Square Law (14) applies.

The Lanchester relations refer to a deterministic description of such a battle. The rate of loss of BLUE forces is proportional to the size of the RED forces, and vice versa:

$$\left. \begin{array}{l} \dot{m} = -k_{mn}n, \\ \dot{n} = -k_{nm}m. \end{array} \right\} \qquad [21]$$

These simple differential equations are referred to as the square law because if one imposes a condition of parity, namely, that the percentage loss rates on either side are equal,

$$\frac{\dot{m}}{m} = \frac{\dot{n}}{n}, \qquad [22]$$

or in mathematical language the logarithmic derivatives are equal, there results:

$$k_{nm}m_0^2 = k_{mn}n_0^2 \qquad [23]$$

as the conditions for expected parity under the initial conditions. Equation 23 expresses the fact that under the conditions described by Equation 21, the effectiveness of the task force as determined by the number of elements that constitute it is proportional to the square of the number of elements rather than to their first power. For simplicity's sake it will be assumed that the constants of proportionality are equal. The stochastic analogue to the deterministic Lanchester battle may be obtained by assigning transition probabilities for the loss of one BLUE element:

$$q = \frac{n}{m + n}, \qquad [24]$$

that is, for a transition from the state (m, n) to the state $(m - 1, n)$. Similarly, the transition from the state (m, n) to the state $(m, n - 1)$ is given by:

$$p = \frac{m}{m + n}. \qquad [25]$$

When one side or the other becomes totally destroyed in this battle, the battle ends. Thus the axes of the plot in Figure 3 represent the trapped states of the system. The states $(m, 0)$ are those states for which there are BLUE survivors and no RED survivors, and are therefore winning trapped states for BLUE, and conversely the states $(0, n)$ on the ordinate axis represent the winning trapped states for RED, or the losing trapped states for BLUE. The values will be

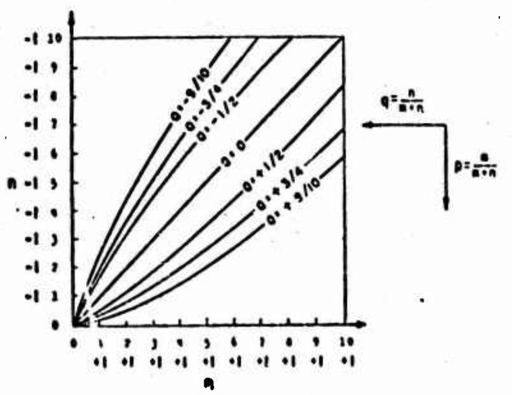


FIG. 4. Ultimate Values in a Stochastic Analogue of the Lanchester Square Law. (Source, Reference 22)

assigned from BLUE's viewpoint. Two cases will be considered.

a. *Case I.* Assume that this battle is for the control of a strategic position. If BLUE can hold, reinforcing task forces will have time to arrive to save the entire situation. Should BLUE lose this battle, RED will overrun the position—gaining an advantage and probably winning the war. Thus the prize won by winning the battle is many orders of magnitude greater in value than the value of the surviving elements. The value of the elements may be ignored in relationship to the value of the prize. Under these conditions, it should be possible to come to a satisfactory agreement concerning the value postulates to be assigned to the trapped states of this battle. By convention the more desirable states are considered to have values more positive. Since the prize of this battle is the entire national integrity or the life of the system from BLUE's viewpoint, it seems reasonable under the circumstances to assign each one of his winning states an equal value and for simplicity's sake the value +1 is chosen. By symmetry each of the losing trapped states is assigned the value −1. It is of course of no consequence whether these values are chosen to be zero or −1. In the present instance, −1 affords a symmetrical presentation. Had the value zero been chosen for the losing trapped states then as already seen from Equation 9 the value of the intermediate states would be exactly equal to the probability for BLUE's winning the battle.

The value Equation 11 is solved essentially in reverse. The state (1, 1) is first selected—the value of this state being given by Equation 11 in terms of the ultimate postulates on the trapped state. This solution will permit a solution of the value for the state (2, 1), etc. by iteration until the complete solution within the framework (m_0, n_0) is obtained. On the basis of these values at these discrete points, contours of equal value may be erected and Figure 4 represents equal value contours in this situation. The line of parity is the line for which the state value is equal to zero. This might also be called the condition of indifference.

A system at parity has equal probability

that either side will win. The deterministic description of the battle would be that it proceeded down the line of parity until both sides were completely annihilated. In the stochastic system this particular path has the least probability. As a curious fact the paths having greatest probability are those for which one side is completely decimated and the other side is completely unharmed. That is, the line of parity is the line of instability. A particular path in this phase plot may be called a history of a system, and it may be observed that the expected histories from points near parity have the greatest uncertainty as to the final outcome. As the initial conditions are farther and farther away from the line of parity, the outcome becomes more nearly deterministic. Thus the deterministic Lanchester Differential Equations 21 may give perfectly adequate description of the system far from the line of parity. Close to the line of parity, however, a stochastic model is required.

b. *Case 2.* Now consider another extreme viewpoint for the assignment of values on the trapped states of the systems. Suppose in a war of annihilation between the RED and BLUE task forces the battle between the particular RED and BLUE task forces illustrated here has only marginal significance. One is concerned then with the value of the surviving element. If the war is to continue, the surviving elements are to comprise a smaller task force that will engage other enemy forces whenever it meets them. Hence, the value of the surviving task force, it shall be assumed, is to be weighted in proportion to the probability of its winning its next battle, also assuming that winning a war still remains of paramount value with respect to any particular intrinsic value of the individual elements. It will be shown in Section III C that the methodology of marginal analysis requires the state value to be separable into two functions—one a function of friendly forces alone and the other a function of enemy forces alone. That is, a solution to the value equation is sought such that

$$Q(m, n) = S(m) - S(n). \qquad [26]$$

Moreover, it should be expected that the separate functions $S(m)$, $S(n)$ are themselves

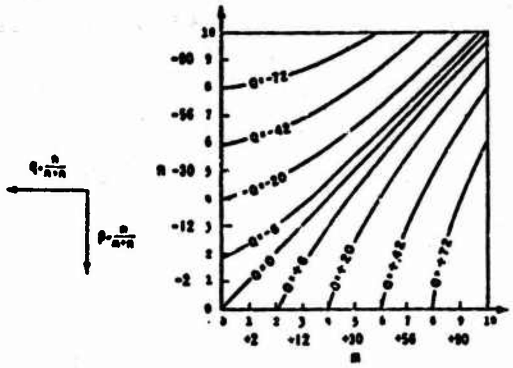


FIG. 5. Marginal Values in a Stochastic Analogue of Lanchester's Square Law. (Source, Reference 22)

proportional to the square of the number of surviving elements in conformity with the characteristics of the Lanchester Square Law.

If the value $m(m+1)$ is assigned to the winning trapped states $(m, 0)$ and $-n(n+1)$ is assigned to the losing trapped states $(0, n)$, then the value of an arbitrary state (m, n) is

$$Q(m, n) = m(m+1) - n(n+1). \qquad [27]$$

The solutions giving contours of constant value for this case appear in Figure 5. It is to be noted that the value function given in Equation 27 is the only nontrivial one that has the form $S(m) - S(n)$. Thus, it appears that in the stochastic analogue the conditions for parity that correspond to the Lanchester square relationship are:

$$m_0(m_0 + 1) = n_0(n_0 + 1). \qquad [28]$$

A third case can be visualized. Assume that this battle between two task forces would be the only battle in which these particular elements could be brought to bear; that they would have no opportunity to enter into a second battle. In this case it might be argued that the intrinsic value of the survivors would be the controlling postulate, and that the trapped state values would be assigned proportional to the surviving elements. It appears, then, that there are many situations, particularly in battles of attrition, in which reasonable and agreeable assignment of trapped state values can be made.

B. Value of a thing

In the discussion thus far the concept of value has been associated with the state of the entire system. This is properly the fundamental and most general way in which values should be viewed. It is customary, however, in our past experience, to consider that values are associated with things and that the value of the system as a whole is a linear combination of the values of things. It was the purpose of the opening discussions on the semantics of things and the illustrative example of the value of a dollar to emphasize that this viewpoint is in general terms erroneous. Only in very special cases can the value of the system be considered to be composed of a linear sum of the values of the things that compose the system. The fundamental notion of value does not depend upon the things themselves, but upon states of things.

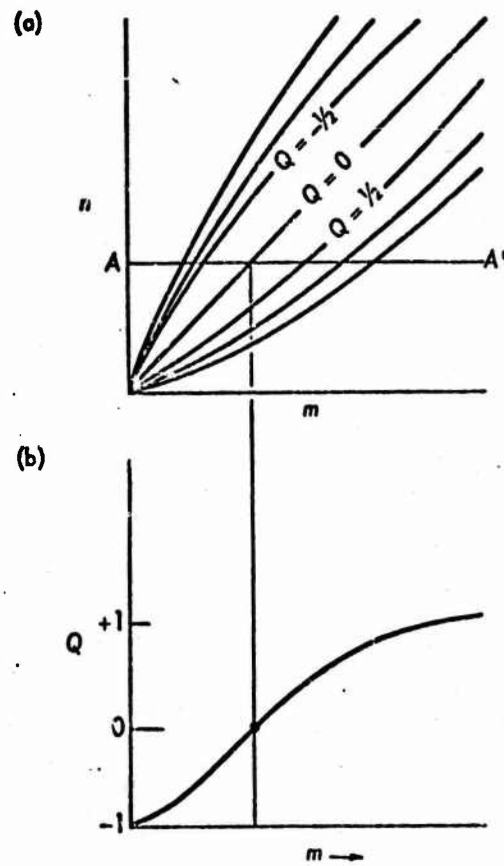


FIG. 6. Illustration of a Cut Through the Value Function.

The value of a thing is here defined as the difference in state values of a system whose states differ by the thing under consideration. This definition is equivalent to defining the value of a thing in terms of the partial derivative of the state value function with respect to the coordinate appropriate to the particular thing under consideration. In discrete terminology the value of the thing is given by the ratio $\Delta Q/\Delta x_i$ where Q, the state value function, is a function of the state x at time t and where x is the vector quantity composed of the commodities $(x_1, x_2, x_3, \cdots x_i)$. The value of a thing is thus defined in the neighborhood of the state of the system, and is in general not a constant quantity but a function of the state of the system. The task force whose values are illustrated in Figure 3 may be used to demonstrate this point.

Consider Figure 6a where the plane AA′ passes perpendicularly through the value function for a constant value of n. Consider the intersection of this plane with the value function as plotted in Figure 6b. The value function Q for $m = 0$ has a value of -1, it then rises monotonically to the value $+1$ with increasing m. It has a point of inflection at the point $Q = 0$ corresponding to the condition of parity. At the point $Q = 0$ the probability that BLUE will win this battle is equal to ½ and the outcome is completely uncertain. This function, of course, is defined only for discrete values of m and n and is illustrated here as a continuous function only for matters of convenience. By definition, the derivative of the function illustrated in Figure 6b is the value of a single element of the BLUE task force as defined with respect to the immediate neighborhood.

This derivative is illustrated in Figure 7. The value of a single element of m for $m = 0$ is equal to zero, it rises to a maximum value that occurs at the condition for parity—that is, the condition at which the outcome of the battle is uncertain—and then falls to zero. The portion of the curve in Figure 7 labeled as I can be taken to illustrate the Law of Diminishing Returns. Whenever the probability of success is greater than ½ and therefore success is reasonably certain, the

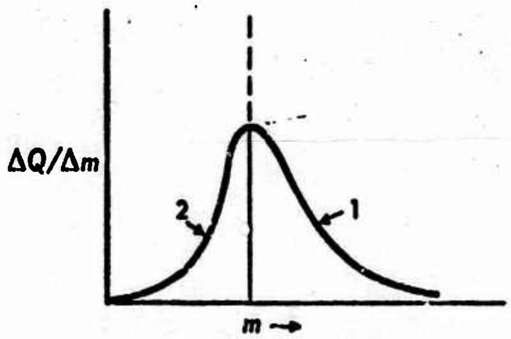


FIG. 7. The Value of a Thing.
(The Derivative of the Value Function)

addition of a single element to the task force adds less value than was added by the preceding quantity, and so on. That is, the value of the things diminish as they are added to the system. Note again that the condition for the applicability of a law of diminishing returns is that the probability of success is greater than ½; i.e., that success is expected. Thus one expects the law of diminishing returns to apply to a system whose chance of survival is somewhat greater than even.

On the other hand, in respect to that portion of the curve in Figure 7 labeled 2 one might propose a Law of Increasing Returns. Here an element added to the task force has a value greater than that added by the preceding element. Values are increasing. Note also that this condition exists whenever the values are below parity; i.e., for conditions under which the survival of the system has less than an even chance. It then applies to very desperate situations. It is reasonable to suppose that one seldom hears this law expressed because it applies to systems whose death is imminent and that do not exist long enough for consideration. There is an old saw to the effect that "a drowning man will grasp at a straw"—essentially a statement of the Law of Increasing Returns, applying to conditions where death is highly probable and situations are very desperate.

The Law of Increasing Returns is the basis of exploitation of desperate people by those who are not in such dire circumstances. This effect may be considered to be true in general. As the system approaches the undesirable state the values of things takes on an augmented and unusual level and the decisions made in the system may appear as highly erratic and irrational to an external observer who himself has relative security and plenty.

C. Marginal analysis

Most analytical procedures presently in use for application to decision purposes employed by operations analysts and other professional groups in the terminology of this theory are marginal analyses. A marginal analysis is defined as that pertaining to a projected change in a system of such a small magnitude that the change in state value produced as the result of a given action can be considered infinitesimal compared to the difference in state value between the least desirable and the most desirable states. In other words, the action contemplated affects the ultimate goals with respect to the system in an incremental or marginal manner. A second condition imposed is that the transition probabilities be homogeneous in time. Under these conditions the state value function $Q(w)$, possessing the proper analytical properties may be expanded in a Taylor series in the phase space defined by the coordinates of the state w around that state w as an origin. Such a Taylor expansion written in difference notation would have the following form:[3]

$$\Delta Q(w) = \sum_i \frac{\Delta Q(w)}{\Delta w_i} \Delta w_i + \sum_i \sum_j \frac{\Delta Q(w)}{\Delta w_i w_j} \Delta w_i \Delta w_j + \text{etc. higher-ordered terms.} \quad [29]$$

In the expansion of Equation 29, the terms $\frac{\Delta Q(w)}{\Delta w_i}$ represent the value of the i^{th} thing evaluated in the neighborhood of the state w, and the terms Δw_i represent the number of things of the i^{th} type consumed by the projected action. The higher-ordered derivatives contain cross-product terms that represent the change in the value of the i^{th} com-

[3] The second-order terms may be used to define a "mutual" value distinct from a "self" value (first-order terms) and may be used to correct first-order marginal analysis.

modity as affected by the change in the value of the j^{th} commodity, etc. By the conditions assumed for the marginal situation all terms of higher order than the first linear set may be neglected, yielding:

$$\Delta Q(w) = \sum_i \frac{\Delta Q(w)}{\Delta w_i} \Delta w_i \qquad [30]$$

as the fundamental value equation applied to marginal situations. Consider now the application of Equation 30 to a battle or game between two contestants. The summation that represents the sum of the product of the value of each type of thing by the number of things consumed may be divided into two classes: the friendly things consumed and the enemy things destroyed. Or, in an economic situation, it could represent raw products consumed and a second class representing final items produced. This leads to:

$$\Delta Q(w) = \underbrace{\sum_i \frac{\Delta Q(w)}{\Delta w_i} \Delta w_i}_{\substack{\text{friendly}\\ \text{weapons}\\ \text{consumed}}} + \underbrace{\sum_i \frac{\Delta Q(w)}{\Delta w_i} \Delta w_i}_{\substack{\text{enemy}\\ \text{weapons}\\ \text{destroyed}}}. \qquad [31]$$

Notice that if higher-ordered terms had been included in this expansion, it would not have been possible to separate the change in state value into two terms associated with these two classes of commodities: friendly weapons consumed, enemy weapons destroyed. *The negative of the first term is defined as the cost associated with the projected action and the second term is defined as the effectiveness of the projected action. Only in a marginal situation does the concept of cost and effectiveness as separable quantities have any meaning.*

As an example of the breakdown of the concept of cost and effectiveness, consider an analysis of decisions pertaining to the establishment of an outpost. It shall be assumed that the mission of the outpost is to delay an enemy's advance for a period of, say, 10 days, or a sufficient time to enable the mobilization of forces in the interior. The decision problems concern questions of magnitude of resources and personnel to be placed in the outpost. The quantities of such personnel and material in a sense might be considered the cost of the outpost. But assume that an outpost of insufficient strength is established, that the enemy overruns the outpost in less than the specified time and catches the interior unprepared and conquers it. In such a situation the cost of the outpost is to be reckoned not only in the commodities that went into its establishment, but also the losses incurred depending upon the degree to which it fulfills its mission. In the hypothetical case the loss was extreme. The entire national integrity was lost, hence the cost of the outpost in which insufficient resources were placed becomes the loss of the entire national values, and not just the resources placed into it. That is, the cost cannot be considered independently of the effectiveness of the outpost. Since cost and effectiveness are not separable quantities in such a situation, they cease to be useful concepts. Such a breakdown occurs when nonmarginal values are considered. In the illustration cited, the values instead of being marginal were total in that they affected the ultimate goals of the system.

Assume that the decision problem has to do with a selection between various ways of accomplishing the marginal task. In comparing the various actions, one may either keep the cost constant and compare the effectiveness, or keep the effectiveness constant and vary the cost. The former is the usual methodology employed by operations analysts in the field where the concern is with the maximization of results with a given weapons system. The weapons system —that is the cost—remains fixed and is compared to the effect produced. On the other hand, in problems involving an initial selection of a weapons system, a given military requirement is selected as a common measure of effectiveness and the cost of various weapons systems required to produce this constant effectiveness is compared, in order to decide between them.

1. An example: intrinsic values and the allocation of common costs in a marginal analysis (22). Value theory has been successfully employed to give solutions to two very puzzling problems in cost analysis. Suppose that the same organization or facility is employed in two different types of missions to produce two kinds of products, say type A and B. There are costs in the organization

and/or facility that accrue irrespective of the quantity of A or B produced. These are so-called common charges. What fraction of the common charges is to be assigned to the cost of production A, and what fraction to the cost of production B? With no further restrictions than those stated, this allocation is perfectly arbitrary.

For example, in the TVA systems the cost of the dams and lakes is chargeable to three missions: flood control, rehabilitation of land, and the production of electricity. How much of the common cost of the dam construction is to be charged to the production of electricity? It may be argued that the production of electricity is purely a by-product and that none of the fixed common charges should be charged to the production of electricity. The net result would be a minimum cost of electricity delivered to the consumer. On the other hand, a competitor to the TVA system might argue that the commercial electricity produced should be required to completely pay for the TVA system. Such an allocation of fixed charges would result in a maximum cost to the consumer of electricity produced. In the actual case, neither extreme is taken.

Another very important situation is encountered at all levels of the decision process and may be included in the illustrative example above, and this concerns the desirability or necessity of the introduction of *intrinsic value* as an *imponderable* into the decision process. In order to set up the problem, let it be assumed that there exists a strategic material involved in the production of A that is not utilized in the production of B. This strategic material is not only relatively costly, but it is in short supply; there is not enough material to produce all the commodities of type A which could be utilized. But the effectiveness of A far exceeds that of B although they are used for the same purpose. The cost-effectiveness ratio for type A is in fact much less than that of type B. Thus, in order to supply the requirement, it is necessary to produce all the commodities of type A as permitted by the quantity of strategic material and to satisfy the remainder of the commodities by producing that of type B. The decision problem that is faced has to do with the methods of production of commodity type A. This is essentially a suboptimization (9). There are various ways that this can be done. What is the cheapest method?

One might, for example, select a method by which the production of a single item of type A, irrespective of the larger problem, was cheaper than by other methods. Yet its effectiveness was so reduced that it materially increases the requirement for the commodity of the less efficient type B. Hence, the suboptimization of the method of producing type A cannot be considered independently of the cost of producing the commodity of type B. *The introduction of the intrinsic value of the strategic material type A into the suboptimization of the production of the commodity A is a device by which the suboptimization of production of commodity A may be conducted in such a fashion that total optimization results simultaneously.* In general it may be stated that whenever intrinsic values or imponderables must be introduced into a decision process it is indicative that a suboptimization is being conducted. But the real problem requires an enlargement in scope of factors considered. One may introduce intrinsic values purposely in order to conduct a suboptimization with a maximum degree of awareness for the total optimization problem.

The following notations will be employed:

n_{ai} = number of commodities of type A produced in the i^{th} manner;
a_i = cost of production of one unit of type A by the i^{th} method, including the cost of producing the strategic raw material consumed;
n_{bi} = number of commodities of type B required to fulfill over-all requirements of A, when A is produced by the i^{th} method (B is always produced by same conventional method);
b = cost of production of each commodity of type B;
$f_{ai}^{\dagger}$ = fixed costs directly traceable to i^{th} method of producing commodity of type A;
$f_b^{\dagger}$ = fixed costs directly traceable to production of commodity of type B;
f_i = common costs incurred when A is produced by i^{th} method;
f_{ai} = fraction of common costs charged to production of A by i^{th} method;
f_{bi} = fraction of common costs charged to pro-

duction of B when A is produced by i^{th} method;

v_{si} = intrinsic value of a unit of strategic material consumed in the production of one item of type A by the i^{th} method;

E = total effectiveness, a constant requirement;

E_{Ai} = effectiveness accomplished by all commodities of type A, when A is produced by the i^{th} method;

E_{Bi} = effectiveness accomplished by all commodities of type B, when A is produced by the i^{th} method;

S_i = relative adequacy of strategic stockpile with respect to meeting requirements entirely by commodity of type A when produced by the i^{th} method, $S_i = E_{Ai}/E$;

U = total stockpile of strategic materials;

u_i = amount of strategic material used in a single item of type A when A is produced by the i^{th} method.

This problem is illustrated in Figure 8. The closed contours represent the respective costs. At the top are the traceable noncommon variable costs that are proportional to the number of items produced, in the second layer are the traceable noncommon fixed costs, and in the third layer the common fixed costs that must be arbitrarily divided and allocated among method A and method B. To these have been added fictitious intrinsic value costs, which total zero.

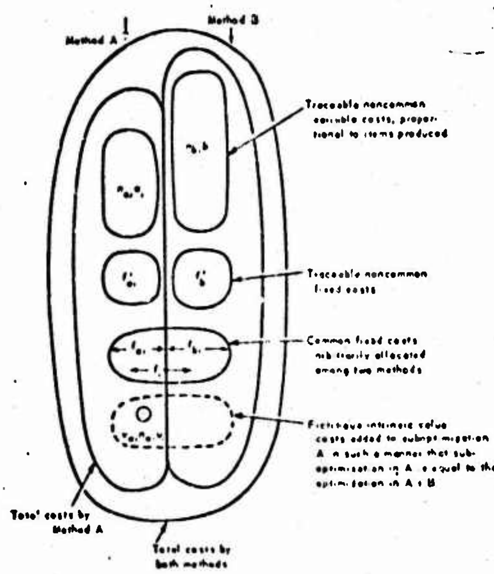


FIG. 8. Illustration of Intrinsic Value Cost, Common Cost, and Traceable Variable Costs in Suboptimal and Total Optimal Analyses. (Source, Reference 21)

The concern here is the portion of these added to the cost of production of commodity A. There exists a unique solution to the assignment of intrinsic value of the scarce strategic material consumed in the production of A, and to the division of the common fixed costs only if one imposes certain reasonable side constraints. Otherwise, the solution is completely arbitrary.

Because of limitations of space, the theorem giving the solution to this problem will be stated without proof:

A unique assignment of intrinsic value of a strategic material and the allocation of common costs may be made if:

(a) it is required that the suboptimization of production of commodity A,—that is, the selection of the i^{th} method—be made in such a fashion that the same selection would be made if the combined production of A and B (that is, the total optimization) were considered;

(b) the intrinsic value costs over and above the actual cost that is assigned to the strategic material must go to zero whenever the strategic material is no longer scarce (that is, whenever the total requirements can be supplied by the commodity A); and

(c) the allocation of common costs is made according to the same rule whether or not an intrinsic value is introduced and conversely, the rule for assigning intrinsic values is the same whether or not the allocation of common fixed cost has to be faced.

Under the condition of these constraints the common fixed cost is to be divided in proportion to the ratio of total effectiveness accomplished by commodities A to that accomplished by commodities B:

$$\frac{f_{ai}}{f_{bi}} = \frac{E_{Ai}}{E_{Bi}} \qquad [32]$$

$$f_{ai} + f_{bi} = f_i$$

and the intrinsic value assigned to the strategic material is proportional to the cost of replacing commodity A by commodity B multiplied by the fractional inadequacy of the stockpile of strategic material:

$$v_{si} = \frac{1}{u_i} S_i(1 - S_i) \left[\frac{E_A}{E_b} b_i - a_i \right] \qquad [33a]$$

$$= \left(1 - \frac{E_A}{E}\right)\left(\frac{D_{bi} - D_{ai}}{U}\right) \qquad [33b]$$

The theorem stated in Equation 32 yields a powerful tool for the study of cultural values. In the example of the TVA dams, the employment of this theorem states that if the common costs are to be divided among

its three missions in such a fashion that total optimization for the nation results, then this division is to be made in proportion to the value of the three separate missions to the nation. What is the value of flood control with respect to rehabilitation of land? With respect to the production of electric power to the nation? Such national values are largely unknown. When facing such a decision for the first time the theorem has no usefulness a priori. It does, however, have a usefulness a posteriori. One may use the actual decision reached as to the allocation of common charges and essentially utilize the theorem in reverse; namely, that past decisions imply certain values. Thus, the actual manner of allocating common charges by inference determines the national values that in effect are held. Similar problems arise in the setting of rates for interstate transportation. What fraction of the cost of erection and operation of railroads should be allocated to the cost of passenger transportation, and what fraction should be charged to the cost of freight transportation? The theorem would state that this allocation of charges should be in proportion to the national value of freight to passenger transportation. In an actual case, these values may be unknown. One uses the theorem in reverse and the actual allocations that are made are used to determine the national values that exist in effect.

2. Cost-effectiveness ratios. One sees first of all that the concept of intrinsic value that would appear essentially as an imponderable in the suboptimization of systems A, was introduced in such a fashion that the problem of larger scope was in effect considered. If the initial attack to the problem had been from the viewpoint of the problem of larger scope, there would have been no necessity for the introduction of the concept of intrinsic value of the scarce commodity. Thus it appears that whenever the analyst feels that intrinsic values and imponderables must be introduced into his problem to give reasonable weighting to the solution, it is indicative that the study was not conducted on a sufficiently broad scope. The use of intrinsic values approximately determined by problems of larger scope is a technique that can be employed in suboptimal problems under conditions for which there is no free market place to which appeal could be made by observation to determine the level of the intrinsic value.

One also sees in this methodology a technique for continually broadening the scope of an analysis with respect to more and more diversified uses for a strategic material as well as for the introduction of other factors.

It is probably good technique to carry out each suboptimization with due consideration of the next broader degree of optimization. In this manner one can proceed by iteration between studies of adjacent areas to successfully broader states of the analysis of very complex and broad problems. For example, assume that the stockpile of strategic material considered in Equation 33 is only the portion of the total stockpile allocated to partially meet the required objective given in this particular example; that there are other uses for the strategic material in which it is in general in short supply. Each problem may be treated separately and the intrinsic value of the strategic material determined in each specific category of use. The stockpiles allocated among these varied uses can then be adjusted until the intrinsic value of the strategic material is the same for each possible use. *When such a condition is reached the allocation is optimal.*

Indeed, it may be stated as a general result that the value-to-cost ratio of the expenditure of resources, or as more commonly expressed, the effectiveness-to-cost ratio of expenditure of resources, is in the optimal equilibrium system equal for all commodities involved. This means, then, that in situations in which the total effectiveness is held constant, the effectiveness-cost ratios may not be used as an index leading to preferred actions. Instead, the difference between the effectiveness and cost must be considered. This law may be proven independently or may be considered as the corollary to the theorem on intrinsic values. It extends not only into economic matters, but also into tactical weapons systems. In a stable tactical weapons system the effectiveness-to-cost ratio of any one component of the system is equal to that of any other. Thus, if an

entirely new weapon that has an unusually high effectiveness-to-cost ratio is introduced into the system, it may be predicted that the environment will react to the introduction of this weapon until equilibrium is again established.

Another use for the intrinsic value concept is that it permits a more rapid convergence in the iteration of solutions of problems in adjacent areas such that each problem is optimized with a reasonable (but not completely detailed) consideration for the solution of the adjacent problem.

D. Values in immortal and mortal systems

In general, real systems are not as simple as the ones described above. In the first place, real systems are not always sharply defined. The original definition of system stated that it was the set of all states that were connected by nonzero transition probabilities. This definition is strictly applicable only to highly simplified models that have some of the characteristics abstracted from the world of experience. Real systems are never completely closed. They are generally embedded in an environment to which they are coupled in various degrees, as Miller (16) and his associates observe in referring to them as "open systems." One must continually remind himself that his rational operations pertain to a *model* having characteristics abstracted from the wide range of his percepts. The decision problem itself must generally act as a guide for the formation of a model. In principle it is desired to abstract all of those qualities that affect the decision importantly and if at all possible, omit all of those qualities that do not affect the decision importantly (20).

It is also found that our knowledge of the system is often incomplete. Since value judgments constantly refer to future states, it is not always possible to be fully cognizant of all the possible future states of the system. Who in 1935, for example, projecting his viewpoint to 1945, would have predicted the existence of atomic weapons? Developments constantly introduce new conditions that cannot be anticipated into real systems. New developments not only in technology, but also in methodologies, in language, and in theory, constantly enlarge the volume of the known universe in a set of possible states. Some systems must be considered in which the winning states do not have significance. This is particularly true in looking at national values as the nation develops from one point to another. During a war the system considered is a set of two nations at war. When a war is ended that system ends, but the nation—at least the winning nation—continues. At the beginning of the war it is not unreasonable to make decisions according to the principle that as long as the war is won any final state is to be considered to be of the same value as any other. However, as the end of the war approaches and it is seen that it will probably be won, more and more consideration is given to the state of the nation after the war. It is then customary to assign intrinsic values to the surviving element after a battle of attrition. Yet, for these intrinsic values one must look further into the future to the utilization of these elements for individual goals in future situations. What is the value to be assigned to the winning state, for example, in which a nation is left so weak that it could readily be overcome by a third one? Hence it appears that the values of the final states of one system have to do with the success or failure of a system formed of the surviving elements of that particular system, and so on. In the case of a nation, national integrity itself may be considered to constitute a prime value of a system. During a war the nation is a subsystem of the combined nations at war. During periods of peace it may form coalitions with other nations, and thus appear as a subsystem in many other different supersystems. Hence in every system there appears to be a residuum of values that refers to the next future problem of overcoming stresses, etc. ad infinitum.

It shall be attempted in this section to state some postulates that are believed to be basic and fundamental to all systems. It should constantly be kept in mind that these postulates are intuitive in character; that it is not the attempt of the author to say that systems should adopt these postulates as their own. It is the attempt of the author on observing systems, to postulate that they

acted *as if* they had these postulates in mind. Whether or not these postulates satisfactorily explain the actions of individuals and systems must be left to experimental observation, historical study, and to the reader's own intuitive approach to the same problems. The statements of value postulates to follow may not necessarily be the best ones, and most certainly not the only ones; it is believed that they contain some of the most elemental ones. They are presented here in order to stimulate the investigation into this important problem and to serve as a basis for discussion.

1. Survival. The peacetime values of a nation are not so sharply defined as those it possesses when engaged in a war. This may be true, however, only because the possible stresses are numerous and the particular stress for which it should prepare itself is uncertain. Over the long run, it appears that a basic and fundamental value is associated with survival. Postulate 1 is therefore proposed:

Postulate 1: *All systems strive to perpetuate themselves.*

In terms of this postulate alone and within certain qualifications to be discussed under the topic of "Mortal Systems," the value to be assigned to the surviving states is a linear function of the survival probability with respect to the stresses imposed upon the system. For simplicity one may adopt the convention that the death state of the system is assigned a zero value. Thus in terms of Postulate 1 the value to be assigned any other state in a system is the survival probability.

Systems are often stressed simultaneously from many causes—both external and internal. For example, it may be possible for a government to make decisions concerning national action that would enhance its probability of winning a potential war with an enemy. The consequences of these decisions might being about internal stresses within a nation; that is, within the values adopted by the subsystems within it, such that these subsystems would repudiate the authority of the government and cause it to meet death from internal stresses. Thus, in assigning values to future states of real systems, one must consider the time-wise application of all the possible stresses.

It is not the purpose here to go too deeply into time-dependent value theory. We shall assume, in the notation below, that conditions exist (See III A) under which values can be expressed independently of the time variable and shall conclude as the result of Postulate 1 that the value associated with state w can be expressed:

$$Q(w) = \sum_x P(w, x, t - s) \prod_i Q_i(x), \quad [34]$$

where $Q_i(x)$ represents the survival probability with respect to the i^{th} stress. The equation reads, in words, that the value of the state w is given by the product of the survival probabilities of the state x with respect to all possible future stresses, multiplied by the transition probability from the state w to state x in time $t - s$.

Equation 34 successfully transfers the problem of value determination from the present state w to future states x. It would be necessary to reapply Equation 34 at the future states x, referring to even more remotely future states y etc, the series never being finished. In the case of the simple attrition model the very happy condition existed that enabled the assumption that the system would ultimately come to rest either in a winning or a losing or a draw situation; i.e., it would ultimately and surely become trapped. It is necessary to add another postulate before successful solution of Equation 34 can be achieved. This postulate must take the place of the assumption of ultimate trapping in the case of the attrition system.

2. Unpredictability. Not only do real systems differ from idealized model systems in that one may not be aware of all the possible future states of the system, but also as one projects himself further and further into the future the ability to estimate transition probabilities diminishes. Transition probabilities may be used to express the condition of ignorance as well as a bona fide branching probability in the presence of complete knowledge. Thus a rational action in the absence of any information might be to assume that all outcomes have an equal probability. Should a commander attack the enemy on its right flank or its left flank?

In the absence of any information whatsoever concerning the enemy's tactical doctrine or his position or deployment, it would be reasonable to suppose or to act as if the probability of winning the battle was the same whichever flank were attacked. This leads then to a second postulate that will permit the termination of the apparently endless process of referring to future and more remotely future situations.

Postulate 2.: *There exists a limiting state probability distribution for any system as it is considered further and further into the future. This limiting distribution is brought about by the increasing unpredictability of the system states and the increased variance of the transition probabilities. A corollary to this postulate, when combined with Postulate 1, states that there exists a limiting state value as the conditions for unpredictability are approached.*

This postulate is stated rather vaguely, primarily because the consequences of increasing unpredictability have not been thoroughly investigated and must remain a problem for future research. It has been observed by von Neumann (27) and Watanabe (28) that the antecedent problem of retrodiction has a diffuseness, increasing as more and more remotely prior times are considered because of the degradation of information. Postulate 2 here states that there is an equally increasing diffuseness introduced into the consequent problem (prediction), which increases with increase of time ahead of the present. The best decision model can be expected to diverge increasingly from a real situation as time progresses. This introduces a diffuseness in prediction over and above that introduced by the stochastic nature of the process. Rationality can exist only in a small island in time and space around the present—fading into uncertainty and intuition in the past and future. The implication of Postulate 2 as stated is that the completely unpredictable system may be considered in effect a random walk of the phase point in the space defined by the system which is uniform except for the existence of a set of death states. Some very simple systems have been considered in which it appeared that the probability distribution was uniform over all possible states and that the probability of survival was linear with respect to the state density. Assuming this conclusion to be correct, the limiting value under conditions of unpredictability would be a value of indifference.

The effect of increasing the variance of the estimate of transition probabilities, even for a system whose states are known, is to move the expected transition probability from any particular value to that corresponding to uniform probability distributed among the possible immediate outcomes. One is concerned here in physical terminology with the entropy of information concerning the future system. The effect of the increase-of-information entropy will be to consider the system to approach more and more that of the completely random system. More investigation is required to clarify this concept. The net effect of the general idea introduced by Postulate 2 is that the continued mathematical induction toward future and more future states can be terminated. This termination is needed in order to fix the solution of the value equation.

5. Values in mortal systems. Although it was not explicitly stated in the preceding section, it becomes clear that the first two postulates taken alone refer to systems that have a finite chance of survival ad infinitum; that is, they refer to *immortal systems*. What happens to the values in a system that sooner or later must meet a stress that it cannot successfully overcome? An examination of Equation 31 reveals that the value in an immortal system is proportional to the product of the survival probabilities with respect to all the stresses to which it is subjected. *If any one of these survival probabilities is zero the entire product is zero, and the estimated values collapse everywhere to a value of zero.* Indeed, the entire value structure becomes trivial and useless. By implication if values are destroyed, rationality is destroyed, and ability to reach decisions is lost.

If any of the survival probabilities with respect to future stresses is zero, then the system is no longer immortal, but transient, and will be referred to here as a *mortal system*. The simple attrition systems previously considered were such systems. It is proposed

therefore, to consider some of the consequences of the assumption of mortality. Under what conditions can values exist? Postulate 3 is therefore proposed:

Postulate 3: *Ultimate death of a system is inevitable.*

Individuals, nations, even whole cultures go through a cycle of adolescence, maturity, decline, and ultimate destruction. The sun and the stars go through their own cycle of birth, evolution, maturity, decline, and death. Entire galaxies of stars have their own history of evolution. Atoms, even fundamental particles, are subject to change. It appears that all things that are identifiable as systems are transient in character. It is the purpose here to discuss the consequences of the postulate of immortality. Therefore, such religious doctrine as concerns the existence of an immortal state and the indestructibility of the human soul will not be discussed, although these exist as very important postulates that lead to the establishment of values. In particular, they lead to value systems of the type previously discussed. What is the consequence of Postulate 3? Can an ethical system exist in the presence of the inevitability of ultimate destruction?

Throughout this discussion, the systems considered have been limited to those for which the Chapman-Kolmogorov Equation holds, and in particular, the concept of value has been associated with states of the system. It is a consequence of the postulate of mortality that the definition of value must be broadened or no value structure may exist. Postulate 4 is therefore proposed:

Postulate 4: *The value of a state of a mortal system depends in general upon the path taken to the ultimate, inevitable death.*

In general then, one is led in mortal systems to propose non-homogeneous value systems. A few simple cases will be considered and it will be seen that in portions of the history of mortal systems and for certain simple conditions a return to homogeneous values is possible.

a. *Case 1. Specific desired goals.* A simple manner of weighting the paths (the histories) of the system from a particular state to the death states exists whenever there is a set of

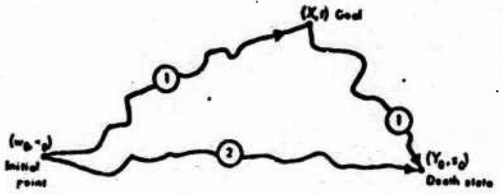


FIG. 9. Schematic Representation of a Stochastic System Containing a Desired Goal and a Death State.

Path 1 is a desired path and is weighted as value of +1.

Path 2 is an undesired path and is weighted as zero.

goals that are desired to be reached before the death of a system. A path that does not lead through a desired goal might be given, for example, a value of zero, whereas a path that leads through any of the desired goals might be given a positive value—say unity. Figure 9 illustrates the desirable and undesirable paths for the case of a single goal. The probability that the system proceeds from the state w_0 at time s_0 to the desired goal X at time t is given by the probability kernel $P(w_0, s_0; X, t)$. If the state X is a *null* state (i.e., if the mean time of the system once at X to return to X is infinite—the goal is reached only once in the lifetime of the system) and if, furthermore, there is no concern over the amount of time consumed in reaching a goal and it is valued equally independently of the time required to reach it, the value associated with the state w at time s is given by Equation 35:

$$Q(w, s) = \sum_X \int_{t=s}^{\infty} P(w, s; X, t)\, dt \quad [35]$$

where the summation is included to extend considerations for paths through other goals equally weighted. Or, if the other goals are not equally weighted, an arbitrary weighting function may be introduced as in[4]

$$Q(w, s) = \sum_X \mu(X) \int_{t=s}^{\infty} P(w, s; X, t)\, dt. \quad [36]$$

The weighting of the desired goal, of course, may be made a function of time. A simple

[4] If X is an ergodic state, i.e., if the reoccurrence time is finite, Equations 35 and 36 must be replaced by more complicated expressions. (See Feller, Ref. 8, p. 320.)

weighting would be that imposed by a deadline after which the attainment of the desired goal is of no value. In this case the integral in Equation 36 would be cut off at the deadline time. Note the similarity between Equation 36 and the former definition of value given in Equation 8. The limiting process in Equation 8 has been replaced by the integral in Equation 36, the fundamental difference being that in Equation 8 the system was assumed to terminate when the trapped state was reached, whereas in Equation 36 the system simply passes through the desired states X. If the definition of Equation 36 is now combined with the Chapman-Kolmorogov Equation 3, there is obtained again Equation 11, the fundamental value equation. Thus the conclusion is reached that in systems having desired goals that portion of their history that leads to the desired goals—or to the death state—may be considered exactly as it was under the discussion of attrition systems. In particular, if the transition probability is time-linear, then as before, the state value equation results in values that are independent of the time. A desired goal is to be treated exactly as if it were a desired trapped state. Once the system has passed through the desired goal, however, this value system collapses and must be replaced by some other value system appropriate to transient conditions.

It is possible, of course, that as soon as one set of goals is reached a new set of goals may be established and a system may proceed from one set to another, each time establishing a new set of homogeneous values as each set of goals is accomplished, and so on, until the ultimate fate is reached. It must be realized that the value structure changes each time a goal is reached. It is a common experience of individuals and organizations to establish in advance a series of objectives. As the successful attainment of one objective is assured, values gradually shift to those appropriate for the next one.

b. *Case 2. Longevity.* Another simple case of mortal systems to be considered are those whose values are concerned with the maximum possible duration of its existence, in which case the value associated with the state w may be defined to be proportional to the expected lifetime, $l(w, s)$, of the system from that state at that time:

$$(w, s) \sim l(w, s) \qquad [37]$$

The resulting value structure is not homogeneous. The simple operations that are permitted by value structures previously discussed do not generally apply. One may restore some semblance of homogeneity by selecting goals or a series of goals such that to pass through them would yield a maximum lifetime. The values placed upon the attainment of these goals might be made proportional to this expected life.

c. *Case 3. Value proportional to magnitude of a system coordinate.* Another possible way of postulating values and mortal systems is to let the value of a state be proportional to the magnitude of one of the coordinates associated with that state. One is interested not only in survival and longevity, but in the most "abundant" life. This abundance might be measured in terms of riches, or other possessions amassed. A nation might set values proportional to a standard of living or a gross national product. Such values may or may not lead to a homogeneous system.

d. *Case 4. Preferred decision codes.* One can visualize systems in which values are placed upon those paths that are determined by more abstract constructs—e.g., the saving of "face," "playing the game," by following the socially established rules of conduct, or in general selecting those paths for high evaluation that conform to established moral constructs.

4. Unpredictability in mortal systems. Consider now what happens to values in a mortal system with such complication and diffuseness of predictability that its death, though certain, lies in the unpredictable future. It has been discussed that unpredictability implies in many respects that the system may be considered ultimately as a random walk, perhaps with some absorption states. It is well known that in such systems, if they consist of more than two dimensions, and if furthermore the death states of the system can be contained in a closed surface, that there is a finite probability that the

state of the system may wander away from the neighborhood of the death states and never return. This is equivalent to saying that under conditions of unpredictability such systems, though mortal, may be treated as if they had a finite chance of immortality. This is proposed here by way of suggesting the fifth postulate:

Postulate 5: *If death of the system, though inevitable, is in the completely unpredictable future, values are assigned as if immortality has a finite probability.*

The application of Postulate 5 to mortal systems whose death is not in the predictable future has the effect of restoring the value systems that are determined wholly by survival—that is, value systems appropriate to a doctrine of immortality.

Thus the implication of immortality is found to be inextricably entwined in the value structures of mortal systems. Since the homogeneity of the value system is so important to decision procedures, this may lead naturally to speculations concerning the decision efficiencies as affecting the technological advancement of cultures whose religious doctrines contain the concept of immortality. Fatalistic societies will find the mechanism of decision much more difficult and thus should not be expected to make the "progress" that the nonfatalistic cultures exhibit.

5. Role of the awareness of mortality in quasi-immortal systems. It has been postulated and reasons have been proposed in support that mortal systems for a period of their histories may act as if they were in reality immortal systems; i.e., are quasi-immortal systems. There will come a time, sooner or later, that the inevitable death emerges from the region of unpredictability. If the values of the individual system have been made solely on the presumption of immortality, the values of such a system will be expected to collapse as soon as such emergence occurs and the individual is left without means of making decisions. The same effect occurs for mortal systems that have just passed through a state designated as a desired goal. The state values must be entirely reoriented; and in the case of the predictable approach of death, the reorientation must be from values appropriate to an immortal system to those appropriate to a mortal one. If the concept of immortality is strongly adhered to under these circumstances it may be expected that increasingly desperate and "irrational" (i.e., irrational to one not facing the situation) measures will be taken as death approaches.

In the terms of value theory this effect has a close correlation with the observation of moralists (10, 11) that the well-balanced individual continually has in mind a sense of the tragic (i.e., mortal and transitory) component of life. The application of the consequences of Postulate 5 in the moral realm would emphasize the need for anticipating inevitable shifts in value systems—as the system previously assumed to be immortal approaches its end.

IV. SOURCES OF DISAGREEMENT IN VALUE POSTULATES

In the description of values thus far, it has been stated that the values postulated for the ultimate states of a system are to be considered arbitrary. In some highly simplified examples, values for the ultimate states have been assigned in a manner in which it can be expected general agreement would result. In this section the important sources of disagreement in the assignment of values of ultimate states are to be considered.

A. Differences of intuition

If ultimate values are postulated in a purely intuitive fashion it is reasonable to suppose that the intuition exercised by different individuals will result in different value postulates. Referring to Figure 10a the ellipse represents all the possible states of a system composed of two nations at war. If the system is presently in the state A and an action is contemplated that might move the system to state B, an attempt must be made to evaluate B. In real situations, this evaluation is normally done by very approximate means as to the probability of achieving state B, and the value of state B is estimated on an intuitive basis. Since these estimates depend upon the judgment and experience of the individual and since experience differs this will result in an important

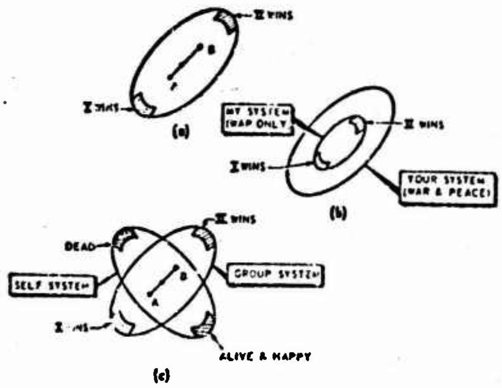


FIG. 10. Schematic Illustration of the Sources of Confusion and Disagreement in Setting Values to the Final States of an Episode. In (*a*) disagreement may result in an estimation of the change of value in moving from state A to state B because the solution is made intuitively. In (*b*) disagreement between the assignment of values of the trapped states may result in differences in areas of closure of systems considered. If I am concerned only with winning the war, no matter what else may happen, then I assign the values of all winning trapped states equal to, say, +1, all losing trapped states −1. On the other hand, you may be considering the reconstruction following the war and wish to value the end of the war proportional to the number of men remaining. In (*c*) is a schematic illustration of a state of conflict in the mind of a tank driver who must choose either to maximize the expectation of his group as a whole, or his personal expectation. (From Reference 22).

source of disagreement. From the theoretical viewpoint however this class of disagreement may be considered a trivial one.

B. Difference of area of closure

Figure 10b illustrates another source of disagreement brought about by the scope of factors included in the consideration of the action. The small ellipse represents the state involved while the state of war exists between two nations. The larger ellipse represents those states in the peaceful reconstruction in addition to the states during the war. One individual may approach the contemplated action as if his sole concern is to win the war, and he values any winning state equal to any other. A second individual concerned not only with winning the war, but also with the peaceful reconstruction, will disagree, and will argue that intrinsic values should be assigned to the survivors.

1. Intrinsic and extrinsic values. This viewpoint leads to an interpretation of the difference between extrinsic and intrinsic values of things. The value of a thing as defined can be a combination of both types of values. If the values of final ultimate states of the system are not weighted proportional to the individual elements surviving, and if all states yielding the prize are weighted equally, then the value of the element in the interior states may be said to be wholly extrinsic; i.e., its value is wholly determined by its utility with respect to the goals of the system. On the other hand, if in addition to the value placed upon the goal, values are added in proportion to the number of surviving elements this value may be considered to be an intrinsic value—intrinsic values with respect to consideration of wider scope. As the scope under consideration becomes greater and greater and approaches the universe of all known factors there may or may not be a residuum of value associated with the individual element (depending upon the postulates of the value system). If it is postulated in this universal state that residual values are still assigned to the individual elements, one has what is normally referred to by philosophers as intrinsic value.

C. Differences associated with states existing simultaneously in several systems

A very important source of differences and uncertainty in the assignment of values occurs whenever the states involved by the given action exist simultaneously in more than one system. Consider a hypothetical example: the dilemma faced by the tank commander in Figure 11.[5] He is facing an

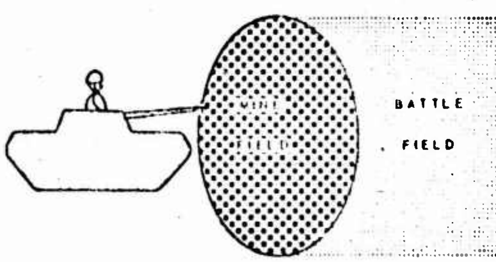


FIG. 11. The Tank Commander's Dilemma.

[5] Taken from a game suggested by D. H. Blackwell.

area in which mines are scattered in unknown places. On the other side of the mine field is a battlefield. The tank commander has been ordered to proceed at all haste through the mine field in time enough to engage the enemy in battle within one hour's time. The mine field is so extensive that by the usual methods of mine detection it would require at least two hours to traverse the mine field safely. This original problem arose in connection with the enemy strategy concerning the layout of the mines and the search strategy pursued by the tank commander.

The question faced here is that of postulating the relative values involved. Three outcomes can be foreseen by the tank commander. Since he has one hour in which to search for mines, he can search for one hour and then proceed without caution through the remainder of the mine field and, arriving there safely, enter into the battle. On the other hand, he might search his way through the entire mine field, consuming so much time that he arrives at the battlefield too late to give aid and assistance to his comrades-in-arms. Lastly, the tank may strike a mine and get blown up. What are the values to be associated with these outcomes? The greatest military value will be assigned to the first outcome—that in which the tank penetrates the mine field safely in time to engage the enemy. If the tank searches too long and does not arrive at the battlefield in time to engage the enemy, the outcome is considered of near zero value since this is an important battle. On the other hand, if the tank gets blown up by the mine field, it is a net loss and cannot be utilized in any future battles.

While thinking this over the tank commander begins to consider the situation in terms of his self-system. Suppose he does get through the mine field safely even though taking his chances in order to arrive at the battlefield on time. He must engage in a very hazardous battle in which there is an appreciable probability that he will become a casualty. So he values this outcome zero in his self-system. In the second place, if he can malinger and take so much time in searching through the mine field, he will not only protect himself against mines, but he will arrive at the battlefield too late to engage in the hazardous battle. This is valued in his self-system as some positive value a'. He will agree (within the frame of his self-system) with the military experts that if his tank gets blown up a net loss results and will value the final outcome as $-b'$. On the basis of the elementary criterion of decision and the military system of values, the tank commander will favor course of action a. He will search in the mine field for as much time as he can spare, take his chances on penetrating the remainder, and hope to get through to assist in the battle. If he is maximizing his self-values, he will malinger, search for mines, and avoid the battle. Thus one course of action in one system of values and a second course of action in the second system of values are indicated. This is shown in Table 1. Where states are common to two or more value systems, sooner or later such a situation is reached. *This is a typical illustration of conflict in decision.* In terms of the ordinary decision criteria there is no way out of this dilemma. Decision appears impossible.

TABLE 1

VALUES ASSIGNED IN ILLUSTRATION OF PENETRATION OF MINE FIELD

Outcome	Military-System Values	Self-System Values
a. Tank gets through safely in time to engage enemy	$+a$	0
b. Tank searches too long—gets through safely but is of no help in battle	0	$+a'$
c. Tank gets blown up by mine	$-b$	$-b'$

This situation is illustrated in Figure 10e by the crossed ellipses. One ellipse represents the military group system that has the winning and losing states of the war, the other system represents the self-system in which the extreme goals are represented as death on one side and survival on the other.

V. THE RESOLUTION OF CONFLICT

Three procedures are suggested for the resolution of conflict. Actually the first sug-

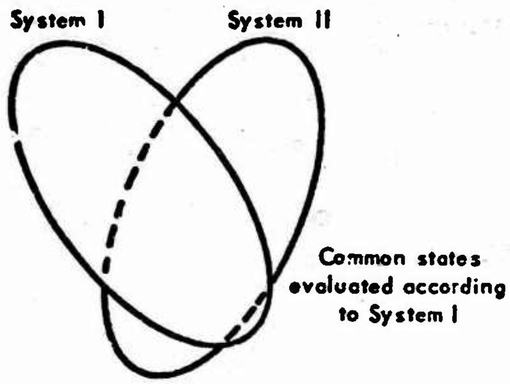


FIG. 12. Dominance and Suppression.

The values of the states shared by the union of Systems I and II are determined by the values of one system only (I) which dominates—the values in the other system (II) being suppressed.

gested method is an extreme special case of the second. These are: "A. Dominance-Suppression," "B. Schism," and "C. Concrescence."

A. Dominance-Suppression

Conflict of values assigned to states that exist simultaneously in two different systems may be resolved by agreeing that the values of one system are to dominate completely over the values of the other system. This situation is illustrated in Figure 12. The dominance of System I may be with respect to all decisions and problems or it may be with respect to certain decision categories only. This situation is a special case of a more general situation to follow and they will be discussed simultaneously.

B. Schism

It may be agreed that one system will dominate only in a portion of the states that exist simultaneously in both systems, that the remainder of this common portion will be dominated by the second system. This situation is illustrated in Figure 13. In effect a boundary is drawn through the disputed union of the systems. A dominance in favor of System I on one side of this boundary, and in favor of System II on the other side results. Should the boundary correspond to the border of either System I or System II a complete dominance-suppression would result.

In general, some type of authority or agreement over or between conflicting systems is required to maintain the schism. This method is a very common means of resolution of conflict, e.g., the whole concept of private property, the jurisdiction of municipal, state, and federal governments. The constitutions of the federal, state, and municipal governments, together with their respective judiciary and police forces, serve to enforce the accepted schisms in our society.

1. Vector values. Underlying schisms in a value structure may exist only for decisions of certain categories. For example, the state and federal governments will have jurisdiction over different types of property, crimes, and civil actions. This is illustrated in Figure

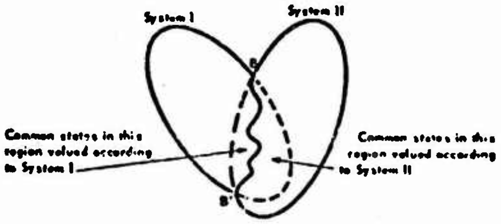


FIG. 13. Illustration of Resolution of Conflict by Schism.

The boundary of BB' divides the states that are common to both systems into two parts. One part is evaluated according to System I, the second according to System II.

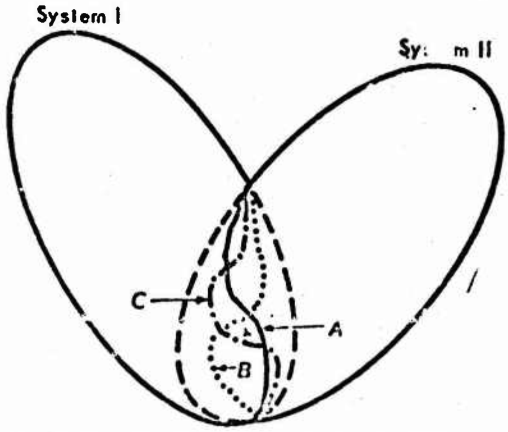


FIG. 14. The Origin of Vector Values by Different Schisms Corresponding to Different Decision Categories.

The boundary A corresponds to one decision category, B a second, C a third, etc. (System I dominates to left of a boundary in that decision category.)

14. The boundary that separates the portion of a common state into two parts such that the values of System I will dominate on the one part and the values of System II will dominate in the remainder is different for the different decision categories. There results for any one particular state in the union of the systems a set of values corresponding to each separate decision category. In Category A the values of state x are determined by System I, in Category B by System II, and in Category C by System II in the illustration. One may confound the situation even further by increasing the number of systems that are involved in the common set of states.

An individual has his self-system, his family system, his community system, his work system, his recreation system, his municipal, federal, and state government system, and his citizenship at large—not to mention such systems as religious groups, social fraternities, professional groups, etc. Complex sets of values are thus built up. The mathematics of multidimensional values will not be discussed here but can be found in papers by Hausner and Thrall on this subject (24).

C. Concrescence

The third and most important process for the resolution of conflict has been given a name borrowed from Whitehead (29)—"concrescence"—and is illustrated schematically in Figure 15. Conflicts develop in the evaluation of states in the union of Systems I and II. Consider that a third system is constructed enclosing not only the conflicting area but the entire systems that enter into conflict. A new set of value postulates is then set up in such a fashion that a new set of scalar values corresponding to the encircled and inclusive system is established.

The word "concrescence" implies synthesis by encirclement. It also implies the growing together of the value systems. For example, consider nuclear physics in the early 1930's. This may be considered to comprise one system. A second system to consider is the field of national politics. In 1933 these systems were almost entirely separate. With the discovery of nuclear fission and the subsequent invention and development of the atomic weapon, both systems became enlarged and overlapping, i.e., they have concresced. Immediately conflicts develop. Should the nation continue to permit free publication of scientific investigation in the field of nuclear physics? Should nuclear physicists be left free to travel over the world if they wish? Is government any longer independent of the properties of atomic nuclei? Such questions can be resolved by any of the three methods of dominance-suppression, schism, or concrescence.

Concrescence is a creative growth process. Although it has been indicated operationally how concrescence may be considered to occur, there exists no formula, no operational directions serving as a guide, that lead uniquely to a set of new postulates in the supersystem that will allow re-establishment of a scalar value. Concrescence is almost an emotional process and for an explanation of the manner in which concrescence occurs one must look deep into the fundamental psychological structure of the individual personality. It is sufficient to point out that concrescence by restoring scalar values automatically relieves the conflict and its concomitant tensions.

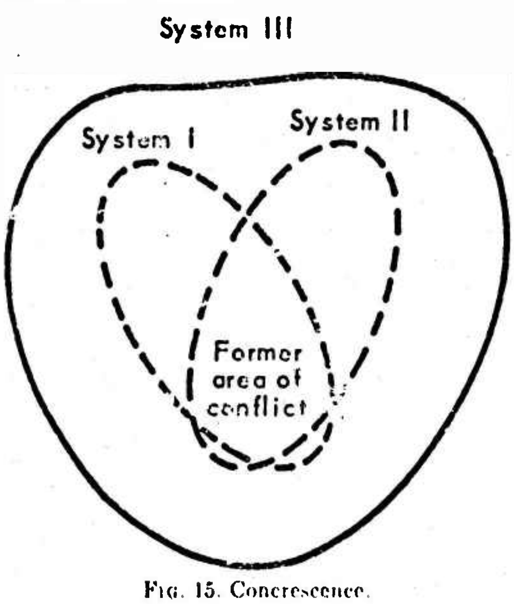


FIG. 15. Concrescence.
A third system enclosing the conflicting systems is constructed; new value postulates are propounded in order to restore scalar values in the total area.

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VI. THE DYNAMICS OF CONCRESCENCE

Concrescence is the means of growth and evolution of a value system, which is never the static quantity that may have been implied in the foregoing discussion. In this section some aspects of the dynamics of the growth of value systems as affected by the concrescence processes will be discussed.

A. Growth of a value system

Consider the evolution of a value system. An individual, say, must begin by adopting a minimum set of values—any values. In the language of a behavioral scientist (16) an initial "coding" is assumed. These values are then used on a trial basis for decision purposes. Sooner or later as the experience of the individual increases, the action indicated by these trial values will lead to conflict. It may be a basic conflict between the indicated decisions and fundamental psychological and physical requirements. A conflict produces a stress, which motivates the process of concrescence—either the old value postulates are changed, or new ones added, in such a fashion that the conflict is removed. Thus one characteristic of the evolution of a value system is that it changes in discrete jumps—in a sense that it may be said to be quantized. Occasionally as the value system evolves with increasing experience and increasing numbers of decisions faced, a set of elemental values will be condensed into a single value construct that reduces the number and complexity of the fundamental value postulates. Actually there has been introduced here, even in a theory that claims to be wholly concerned with the form and not the substance of values, two basic assumptions of postulates concerning value systems.

The first of these is that *the resolution of conflict is to be considered an absolute virtue*. The words "inconsistency" and "conflict" imply the same type of tension and will often be used synonymously. It is frequently stated that consistency is a mark of a small mind. This statement would be explained by saying that consistency in a value system in the face of a conflicting situation is to be considered a lack of growth. Re-establishment of over all consistency by the concrescence growth process is to be considered the highest form of rationality.

In describing the simplification of value systems by the introduction of value constructs, a second basic virtue has been implied; i.e., *the virtue of simplicity*. If two different value systems are equally effective in permitting decisions to be made without conflict the simpler one is to be preferred. In common with usual scientific doctrine we are wielding Occam's Razor.

Although the avoidance of conflict in decision has been described as a fundamental psychological value, it is fairly obvious that this is only one side of a question. In the first place the successful use of values can be accomplished more in a statistical than deterministic sense. Good decisions do not necessarily always lead to the best actions. One must be satisfied with less than perfection—being pleased with a high percentage of successful decisions. Hence any particular value structure must be able to withstand a certain amount of stress and trial and error before any part of it is discarded. The statistical nature of value postulates is acknowledged in the old saying "honesty is the best policy" (not "honesty always pays off"). Individuals learn to adopt a degree of rigidity with respect to value changes. One does not change his fundamental values as a result of the first tension that develops. A stress caused by the conflicting situation must strongly motivate the individual in order to overcome the inertia of rigidity to value change. There seems to be this fundamental built-in conflict that effects the evolution of value systems—the urge to change or adjust values to remove conflict, as opposed to the urge to remain fixed with respect to a value system that has proven successful in the past. Concrescence has been described as a creative and almost emotional process. It is now described as a sometimes painful process. The more a value system has been used successfully in reaching decisions, the greater is the degree of confidence in its workability. This is particularly true if it has successfully withstood conflict stresses in the past without requiring revision by concrescence. Hence one sees a fundamental basic and unavoidable conflict in the dynamics of the

concrescence process itself. Rigidity or pliability may thus become acquired personality traits.

What is the nature of concrescence? How is it accomplished? It is conjectured here that value systems are continually being tested by an imaginative random process. New values are interjected repeatedly on a random trial and error basis (free association?). Rate of occurrence of such imaginative tests is proportional to the tension stimulus brought about by the conflicting situation. We might further conjecture that most of these trials are failures and are rejected. They may be occasionally and accidentally of such a nature that they would remove the immediate conflict but would in turn interject new conflicts in the types of experience already undergone.[6] The object of concrescence is not to remove only the immediate conflict and thereby introduce others, but to remove the conflict without introducing others should the history of the system occur over again. These random trials of new value postulates in some ways are analogous to mutations in a biological evolutionary system where most of the mutations are destructive in character. Occasionally and rarely, however, a trial concrescence is found that relieves the present conflict and introduces no new conflicts. The individual may become suddenly aware that a new value system has been created. Hence the "creativeness" of the process.

Two different characteristics of the value system may be expected to add to the difficulty with which a new concrescence can occur. One of these will be the age of the system (in terms of the successfully accomplished decisions it has rendered). Since any new concrescence must be consistent not only with the immediate situation but with all situations faced in the past history of the system, then new concrescences become increasingly less probable. A second characteristic that may add to the difficulty of performing a new concrescence is the presence of underlying and deeply imbedded schisms in the system. It is not inconceivable to suppose that a complex interlacing of systems deep within a value structure may eventually cause the system to reach a point where further concrescence is no longer possible; i.e., where further concrescence can be reached only through repudiation of the entire value structure, starting anew with a fresh "coding." Such violent and catastrophic behavior of value systems in social cultures can be observed in revolutionary events.

B. Conjecture on "Law of Diminishing Weight"

The preceding section has described how *confidence* is built up in a value system. In particular it is suggested that a measure of this confidence is to be found in the number of cases to which a value system has been successfully employed in decision problems without leading to a situation of conflict. As value systems evolve, some value constructs will be older than others. In the first place there will be the originally "coded" values. These are enlarged upon, and occasional concrescences will occur that add new value postulates or that condense a set of elemental value postulates into value constructs. Consider a step in the evolution of such a system. A series of decisions are made on the basis of a value system until a situation of conflict is reached. Concrescence is needed to readjust the value system. Which one of the many value postulates shall be adjusted first in order to remove the conflict? Certainly it will be that value construct in which the least confidence is placed. Since it has been proposed that confidence exists in proportion to the number of times of usage of the value construct, the first construct to be suspected in approaching a new concrescence is that one which has been subjected to the least usage. In frequent cases this will be the newest construct in the system. Thus, as the value system is reviewed for possible alteration, it is not the long-established values that are adjusted, but those that were established most recently or that for some reason have not been subject to use. This practice will give an overweighting influence on the course of

[6] Although it is conjectured here that concrescence is a trial and error process, there exists the possibility that systematic procedures may be set up which will expedite the concrescent process.

evolution of the value system to the oldest values. Hence, the evolution eventually takes up the form that the old values are seldom tested, only the newest ones. Thus the very first values postulated for the system have an overwhelming weight in the course of evolution of the value system. The newer constructs have an effect on the evolution of the value system that diminishes with their degree of newness. The originally "coded" values in particular will have the greatest weight. This effect may be called a "Law of Diminishing Weight."

1. Reality. Are value constructs real or merely imaginative constructs? Are they arbitrary or absolute? This question will be more fully discussed in the section on the "Epistemological Problem in Value Theory." It is, however, natural to introduce here a concept of *ordered reality. Constructs* shall be considered *real in proportion to the confidence placed in them.* Thus a measure of the reality of a value construct is the number of times it has been successfully (without conflict) employed in decision purposes.

C. Concrescence and the scientific method

In this section the logical congruity between the evolution by concrescence of value systems and the so-called scientific method will be discussed. Consider first the scientific method (15) in building up a scientific theory. In Figure 16 a schematic representation of the scientific method is illustrated.

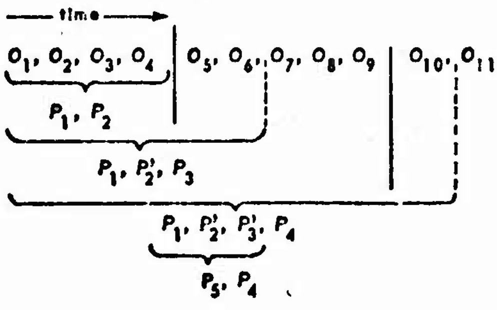


FIG. 16. A Schematic Representation of the Scientific Method.

O_1, O_2, etc. are observations; P_1, P_2, etc. are postulated constructs. The vertical lines represent inconsistencies introduced by new observations, which require revisions of the scientific postulates. P_5 represents a construct of higher abstraction, or order.

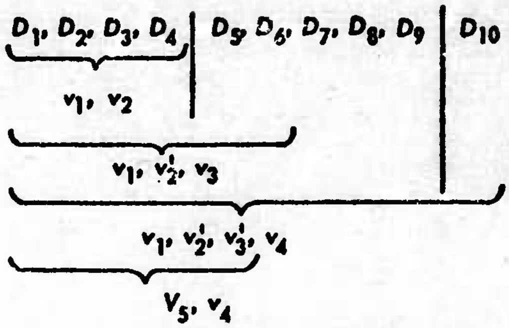


FIG. 17. A Schematic Representation of the Evolution of a Value System by Concrescence.

D_1, D_2, etc. are decision problems. The v_1, v_2, etc. are the value postulates consistent with the decisions. The vertical lines are conflicts (inconsistencies) appearing when a new decision is faced. A new value system is reached by concrescence; the newest value usually being the one adjusted. Occasionally several elemental values are consolidated into one value (moral) construct, (V_5).

A series of observations is first made as indicated from O_1 through O_4. On the basis of these observations, certain postulates are made concerning the nature of the physical world, P_1, P_2.

Sooner or later a new observation, indicated as O_5, is made that gives results inconsistent with the preceding postulates. Thus it may be not only necessary to readjust some of the previous postulates, but to add new ones in order to restore consistency in an ever-widening area of observation, P_1, P_2', P_3. The set of postulates together with the rules for interactions between them constitute the theory. Represented in Figure 16, observations O_7, O_8, O_9, are predicted by the theory. Again, sooner or later, new observations, as indicated by O_{10}, require a new theory or revision of the old theory to restore consistency. Occasionally the theory itself gets so cumbersome with respect to the numbers of postulates and the complexity of relations between them that more general theories or postulates are sought that reduce the number of fundamental notions in the theory, and thus simplify it; e.g. replacement of P_1, P_2', P_3', P_4 by P_5, P_4 in Figure 16.

The evolution of a value system occurs in a similar manner (see Figure 17). A set of

decisions are reached based on a primitive set of values. Sooner or later, as indicated in the figure, a new decision is faced that introduces conflict in the decision. By the concrescence process either new values are added or old values are altered, or both, until consistency is restored and the conflict is removed. In this way one can proceed toward new decisions, with a new set of values. As the history of the system develops, new decisions are faced for which the value system is inadequate, and the value system must be revised by concrescence to remove the conflict.

VII. THE NORMATIVE APPROACH IN PROBLEM SOLVING

(Authors' Note: Tables 15.1-5 ff. suggest the line of development for an example of normative method in application to a trivialgame situation. Accompanying text can readily be furnished in the event the Editor judges this material promising as an effective illustration of method.)

Table 15.1

Example: A NORMATIVE APPROACH TO PROBLEM SOLVING

Situation: A betting game with outcome decided by an unfair two-sided (and edged) coin.

Rules: a. Player I selects an x from interval $0 \leq x \leq 1$ and $y = 1 - x$.

b. Player II selects Heads or Tails.

c. Coin is Tossed.

d. I pays II following amounts:

Selection by II

Results of Toss		H	T
	H	x	$-x$
	T	$-y$	y
	E	repeat toss	

Table 15.2

NORMATIVE THEORY — EXAMPLE

I.	A 1, 2 One state, H $\oplus$ T, random B Two state, random C Two state, sequence 1 D	}	OBJECTIFICATIONS
II.	Sequence $g = \{H, T, T, H, T, \ldots\}$ $(\lambda$ heads, μ trials)		EXTROSPECTION
III.	$P\{g\} = \binom{\mu}{\lambda} p^{\lambda} (1-p)^{\mu-\lambda}$		SYMBOLIC FREEDOM $0 \leq p \leq 1$
IV.	$F(\lambda, \mu, p) = \binom{\mu+1}{\lambda} p^{\lambda} (1-p)^{\mu-\lambda}$		NORMALIZED P $\{g\}$
V.	Expected Gain $Q(\mu+1) = \left[\int_p p \, dF(p)\right] Q(\mu)$		DECISION ALGORITHM
VI.	$k(\lambda, \mu) = \int_p p \, d\, F(p) = \frac{\lambda+1}{\mu+2}$		SUBSTANTIVE PROBABILITY

Table 15.3

OBJECTIFICATION IN COIN GAME

1a: Both sides Heads

1b: Both sides Tails

2a: One side ea. H & T, outcome random, uniform

2b: One side ea. H & T, outcome cyclic with period τ

2c: One side ea. H & T, outcome quasi random, with probability periodic

3a: One side ea. H & T, plus Edge, outcome random, uniform

ETC.

FORMALIZATION

Introduce a symbolic "operating" variable p, defined

Obj 1a: $p = 1$

1b: $p = 0$

2a: $0 < p < 1$

Other Obj: Test each trial for randomness, cycles, patterns, etc.

Table 15.4

EXTROSPECTION

An observed sequence of outcomes

$g = (H, T, T, H, H, T \ldots)$

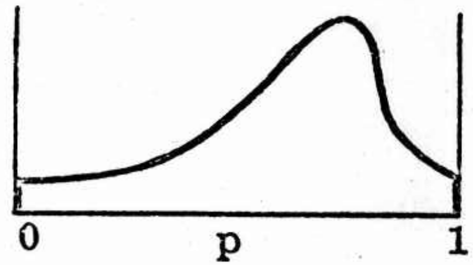
includes	k	Heads
	$n - k$	Tails
out of	n	Trials

n, k may be zero.

Table 15.5

NORMATIVE SETTING IN PROBLEMATIC SITUATION

$$P\{g\} = \binom{n}{k} p^k (1 - p)^{n-k};$$



$$F(k, n, p) = \frac{P\{g\}}{\int_0^1 P\{g\}\, dp} = (n + 1) \binom{n}{k} p^k (1 - p)^{n-k}.$$

VALUE TO PLAYER II

$$dv(H) = (pdF)x - (1 - pdF)y = dv(T)$$

STRATEGY OF II

$$\int_{p=0}^{1} dv(H) = \int_0^1 pdF - y = -\int_0^1 dv(T) \begin{cases} > 0 \text{ select H} \\ = 0 \text{ indifferent} \\ < 0 \text{ select T} \end{cases}$$

STRATEGY OF I: Set $\int_0^1 pdF = \frac{k+1}{n+2} = y.$

$\frac{k \div 1}{n+2} \equiv$ "substantive" probability of H in $n + 1^{st}$ throw.

Chapter 16

TOWARD NORMATIVE ANALYSIS FOR ADAPTIVE SYSTEMS

Despite the perennial dream of attaining an "all-purpose" mode of rational analysis, the conclusion of experience is that each distinctive mode of analysis inevitably generates its own characteristic limitations. So it is with stochastic representation of decision systems. The logical elegance of stochastic models proves extremely advantageous in global representation of macrosystems--for example, a strategic conflict situation in which entire collections of losing v. winning states may be aggregated by gross quantification of value parameters. Yet the effectiveness of this approach is seriously blunted by any demand for representation of the fine-structure of a complex, adaptive, institutional decision system.

FEASIBILITY OF A NORMATIVE ANALYSIS PARADIGM

In the typical social organization--say, a corporate or military organization--composed of many relatively autonomous subsystems, an extensive hierarchical range of <u>interaction</u> significantly affects overall response. The requirement is for multiplex representations, for "total system" models adequately reflecting the intricate connectedness of multi-level, multi-goal processes in organized problem solving. Institutional decision systems are characterized by "distributed" objective functions associated with echelons

of subordinate goals and corresponding echelons of locally responsible decision makers. The control function is similarly dispersed over many decision levels, where positive and negative feedback processes of reprogramming, renormalization, and reorganization are continuously in progress. The modifiability of the characteristics response of adaptive systems has long posed the paramount obstruction for behavioral inquiry in the life and social sciences (though it does not appear that the difficulty has been described heretofore in exact terms). The creative-rational capabilities of individuals cooperatively aligned by social responsibilities further complicate the response of institutional adaptive systems. Thus, while stochastic system models have their definitive uses--as demonstrated throughout Chapter 15--the pristine logic of exhaustive state-description tends to crumble under the demands of practicability in representation of adaptive systems and analysis of optimal adaptive response. The necessity to track continuously changing transition probabilities that are time-sensitive and history-dependent signals the onset of infeasibility. The case is similar, of course, for deterministic models, since no fixed specification of the characteristic response of an adaptive system could purport to yield analytical relationships between initial conditions and consequent states.

The variability of the behavior of adaptive systems is of such order that input-output relations, based on experimental investigation, are typically many-to-many rather than one-to-one. Invariant representations of adaptive systems must therefore be sought in terms of concrete reductions of the hierarchical structure of normative parameters (value-commitments, goals, policies, strategies) suggested by the intuitive notion that an organismic system is in part self-determined in adaptive response. "Concrete" reductions, as against abstract reductions, are representations which preserve distinctions

among all three categories of normative, deterministic, and stochastic aspects of systems behavior. Such a format affords more scope for an attempt to formulate invariant characterizations of adaptive systems. The rationale is as follows: that the operant behavior of an adaptive system--even when replication of input yields a distribution of distinct outputs--may still be conceived as determined by <u>invariant extremalization of system norms</u>.

The intuitive basis of this approach is readily understandable. The emphasis on concrete reduction in system modelling is nothing more than the emphasis with which systems analysis began, i.e., the intention to consider <u>all</u> the significant parameters of decision. Attention to the normative parameters of purpose and goal, policy and strategy, has been the motivation underlying innumerable developments in decision theory, game theory, simulation and gaming, and even in the strictly analytic development of mathematical programming. Value-sensitive decision in general is what systems research is all about. However, the attainment of system models capable of doing justice to the complexity of <u>adaptive</u> value-sensitive decision systems has always required something more than mere technical extension of the conventional scientific format of an abstract reduction. It has required a thematic change of perspective in rational inquiry, keyed to the intuitive notion of an organismic system. It now requires the service of a mode of representation that is capable of mapping in detail the operational function of hierarchical normative control in adaptive response.

In following sections of this chapter, we explore the feasibility of a systems analysis paradigm that is believed to be capable of generating models that are serviceable in the context of the "evolution" of adaptive behavior.

SERVICEABLE SYSTEM MODELS

In the broadest interpretation of the role of systems analysis, a comprehensive scientific-advisory task is openly accepted: the attainment of warrantable conclusions from analysis--sometimes explicit recommendations--that are intended to be immediately serviceable as inputs to a specific command or management decision problem. Two versions of systemic complexity, however, frequently intrude to block acceptance of responsibility for any such ideal total performance. First, the actual context of decision may involve so many levels of organizational structure and so many poorly understood interactions between organizational elements that formal characterization of the actual decision problem (by analytical decision model or operational simulation) cannot confidently be undertaken. Second, any array of value-parameters that might be construed as covering all the relevant aspects of improved organizational effectiveness in the given decision context may well include non-quantitative, incommensurable, intuitive value-measures that--however practically significant they may be--are conventionally excluded from scientific consideration as being "off limits" for objective inquiry.

The Strategy of Reduction

In the face of antithetical demands for comprehensiveness v. rigor, investigators in the emerging disciplines of decision science have understandably tended to take recourse to the longstanding strategem of reductionistic abstraction that has served successfully in earlier sectors of inquiry. With respect to method, the conversion of a realistic but overwhelmingly complicated operational problem into a simplifed abstract representation entails:

(1) Reduction of substantive complexity by decoupling a structural-functional subsystem from the total nexus of its organizational relations; and

(2) Reduction of normative complexity by decoupling quantifiable value-parameters (localized objective functions and immediate measures of effectiveness) from their implicit connection with higher order value-criteria for improved performance of a mission-oriented organization as a whole.

In decomposing an actual problem and representing "essential" components by means of a suitably formalized abstract system of interest, the analyst must intuitively resolve a second-order decision problem <u>of his own</u> concerning selection among (1) alternative strategies of reduction, (2) alternative theoretical schema, (3) alternative decision models, (4) alternative parametric specifications, (5) alternative decision operators and computational algorithms. The formulation of a theoretical model appropriate to the selected problem context directly involves the creative capacities of the analyst <u>as</u> a decision maker. The great range and variability of plausible interpretations for abstract decision-theoretic schema admit of a bewildering number of ad hoc analytical formulations that embody distinctive effects of particular investigators' perspectives and interests with regard to problem definition--and inevitably exclude, as irrelevant, problem aspects that may fail to meet <u>a priori</u> expectations as to significance.

Following the lead of conventional scientific method, this prejudgemental involvement of the investigator has usually been treated covertly. The strategy for obtaining an acceptable reductive model, the demarcation of its range of meaningful interpretation, limiting conditions of uncertainty and constraints on practicable implementation, the criteria for its warrantability as a prescriptive determinant of practical decisions--these are issues that have not generally been regarded explicitly as methodological problems.

This situation is in part the consequence of failure to realize that such considerations have an immediate significance for the decision sciences that they have not had in classical science. But it is primarily the result of a deliberate intention on the part of the technical specialist: to limit scientific-advisory responsibility to just those concerns of logical validity and quantitative prediction that lie within reach of an established expertise, to foreshorten the idealized professional task of comprehensive analysis by relegating to the decision maker all responsibility for valuative judgment. Certain advantages of a detached abstract-reductionist approach are incontestable. Rigorous analytical techniques developed in decision theory and mathematics of optimization are readily exploited in this way. Yet a characteristic pitfall is equally obvious. The crucial issue of the serviceability (interpretability, warrantability, practicability) of analytical results--the issue on which the whole justification for engaging in analysis depends--is consigned to informal exercises of interpretation and appraisal that are notably lacking in the kind of systematic rational control demanded of procedures internal to the analysis. Insistence on separation of responsibilities in analysis v. implementation allows the systems analyst to maintain a reassuring claim to scientific objectivity. It does so, however, at the risk of leaving the procedures of practical implementation and critical appraisal--as to the relevance, applicability, and adequacy of a proffered decision model--sometimes technically or even logically ambiguous and, in any case, <u>untestable</u> in terms of criteria other than belated indications of success or failure by trial and error implementation.

The broader type of assurance that is wanted in a command decision situation is confidence that a proffered decision model is <u>warranted for use</u> in practical decision making. "Warranted" in this sense would mean

(1) that the analytical model has been subjected to explicit test procedures of critical appraisal and (2) that it has been shown to be admissible as a directive to action not only in terms of logical validity and empirical confirmation but in terms of valuative criteria distinctive to managerial concerns: conditions of uncertainty and managerial risk, simplicity and practicability, sensitivity and adaptability to variation of the specific problematic situation, above all, the adequacy with which its implications are related to the overall aims of an actual operational system.

In countenancing the conventional separation of analysis v. implementation, a command or managerial decision maker becomes charged perforce with the whole responsibility for:

(1) Composing a bewildering number of isolated relationships into a coherent pattern of inference having practical significance for decisions affecting his organizational unit as a whole; and

(2) Achieving meaningful connection between valuative consideration associated with:

(a) Alternative trajectories of consecutive situations ranging from a present state to distinguishable future states;

(b) An intersection set of simultaneously relevant value systems with their immediate, mid-term, and long-range goals; and

(c) Incommensurable value-measures assignable to material resources, operational capabilities, time, effort, human life, strategic posture, organizational viability, and the like.

That this type of partitioning <u>over</u>charges the decision maker with responsibility is obvious from the fact that <u>sub rosa</u> strategic commitments--which an analyst must inevitably make in selecting the format of an abstract representation--will already have foreclosed certain of the options that

supposedly comprise the province of the decision maker. Although many scientists would instinctively object to the notion that a managerial type of responsibility is inherent in the activities of problem formulation and choice of analytic modes, every system analyst does undeniably exercise "executive" control over the theoretical component of an overall problem-solving activity and thus participates, however covertly, in valuative judgments that belie any claim to strict detachment.

Systematization

Special significance attaches to the enterpreneurial issue that is raised at this point: To what extent can systematization be introduced into the interdependent intuitive-formal-empirical-valuative procedures involved in scientific investigation and practical implementation of theoretical results in systems analysis? The concept "systematization" in this context broaches the possibility of coherently coupling theory and practice in:

(1) A unified managerial decision process capable of identifying

(2) Minimal-configuration decision systems relative to a particular command control responsibility and resolving

(3) System-specific problem situations via the adoption of

(4) Explicit logical, empirical, and valuative criteria as norms controlling

(5) Theoretical formulation of decision models and selection among alternative models determining

(6) Unambiguous decisions contributing toward improvement of

(7) Operational performance in terms of immediate measures of effectiveness related to

(8) Terminal objectives designated or modified by a superordinate decision system, subject to constraints imposed by

(9) Limits on resources, capabilities, and allowable subsystem stresses.

This interpretation of the overall goal of analysis is, of course, highly generalized--even idealized--for the express purpose of considering long-range significance. On such a scale, it is evident that adequate comprehension and effective exploitation of the complex interplay of theoretical and practical problem solving must entail considerable advance with respect to the rationalization of institutional decision making processes. The practical motivation to systematize disjointed, inexplicit, uncoordinated components in the conduct of analysis opens a far-reaching line of methodological development. The basic enterprise amounts to nothing less than an attempt to organize, formalize, standardize--to "program" so far as may be possible--certain heuristic aspects the problem solving process, in which dependence on trial and error yields results that are not sufficiently coherent, timely, or comprehensive. But this is a fair description of the principal exercise of "rationality" in general: namely, to institute systematic procedures and criteria for testing, redirecting, controlling the artful but insecure operations of intuitive judgment. Stated in uncompromising terms, success must ultimately be predicated on methodological extension of traditional modes of rational analysis--incorporating specification of neglected valuative aspects of "rational" decision--as a means toward systematization and, consequently, warranted applicability of a program of analysis.

The Scope of a Relevant Program of Analysis

It is not necessary to begin by taking overly detailed account of the implications of ultimate goals. An attempt to design research projects

exhaustively covering a range of practical, theoretical, and metatheoretical problems at once may well prove to be premature. Topics of analysis, as presently envisaged, are usually quite sensibly couched in terms of immediately recognizable anomalies or ambiguities--unmistakable obstructions to effective connection of theory with practice--where improvement is clearly a sine qua non condition for rational decision making. On two counts, however, the embedding of subsidiary research topics in at least an outline of the full-scale objective seems to represent not merely a sound strategic approach but, literally, the key to immediately relevant results.

First, consider the essential character of such typical problems as (1) the presence of unresolvable alternative objective functions, (2) their inexplicit connection with accessible measures of effectiveness, (3) the indeterminate effects of covert time-preference judgments. Such problems arise not so much from technical inabilities within disciplinary specialties as from the lack of any overall regimen capable of coordinating many compartmentalized sectors of professional expertise. These problems mark troublesome gaps between the areas of nominal responsibilities of command/management decision makers, systems analysts, theoretical or methodological investigators, and research administrators. For the solution of "interface" problems, cooperation among specialists--with guidance provided by a unifying program concept--is a prerequisite.

Second, reconsider the basic stratagem on which success ultimately depends: that (1) systematization of analysis based on methodological advances will tend to insure, (2) the serviceability of analytical results as directives useable by command/management decision makers. The components for a systematized approach in analysis admittedly exist in profusion throughout the domain of the system sciences. Their assembly into a

comprehensible, manageable process of problem solving, however, presupposes recourse to organizing principles covering generalization, categorization, selection, standardization of analytical resources in codified terms that are specially appropriate to specific practical needs.

It is immediately apparent that "organizing principles" must obtain at a level of generality beyond the scope of methods extant in any particular scientific discipline. They must constitute metascientific commitments adequate to coordinate and direct the coupling relations between scientific and practical decision processes. The necessary incorporation of formal, factual, and valuative aspects of optimal decision outreaches not only the methods but even the aims of objective inquiry. Here again, we encounter a preliminary demand, this time for a comprehensive methodological framework, in order to approach initial research topics with any reasonable sense of perspective.

These two prerequisites (1) an overall program concept and (2) a methodological prospectus--lend meaning to the key term "paradigm." To the root meaning of exemplary pattern or "template," they add the notion of a schematic standard procedure for the conduct of systems analysis. The nature of the exercise in the following section now becomes clear: to envision, in outline, the conceptual and methodological components needed to compose an analysis paradigm that would satisfy requirements for systematization and serviceability.

CONCEPT: SYSTEMS ANALYSIS PARADIGM

In introducing the proposed concept it will be helpful to examine successive stages of sophistication in adaptive control of an institutional command-decision system. Figure 1 presents the basic schematic for an

adaptive decision system, indicating that a representation of any such system must accommodate the dual status of each component as an element of some supersystem and as a collection of more elemental subystems. Expansion of this minimal configuration yields an indefinitely extended hierarchical array which must terminate ultimately at a subsystemic level where mere chanceful <u>interactions</u>--in contrast with controlled responses--indicate the presence of a boundary demarcating the particular system of interest from its environment. It should perhaps be noted that this environment comprises, in fact, just the collection of all "other" systems encountered in competitive or supportive interaction.

As suggested by Figure 16-1, the basic communication-control operations that characterize an adaptive component system at any level of this hierarchy may be analyzed in terms of:

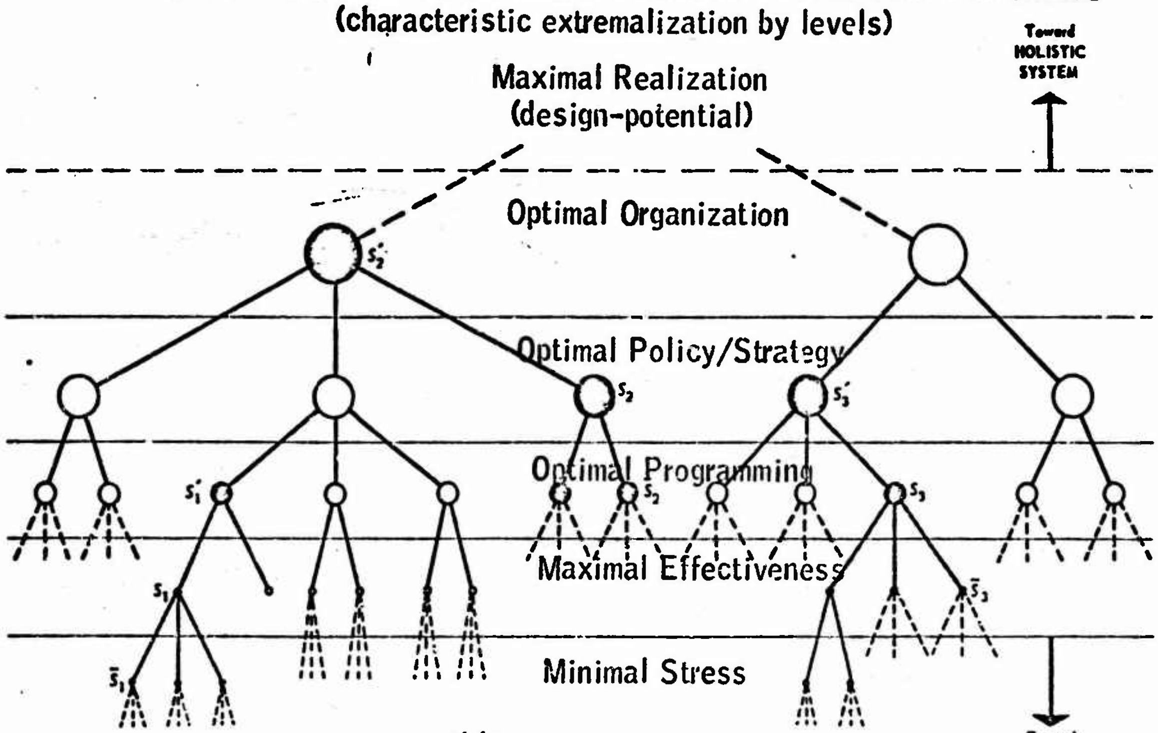
(1) <u>Problematic situation</u>: information indicating disparity between (a) present system state v. system norms or (b) capabilities v. missions;

(2) <u>Decision process</u>: selection among alternative modifications of system organization (design, programs, operations) implemented by modification of subsystem norms (missions, objectives, policies, performance criteria, resource constraints) to achieve

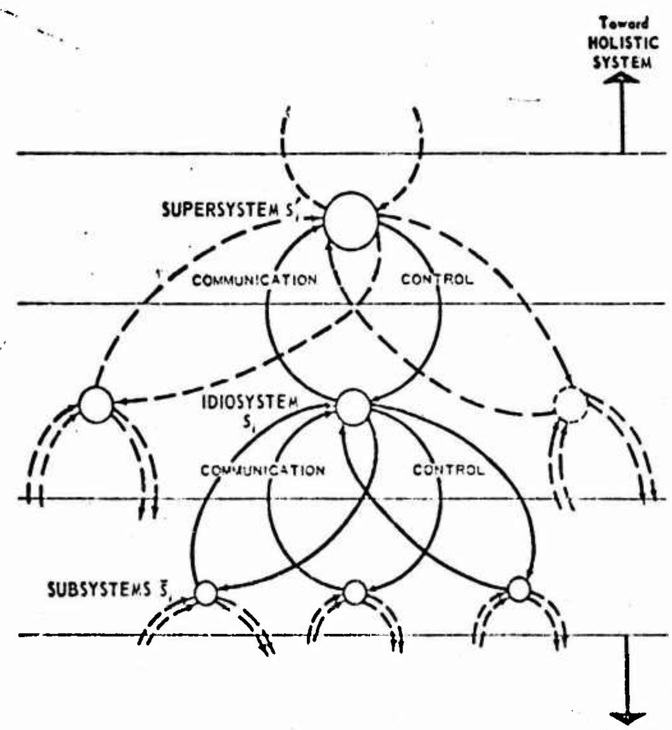
(3) <u>Adaptive response</u>: subsequent information from subsystems (as to outcomes of interaction, present state, and capabilities) indicating an iterative approach toward resolution of the problematic situation (decreasing measures of stress and/or increasing measures of operational effectiveness).

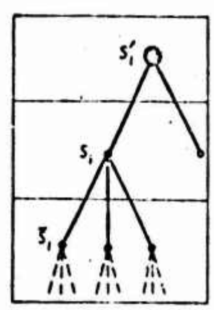
Regenerative (positive and negative feedback) control linkages furnish the rationale of Figure 16-1. Sacrificing detail for clarity, we may represent this total system--in interaction with its environment--as the

HIERARCHICAL EMBEDDING OF DECISION PRINCIPLES
(characteristic extremalization by levels)



16.1a
Fig. 1a—Hierarchy of Decision Systems
Prototype triodic configurations.
S_i, an idiosystem of interest at some level in the hierarchy; $\bar{S}_i$, the collection of subsystems of S_i; and S_i', the supersystem of which S_i is an element or subsystem.





Prototype triedic configuration

16.1b
Fig. 1b—Detail of Hierarchy of Decision Systems
Showing Idiosystem Communication Control
schematic regenerative circuits.

cybernetic loop of Figure 16-2a. Contemporary development and utilization of systems analysis for improvement of system performance may then be associated with the addition of a second-stage feedback loop in Figure 16-2b. Information flow in this secondary loop is directly relevant to the quality of the decision making (command/management) performance in the primary control loop. Improved control in this second stage, however, subsequently yields the possibility of improved operational performance; and it is in terms of this primary criterion that overall improvement must finally be evidenced.

The principal advantage of staged feedback design is its superior ability to provide sufficient conditions for attaining adaptive system response. The secondary loop serves to modify the capabilities of the primary whenever iteration of existing decision procedures proves to be ineffectual in resolving intractable problems. Despite its advantages, however, this system schema cannot be presumed to be unconditionally adequate. Present obstructions to effective implementation of systems analysis already indicate the need to improve, in turn, the performance of the secondary loop of Figure 16-2b. Here the presence of intractable second-stage problems is signalled by ineffectual iteration of a mode of systems analysis in which methodological controls are inadequate to insure the coherence, warrantability, and serviceability of proffered decision models. Extending the stratagem of staged control by addition of a teriary feedback loop, Figure 16-2c presents the schematic design which we would consider minimally sufficient to maintain adaptive response of a command decision system. The intended contribution of the third stage is improvement of the decision making (para-managerial) performance of the systems analyst in terms we have previously associated with "systematization" of analysis.

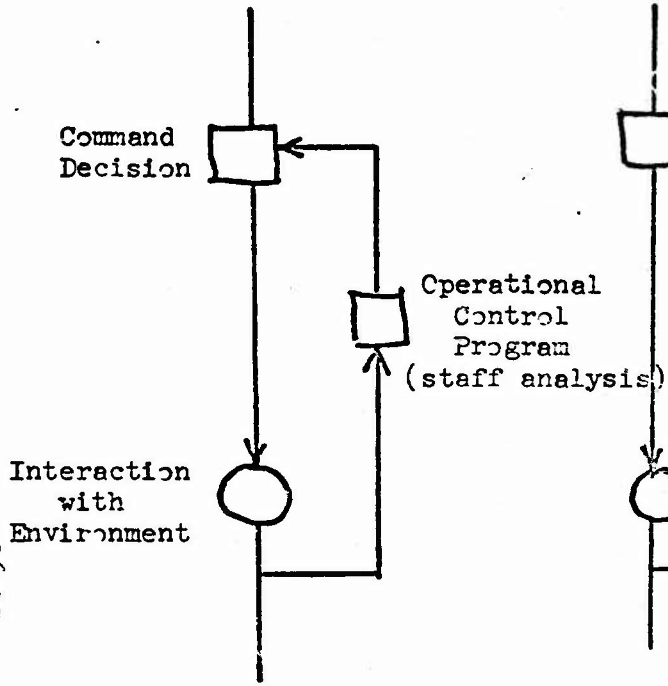


Fig 16.2-a

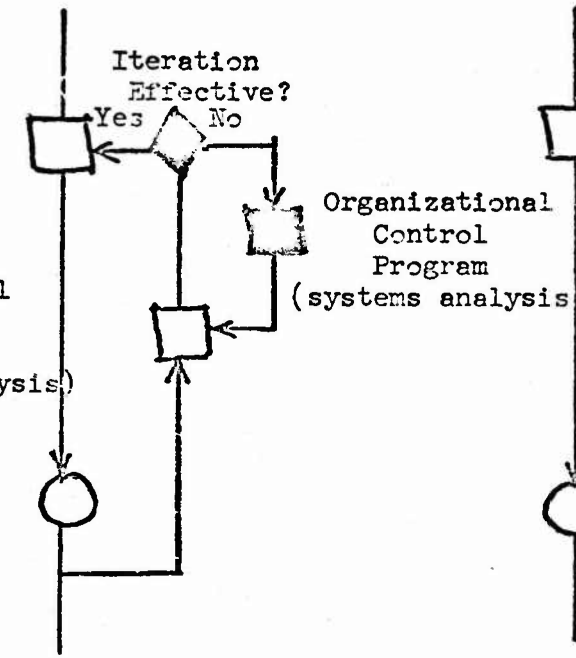


Fig 16.2-b

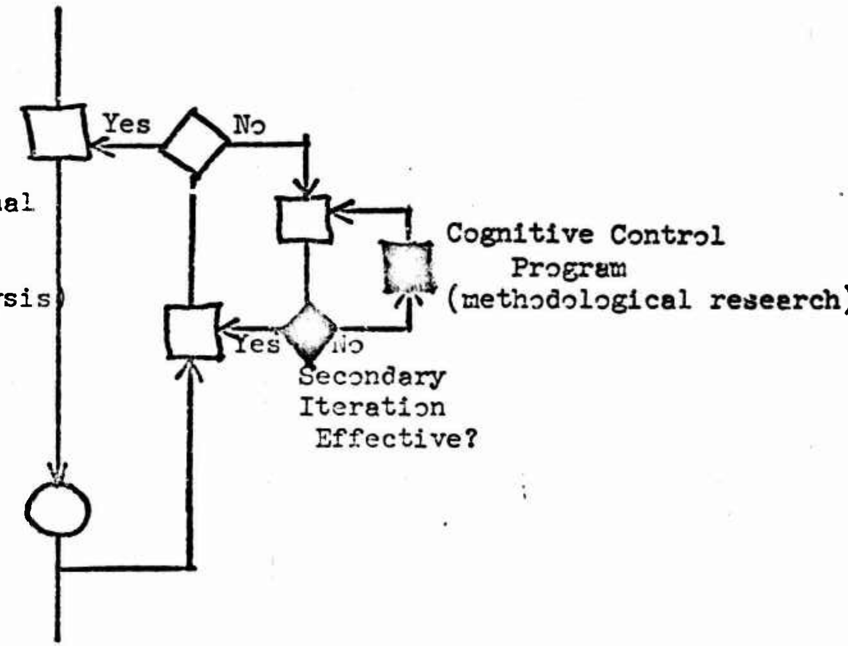


Fig 16.2-c

Adaptive Control in an Institutional Command Decision System

(Successive Designs)

Note: It should be emphasized that the principal service of Figure 16.1 is to prevent misinterpretation of the simplistic diagrams above. We are not dealing here with the engineering concept of a servo-mechanism with a singular error function and a single control function. Institutional decision systems are characterized by "distributed" objective functions associated with several levels of distinguishable goals (and many participants). The control function is likewise dispersed over many decision points throughout the system. Every level of the indefinitely extendable hierarchy of Figure 1 is therefore construed as contributing toward the determination of characteristic response; and it is this combined contribution to which the notion of control "programs" refers in the diagrams above.

Each of the successive stages in the echelon of decision-process control (Figure 16-2b) will be identifiable with a program of activities viz., operational control program (staff analysis and planning),organizational control program (systems analysis), and cognitive control program (methodological research) respectively. Our concern now is to envisage the distincitive program elements that methodological research must furnish in support of the conduct of systems analysis--if adequate cognitive control is to be realized in the third of these decision-process control stages. A general outline can be developed by examining typical procedures of systems analysis and then discerning what conceptual and methodological resources are yet needed in order to insure acceptability of systems analysis output in principle and, particularly, utility in practice.

The characteristic method of systems analysis consists in the following composite of orderly procedures that we ordinarily associate with responsible and competent professional-advisory practice--with the important addition of recourse to advanced mathematical modelling techniques unique to this newest of professions:

(1) Diagnosis of a client-organization's problem situation;

(2) Selection of a theoretic schema and formulation of a reductive representation (decision model) of the actual problem situation;

(3) Constructive specification of a formalized decision problem: quantification of the model and data acquisition, including value-parameters (objective functions, measures of effectiveness, constraints on resources, stress thresholds) as well as deterministic and probabilistic measures;

(4) Manipulation of the model, i.e., experimental simulation, analytical derivation, computation in search for a solution;

(5) Communication of conclusions to the client-organization.

With regard to each of these procedural phases, we outline in Table 16-1 corresponding methodological research projects which are needed to complete an adequate systems analysis paradigm. By means of graph-theoretic devices the general schema of Figure 16-2c might be expanded to display details of the relational logic of a total decision-process control program:

(1) Interrelation of the responsibilities of the several categories of decision makers for command/management, systems analysis, research administration, methodological research;

(2) Detailed procedural phases in the conduct of systems analysis;

(3) Injection of objective and normative process control criteria for admissibility of a system model;

(4) Recourse to conceptual-methodological resources (the metatheoretic commitments and cognitive control principles of Table 16-1) for improvement of system models in terms of interpretability, practicability, warrantability in use.

CONCLUDING REMARKS: THE FUTURE OF PRESCRIPTIVE SCIENCE

Concluding remarks on this line of research must be addressed not to an ending, but to a beginning. A good deal of arduous effort, with helpful critique from many sources,[1] comes only to a "halfway house" culmination in this volume. Although the rationale of normative analysis has recently had notable impact of a major systems study,[2] results of research

1. During the course of this research, we have had great benefit from preliminary presentations of ongoing work before graduate and post-graduate seminars in the following institutions: Case-Western Reserve University (1963), Johns Hopkins University (1964), Industrial College of the Armed Forces (1965), George Washington University (1963-65-69), Lawrence University (1965), University of Maryland (1966), National Institutes of Health (1967-68).

2. SWEM Study (Strategic Weapons Exchange Models), conducted at Research Analysis Corporation, Advanced Research Department, over the period 1966-70.

Table 16-1

SYSTEMS ANALYSIS (Principal Phases)	CORRESPONDING METHODOLOGICAL RESEARCH--PROGRAM ELEMENTS (Re: Systematization and Improved SA Performance)
I. Diagnosis	1. Characterization of Distributed Decision Control by Type-Levels of Command Decision or Managerial Decision 2. Standardized Syntax for Representation of Multi-Level, Multi-Goal Systems 3. Total-System Factorization (by "minimal configuration components specific to decision maker's responsibilities) 4. Decomposition Strategy (decoupling at junctures of least interaction)
II. Model Formulation	1. Taxonomic Classification of Decision-Theoretic Procedures (by "families" of system models based on combinatorial multi-attribute alternatives) 2. Reduction Strategies (including recomposition rules or protocol) 3. Sensitivity Tests: (1) Parametric Variation (confidence interval, resource allocation, stress threshold, type or quality of data) (2) Problem Variation (mission, objectives, range of interpretation, contingency assumptions, risk policy) 4. Detailed Flow Diagrams, Systems Analysis Paradigm
III. Quantification	1. Categorical System Norms and Decision Operators 2. Generalized Classes of Objective Functions 3. Codified Measures of Effectiveness (by system type) 4. Mensuration Principles for Mixed-Component Systems
IV. Solution	1. Tests for Holistic Admissibility of a System Model (1) Formal: syntactical well-formedness and logical consistency (2) Empirical: empirical confirmation and reproducibility (3) Pragmatic: interpretability and serviceability (4) Aesthetic: simplicity or cybernetic "elegance" (5) Evolutionary: meliorative trend or convergence to optimality
V. Communication of Results	Standard Specifications for Warrantability/Serviceability of System Models

Table 16.1: PROGRAM OUTLINE

on theoretical foundations have so far hardly reached the edge of prospects for detailed implementation of normative method in the future of applied prescriptive sciences. Test applications of normative theory and analysis are only now beginning to be considered in areas of (1) military force structure planning and strategic studies, (2) social policy analysis in contexts of technology impact assessment and urban studies, (3) design of interdisciplinary academic programs, (4) national administrative organizational design, and (5) social environment-behavior interaction studies.

It is probably far from evident to a practical minded reader that the original motivation to reach this level of applications should have led necessarily to a long apprenticeship in such investigations as theory of value, cognitive theory, taxonomy of adaptive systems, interpretations of formal duality, and, finally, reconstruction of philosophical commitments. Why did these rarified topics prove to be prerequisites? The answer is: that an intention to analyze the decision problems of a complex social organization--replete with value-sensitive aspects of adaptive response at every level of decision and action--raised a more complicated objective than science had heretofore accepted. A chained sequence of problems--practical, applied scientific, theoretical, and metascientific--was inherent in the nominal problem. And this sequence terminated only with a requirement to undertake one of the more difficult accomplishments in the whole of inquiry: to break out of the situation in which intelligence is locked in by its own abstractions. In this case, the abstractions were those of the value-free mode of objective inquiry, which is so impressively successful in application to inanimate systems yet so defeatingly reductionistic in application to organisms and organizations of organisms.

More adequate techniques of analysis, more realistic system models, more relevant theories of value and decision--all these were needed--but above all we needed a more comprehensive way of thinking about the interconnected world of adaptive systems at large. To demonstrate the interpretability and applicability of a normative scientific mode will be to establish the legitimacy of a new thematic alternative with regard to choice of perspective in rational inquiry. Under this alternative, all the objects of our attention would be conceived as organizations, or organized systems, engaged in competitive and cooperative transactions tending toward extremalization of characteristic variational measures. As the formal equivalent of the "initial conditions and analytical relations" hypothesis of objective inquiry, this law-like regimen is expressly designed to be amenable to purposive, motivational aspects of causation in the context of human behavior. Without sacrificing the mathematical rigor that is the ultimate mark of exact scientific investigation, a normative mode of inquiry accedes to one paramount necessity: to accommodate in science the complexity of the adaptive systems that are of primary human significance.

This is the direction in which Foundations of the Prescriptive Sciences moves toward future applications in decision science; and it is this direction which gives our concluding remarks the sense of a beginning. Inevitably, one pauses over the estimate of laborious and time consuming processes involved in testing and assessing an abrupt modification of the conventional paradigm of "normal" science. At successive levels of theoretic, applied, and practical problem solving, subsidiary innovations cascade from the following changes introduced at the level of primitive commitments: (1) organismic schema, (2) extended canons of rationality,

(3) embedded methodologies, and (4) complementary modes of analysis. In order to carry these strategic innovations into effective practical use, an imposing amount of effort must be devoted simply to explaining precisely what they mean in terms specific to various real-world systems. Despite these "initial costs" of reconstruction, one of A. N. Whitehead's pithiest observations stands as a reminder that, ultimately, the most practical thing in the world may be a philosophy that is adequate to the crucial problems of the society it serves:

> The importance of the theoretical side of science arises from the fact that action must be immediate, and must take place under circumstances which are excessively complicated. If we wait for the necessities of action before we commence to arrange our ideas, in peace we shall have lost our trade, and in war we shall have lost the battle. Success in practice depends on theorists who, led by motives of exploration, have been there before, and by some good chance have hit upon the relevant ideas.

The Aims of Education, Macmillan (1929)

APPENDIX

GEOMETRY OVER A FINITE FIELD

GEOMETRY OVER A FINITE FIELD

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ABSTRACT

The development of certain aspects of a physically interpretable geometry defined over a finite field is presented. The concepts of order, norm, metric, inner product, etc. are developed over a subset of the total field. It is found that the finite discrete space behaves locally, not globally, like the conventional "continuous" spaces. The implications of this behavior for mathematical induction and the limit procedure are discussed, and certain radical conclusions are reached. Among these are: (a) mathematical induction ultimately fails for an finite system and further extension leads to the introduction of formal indeterminancy; (b) finite space-time operations have inherent formal properties like those heretofore attributed to the substantive physical universe, and (c) certain formal properties attributed to continuous spaces cannot be developed from successive embedding in finite space of finer resolution—but must be based on independent axiomatic (non-testable) assumptions. It is suggested that a finite field representation should be used as the fundamental basis of a physical representation.

1. INTRODUCTION

"Mathematics is devised by mathematicians." This tautology contains potentially significant implications. Mathematicians are mortal human beings whose conceptualizing capacity is finite. Acting in a rational mode, or as we shall say, as a "cognitive agent," man communicates at finite rates; employs finite strings of symbols; and has finite data processing and storage capacity. Yet he has devised conceptual mathematical geometries of continuous and infinite spaces. It is reasonable to expect that a man's finiteness qua mathematician will exert a controlling influence on the nature of the concepts he develops. This realization has led us to seek a priori characterizations of these concepts that result from the nature of the cognitive agent who produced them.

Thus, we have set ourselves the task of determining "How do you get there from here." Or more formally, how can a finite cognitive agent develop concepts of continuous spaces and space-time systems as well as the associated mathematical operations. It is necessary to start with the development of numbers—in the finite cognitive system and demonstrate how this leads to operations such as translation and rotation in finite geometries. Then, we examine the meaning of the corresponding processes in continuous spaces in a manner appropriate to the operations admissible to the finite cognitive agent.

The resulting implications of this investigation are in some respects expected;—in other aspects quite radical. The findings of this paper are

2. HEURISTICS

The goal of this effort is to establish that one can perform all legitimate arithmetic and algebraic operations solely within the context defined by a primitve commitment to finitism, etc.[3] In order to achieve this demonstration, we must adopt certain mathematical structures that can serve as the foundation of the various operations. In this section we shall present a series of metatheoretical and motivational arguments that seek to establish the concepts and development that are employed. It is clear that uniqueness and necessity of a representation of experience cannot be proven unless the universe of discourse is closed, i.e., uniqueness and necessity are always with respect to a given context. Hence a system purporting to represent or at least be consonant with experience is perforce backed "only" by sufficiency or demonstrable adequacy.

To insure that arithmetic can be carried out we shall require that the two basic operations of addition and multiplication be defined. Also the inverse operations of subtraction and division will be required. To insure that we satisfy the requirement of ontological parity[3] we shall demand that the basic set be closed under the four primary operations. Nonambiguity similarly implies that the results of these operations be unique. We shall also seek as much procedural invariance as we can by requiring associative and commutative multiplication and addition. Furthermore, the combination of these two operations will be such that the appropriate distributive laws are valid. These conditions are sufficient to define a field as the underlying mathematical reservoir

in opposition to certain of the conventional conclusions and assumptions of mathematics and we are aware of their heretical nature. Thus we ask the reader to consider the arguments in the context of the philosophical viewpoint upon which they are based.

It is found that there are constraints that limit the cognitive agent in actu and contribute certain formal properties to his admissible concepts. In particular, we look at mathematical induction and the process of going to the limit; examples are presented that are physical illustrations of our ideas. It is proposed that certain of the presumed external physical postulates are in fact formal properties of finite spaces and their reintroduction as physical properties results from our using infinite field mathematics. We suggest, and present examples, to show how these postulates arise to constrain the mathematics of infinite fields to represent "experience."

In this paper, we explore a finite field (Galois Field) representation and attempt to formulate a physically interpretable geometry over this field. Thus we seek to define the basic objects of geometry solely in terms of the finite field concepts. We have developed some formal aspects of a vector space defined over a finite scalar field. Kustaanheimo and Järnefelt[1] did extensive work to formulate such a geometry in order to provide a structure that was consistent with the apparent finiteness and discreteness of the physical universe; since then there have been additional efforts to refine the mathematics.[2]

for our primitive operations. Therefore, as an immediate consequence of our commitment to finitism, we are led to consider a finite field (Galois field).[4]

If our mathematical system is to serve as a suitable basis for the many computational operations, then it must admit of many other operations that are to be considered legitimate. There are certain such operations that do not always lead directly to a formal answer because of the severe limitations imposed by the restriction of finite resources and capabilities. However, in any actual calculation one always has finite and greatly limited resources and that never becomes a deterrent or causes termination of the logical procedures. One simply replaces the problem for which there is no formal solution by some solvable problem taken from the given field. Thus, for example, when computing the square root of two, one "truncates" the calculation at the desired level of resolution. Clearly this is tantamount to introducing a replacement problem. If we seek a resolution of one decimal place in the answer, then we look to a neighboring perfect square, 196×10^{-2} that is "close" to 200×10^{-2} and declare the answer to be 14×10^{-1}. In this way, replacement permits our mathematical operations to continue and avoids cessation due to uncomputability or lack of performable instructions. One could also construct his system to reset itself to some arbitrary point—say zero—whenever an impasse is reached. However, the rationale for replacement is clearly preferable because it seeks a "nearby" problem and we shall adopt it.

Let us point out that we have described replacement with a "neighboring" problem or "best" approximation but the bare algebraic structure does not yet have any procedure for determining such a "best" replacement. Thus we need some measure of proximity or closeness in order to determine that which is the appropriate substitute. If we are to define a vector space over the finite field, then a metric can fill this requirement. However, even if some value can be associated with the "distance" between points, we still require a mechanism for comparing different distances. In short, the underlying number field must have some ordering relation. Since our primitive commitments do not demand global operations but merely a suitable local definition, we shall seek—as a minimum—an irreflexive[5] binary relation. Clearly there is no way to define a meaningful transitive order throughout a finite field and still retain the other properties of uniqueness, nonambiguity, irreflexivity, etc.

In defining a metric for a vector space, we will encounter the square root operation. If we restrict ourselves to a ground field GF(p), then there will be formal square roots for half the elements of the multiplicative group, i.e. for (p - 1)/2 elements. This behavior is reminiscent of the analogous property of the real number system in which only "half" the elements (positive elements) have square roots in the field. If we wish to institute an extension of GF(p) in order to generate a square root for every element of GF(p), then we may do a

similar thing to that done in conventional analysis, viz. embed our system in a "complex plane" obtained by expanding GF(p) via $x^2 + 1$ as a prime ideal. In this way all "real" numbers (i.e. elements of GF(p)) will have formal square roots. Unfortunately, we have merely set the problem back one stage for only $(p^2 - 1)/2$ elements of $GF(p^2)$ have square roots in $GF(p^2)$. We can establish a replacement technique for these nonsquare elements of $GF(p^2)$, thereby closing our system. This procedure does generate a formally satisfactory system for all elements of the ground field. Actually, we will find that even these hard won formal square roots for GF(p) do not in general behave as desired and we are forced to introduce still another replacement procedure to rectify the situation. This is necessitated by the additional demand that square roots of ordered numbers lie in the same order. With these many qualifications and extensions, we will find that certain general properties of geometry in vector spaces can be realized.

3. AN "ORDERING" RELATION

In this section we shall begin work upon the explicit development of geometry defined in a finite and discrete space. For the early and classical work on this topic see reference 1.

Consider GF(p) with $p = 8 \prod_{i=1}^{k} q_i - 1$, where the q_i are the odd primes. We know that in such a field the elements $1,2,\ldots,q_k$ are all square residues and -1 is nonsquare.[1]

Definition(1) Let $x,y \in GF(p)$ with p given above. If $x - y =$ square residue, then x is said to exceed y, in symbols $x > y$. If $x - y =$ nonsquare residue, then x is less than y, $x < y$.[6]

Theorem (1). Let p be as above. If x is square, then -x is nonsquare and vice versa (here -x is the additive inverse of x, i.e. $x + (-x) \equiv 0 (\mathrm{mod}\ p)$.

Proof. If p is an odd prime and w a primitive root of GF(p), then $w^{(p-1)/2} \equiv -1 \pmod p$.[7] Let $x = w^n$ and $-x = w^m$, where n an even integer and m an integer. We have

$$x + (-x) = w^n + w^m \equiv 0 \pmod p .$$

Now, either $n > m$ or $m > n$. Assume, for definiteness, $m < n$. Then

$$w^m(w^{n-m} + 1) \equiv 0 \pmod p .$$

Since $w^m \not\equiv 0 \pmod p$, we have $w^{n-m} + 1 \equiv 0 \pmod p$. From above, this gives $w^{n-m} \equiv w^{(p-1)/2} \pmod p$. From this we obtain $n - m \equiv (p - 1)/2 \pmod{p - 1}$. However, for a p in the form given above, we see that

$$(p - 1)/2 = 4 \prod_{i=1}^{k} q_i - 1$$

which is always odd. Therefore n and m have opposite parity.

Theorem (2). If $x,y \in GF(p)$ with p given above, then $x > y$ iff $y < x$.

Proof. Assume $x > y$. Then $x - y$ = square residue and $y - x$ = -(square residue). In such a GF(p), the additive inverse of a square residue is nonsquare and vice versa. This follows because in such GF(p) -1 is a nonsquare residue and if x is a square residue, then $(-1)(x) = -x$ is a nonsquare residue. (See Theorem (1)). Hence $x > y \Rightarrow y < x$. The converse is proved similarly.

Theorem (3). If $\alpha \in GF(p)$, then $\alpha^2 \equiv (-\alpha)^2 \pmod p$.

Proof. From the above theorem, we know that if $\alpha = w^n$ and $-\alpha = w^m$ then $n - m \equiv (p - 1)/2 \pmod{p - 1}$. Hence, since $\alpha^2 = w^{2n}$, $(-\alpha)^2 = w^{2m}$, we have $2n - 2m \equiv (p - 1) \pmod{p - 1} \equiv 0 \pmod{p - 1}$.

Definition (2). Define an "absolute value" function in the following way. If $\alpha \in GF(p)$ with $p = 8\prod_{i=1}^{k} q_i - 1$, then $|\alpha| = \alpha$ if α is a square residue and $|\alpha| = -\alpha$ if α is a nonsquare residue. Theorem (1) provides the justification for this definition.

Theorem (4). Let $x \in GF(p)$ $(p \neq 2)$ be a square residue. Then there exist two elements $a,b \in GF(p)$ such that $a^2 \equiv b^2 \equiv x(\text{mod } p)$. Furthermore, if $p = 8\prod_{i=1}^{k} q_i - 1$, then a and b are of opposite parity and are additive inverses of each other, $a + b \equiv 0(\text{mod } p)$. One of the two, say a, is square and the other b, is nonsquare. a is called $+\sqrt{x}$.

Proof. Let us first prove there can't be three elements all of which square to the same value. Assume $\exists$ three distinct elements $a,b,c \in GF(p) \ni a^2 \equiv b^2 \equiv c^2 \equiv x(\text{mod } p)$. Then $a^2 - b^2 \equiv 0(\text{mod } p)$ and $a^2 - c^2 \equiv 0(\text{mod } p)$, or $(a + b)(a - b) \equiv 0(\text{mod } p)$ and $(a + c)(a - c) \equiv 0(\text{mod } p)$. Since a,b,c are distinct, we have $a + b \equiv 0(\text{mod } p)$ and $a + c \equiv 0(\text{mod } p)$. But in a field, the additive inverse is unique, so $b \equiv c(\text{mod } p)$ which violates assumption that a,b,c are distinct.

Now prove $\exists$ two elements $a,b \ni a^2 \equiv b^2 \equiv x(\text{mod } p)$. There are $p - 1$ distinct nonzero elements and $(p - 1)/2$ distinct squares in $GF(p)$, $p \neq 2$. Since there aren't three elements having same squared value, there must be two such distinct elements for every square x.

From $a^2 - b^2 \equiv (a + b)(a - b) \equiv 0(\text{mod } p)$ and $a \not\equiv b(\text{mod } p)$ we see that a,b are additive inverse. If $p = 8\prod_{i=1}^{k} q_i - 1$, we may invoke Theorem (1), to conclude they are of opposite parity.

4. LOCAL "ORDER"

Let us consider some arbitrary $x \in GF(p)$. We know that $(x + 1) - x = 1 =$ square residue; hence $x + 1 > x$. Similarly, $x - (x - 1) = 1$, or $x > x - 1$. Also, $(x + 1) - (x - 1) = 2 =$ square residue, so that $x + 1 > x > x - 1$. Clearly this process may be continued for q_k consecutive elements to generate the following order relations:

$$x - (q_k - 1)/2 < x - (q_k - 3)/2 < \ldots < x + (q_k - 1)/2.$$

Let us designate this set of q_k consecutive transitively ordered elements that is centered about x by Toss (x, q_k). We shall consider $x + 1 , \ldots , x + (q_k - 1)/2$ as all "positive" with respect to x while $x - 1 , \ldots , x - (q_k - 1)/2$ are "negative" with respect to x. It is important to realize that the terms positive and negative express a relation that is referred to some specific point, not necessarily the additive identity 0. In order to perform calculations we must be sure to refer to this central point x. This is done by counting the number of steps "above" or "below" x for any member of Toss (x, q_k). Thus, if $a, b \in$ Toss (x, q_k) , we have the sum as $(a - x) + (b - x) + x$, etc. Clearly this is the well-known transformation of linear translation. Thus, with the above identifications and definitions, we see that any point $x \in GF(p)$ may serve as the center of a Toss (x, q_k). Thus whatever "geometry" can be done at one point can be done at any point. Therefore we have shown that

Theorem (5). Any point $x \in GF(p)$ can be the center of a Toss (x, q_k) and "geometry" can be done locally within this set.

To simplify calculations we may assume that $x = 0$ is the chosen center, thereby avoiding the extra terms of $a - x$, etc. However, we must remember that the choice of center point is arbitrary and the geometrical results obtained in one Toss are equivalent to those found in any Toss in the field.

Since we are defining a vector space over $GF(p)$, we may generalize this discussion for n-dimensional vectors and let the center become a vector $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$. Essentially this is a succinct formulation of n distinct centers, one for each component.

5. EXTENSION OF THE FIELD[8]

Let $GF(p)$ be a Galois field. We know that $(p - 1)/2$ elements are square and $(p - 1)/2$ are nonsquare (see Theorem (4)). The $(p - 1)/2$ square elements all have two square roots in $GF(p)$ whereas the nonsquare elements do not have a square root in $GF(p)$. If $p = 8 \prod_{i=1}^{k} q_i - 1$, then the two roots of the square elements are related by $+\sqrt{x} + (-\sqrt{x}) \equiv 0$ and $+\sqrt{x} > 0$, $-\sqrt{x} < 0$. To obtain square roots for the nonsquare elements we must embed $GF(p)$ in a larger field $GF(p^2)$ which is the extension of $GF(p)$, i.e. we obtain $GF(p^2)$ from $GF(p)$ by adjunction of a root of $x^2 + 1 \equiv 0 (\mathrm{mod}\ p)$. $GF(p^2)$ becomes isomorphic to the set of first degree polynomials

$a + bx$ where the coefficients $a, b \in GF(p)$. Clearly there are p^2 elements in $GF(p)$. To simplify comparisons with "ordinary" mathematics, let us denote the indeterminant by i. Since $i^2 + 1 \equiv 0$ we have $i^2 \equiv -1$, and $a + ib \in GF(p^2)$. Let us now find square roots of the negative elements of $GF(p)$. Let $x \in GF(p)$ $p = 8 \prod_{i=1}^{k} q_i - 1$ be nonsquare. Assume an element $a + ib \in GF(p^2)$ as the square root of x. Hence $a + ib \equiv \sqrt{x}$ or $x \equiv (a + ib)^2 \equiv a^2 - b^2 + 2iab$. From this we see that $ab \equiv 0 \Rightarrow a \equiv 0$ or $b \equiv 0$. Also have $a^2 - b^2 \equiv x$. If $b \equiv 0$, then $b^2 \equiv 0$ and $x \equiv a^2$ which violates assumption that x is nonsquare. Hence $a \equiv 0$ and $x \equiv -b^2$ and $b^2 \equiv -x$. We know that $x < 0 \Rightarrow -x > 0$ so $b \equiv \pm\sqrt{-x}$. Hence $\sqrt{x} \equiv \pm i\sqrt{-x}$ which conforms to our prior expectations. Thus $\forall\, x \in GF(p)$ $\exists\, z \in GF(p^2) \ni z^2 \equiv x$.[9]

Since $(p^2 - 1)/2$ elements of $GF(p^2)$ are square and $(p^2 - 1)/2$ are nonsquare, we see that a square root for elements of $GF(p^2)$ can be found for some of the elements ($(p^2 - 1)/2$ of them). This can also be seen since there are two square roots for each $x \in GF(p^2)$ $(x \neq 0)$ and this—due to uniqueness properties—implies that only half the nonzero elements can have square roots in $GF(p^2)$. This is yet another way in which the finite field differs radically from the continuous field where every complex number has two square roots in the complex plane. In finite field mathematics we are able to count according to the customary rules without encountering the unusual characteristics of the transfinite arithmetic.

Theorem (6). If $p = 8 \prod_{i=1}^{k} q_i - 1$, then $x^2 + 1$ is irreducible over $GF(p^2)$.

Proof. Assume $x^2 + 1$ is reducible over $GF(p^2)$. Then $\exists\, a,b \in GF(p) \ni$ $x^2 + 1 \equiv (x + a)(x + b) \equiv x^2 + ab + x(a + b)$. For this to hold, we must have

$$ab \equiv 1 \quad \text{and} \quad a + b \equiv 0.$$

This implies $a^2 + ab \equiv a^2 + 1 \equiv 0$. In $GF(p)$ of the above form, there exist no solution to this because -1 is nonsquare and $a^2 \equiv -1$ cannot be solved.

<u>Theorem (7)</u>. If $p = 8 \prod_{i=1}^{k} q_i - 1$, then $x^2 + 1$ is not primitive in $GF(p^2)$.

<u>Proof</u>. For such a p, we always have $p^2 > 4$, yet $x^2 + 1$ divides $x^4 - 1$; hence $x^2 + 1$ cannot be primitive because its order is less than p^2.

Beltrametti and Blasi[10] have shown that for a p of the above form, $i^p = -i$; if $a,b \in GF(p^2)$, then $(a + b)^p = (a^p + b^p)$. Therefore if $a,b \in GF(p)$, then $(a + ib)^p = a - ib$; hence complex conjugation can be associated with the p^{th} power of a "complex number." Define Z^* such that $\forall z \in GF(p^2)$, $z^* = z^p$.
We follow reference 4 and define the absolute value in an obvious way, viz. $\forall z \in GF(p^2)$, $|z| = \sqrt{z^* z} = \sqrt{z^{p+1}}$. If $z = a + ib$, $a,b \in GF(p)$, then $|z| = \sqrt{a^2 + b^2}$ and we see that $\forall z \in GF(p^2) \,\exists\, |z| \in GF(p^2)$. This is somewhat unfortunate because the absolute value function is generally considered to be a mapping from the complex plane onto the positive real line. In our case, this becomes a mapping from $GF(p^2)$ onto the square residues of $GF(p)$. As before, we can achieve such a condition over a subset. Let $S(x,q_k) = \{y : y \in \text{Toss}\,(x,q_k) \text{ and } 2y^2 \in \text{Toss}\,(x,q_k)\}$ (remember $2y^2$ to be performed with respect to the center, x). For simplicity, consider $S(0,q_k)$. Define $C(S(0,q_k)) = \{z : z = a + ib\,,\ a,b \in S(0,q_k)\}$. Then $\forall z \in C(S(0,q_k))$ $\quad |z| \in GF(p)$ and $|z| =$ square residue. We may generalize our definition to include the

replacement square roots to find (see section 7).

$$|z|_R = \left(\sqrt{z^* z}\right)_R \ .$$

In terms of this definition, $|z|_R$ is ordered, etc. over an appropriate subset. Thus, we can introduce a length notion over part of $GF(p^2)$.

6. SQUARE ROOTS AND INCOMPLETENESS

We have seen that for any GF(p), there are (p - 1)/2 elements that do not have square roots in GF(p). If you embed GF(p) in GF(p^2), then each of these (p - 1)/2 elements has a square root in GF(p^2). However, in GF(p^2), there are (p^2 - 1)/2 elements without square roots in GF(p^2). This process continues for all finite fields, the richest always failing to contain square roots for about half its members. This is a form of incompleteness that is somewhat reminiscent of the Godel type incompleteness. Godel showed that within any formal system at least as rich as arithmetic, there always exist statements whose truth or falsity depends entirely upon the truth or falsity of a meta statement.[11] Hence the status of certain statements cannot be determined within the system. The square root situation is much the same, for every system (field) is dependent upon the embedding field (meta field) for its square root completion. It should be noted that an infinite field is not considered to display such behavior. In fact the complex plane is purported to contain the square root of every one of its elements. This is another example of the curious counting results one encounters when dealing with infinities of numbers. However, if the infinite field is obtained from a limiting process of successive imbedding, of finite fields, the cited property of the continuous complex plane will not appear for any finite part of the limiting process.

7. REPLACEMENT TECHNIQUE FOR SQUARE ROOTS

Let us concentrate our attention upon the ordered subset centered about 0, i.e. Toss $(0,q_k)$. We are going to be concerned with those elements in Toss $(0,q_k)$ that in conventional number theory are known as perfect squares, viz. 0, 1, 4, 9, 16, ... Let $\text{Toss}^+(0,q_k) = \{0,1,2,\ldots,(q_k-1)/2\}$, i.e. the "positive" elements of Toss $(0,q_k)$.

Let us construct a set $\Gamma(q_k)$ as follows. Let $0 \in \Gamma(q_k)$. Then let $(0+1)^2 \in \Gamma(q_k)$ if $(0+1)^2 \in \text{Toss}^+(0,q_k)$. Continue in this until the first time that $(0 \underbrace{+ 1 + \cdots + 1}_{n+1 \text{ times}})^2 \notin \text{Toss}^+(0,q_k)$. Then the n elements

$0^2, 1^2, 2^2, \ldots, n^2$ will constitute $\Gamma(q_k)$. Let us arrange and number the elements of $\Gamma(q_k)$ so that $\gamma_0 = 0^2$, $\gamma_1 = 1^2$, etc. Then we have $\gamma_0 < \gamma_1 < \gamma_2 < \cdots < \gamma_n$ where γ_n is the largest "perfect square" in $\text{Toss}^+(0,q_k)$. Let $S(q_k) = \{x : x \in \text{Toss}^+(0,q_k) \text{ and } x \leq \gamma_n\}$. Thus for the elements of $\Gamma(q_k)$ we have the square roots lying in the same order as the squares, clearly a desirable situation. Unfortunately the formal square roots of the elements of $S(q_k)$ not in $\Gamma(q_k)$ do not exhibit this property. We shall impose the additional condition that the squares and square roots of $S(q_k)$ lie in the same order. Since we cannot obtain an acceptable solution—acceptable with respect to the criteria established above—we shall replace the problem by one that we can solve within the framework.

Let $x \in S(q_k)$, $x \notin \Gamma(q_k)$. Problem (1) is to find $y \in S(q_k) \ni y^2 \sim x$. If $x \in S(q_k)$ and $x \notin \Gamma(q_k)$, then $\exists\, i\ (i \in \{0,1,2,\ldots,n-1\}) \ni \gamma_i < x < \gamma_{i+1}$. We shall replace problem (1) with $\sqrt{\gamma_{i+1}} \in S(q_k)$ and designate the replacement by $(\sqrt{x})_R$. Clearly, if $x \in \Gamma(q_k)$, then $(\sqrt{x})_R = \sqrt{x}$ (where $\sqrt{x}$ has its ordinary definition.).

We have replaced problem (1), which does not have an acceptable solution in GF(p), by another problem that does admit of solution. We again see that going beyond Toss $(0,q_k)$ leads us into a realm of uninterpretable results. In effect, this corresponds to going beyond the capabilities or resources of the given GF(p).

Theorem (8). 1. $\forall\, x \in S(q_k)$, $(\sqrt{x})_R \in S(q_k)$

2. If $x,y \in S(q_k)$ and $x \leq y$, then $(\sqrt{x})_R \leq (\sqrt{y})_R$

3. If $x,y \in S(q_k)$ and $(\sqrt{x})_R < (\sqrt{y})_R$, then $x < y$.

Proof. Property 1. follows immediately since the γ_i were chosen to be those elements for which $\sqrt{\gamma_i} \in S(q_k)$. For property 2., let $\gamma_i = \min_{\gamma \in \Gamma(q_k)} \gamma \geq x$. Then $\gamma_{i-1} < x \leq \gamma_i$, and $(\sqrt{x})_R = \gamma_i$. Since $y \geq x$ we know that either $x \leq y \leq \gamma_i$ or $y > \gamma_i$. If $x \leq y \leq \gamma_i$, then $(\sqrt{x})_R = (\sqrt{y})_R = \sqrt{\gamma_i}$. If $y > \gamma_i$, then $(\sqrt{y})_R = \sqrt{\gamma_j}$ where $\gamma_j > \gamma_i$; hence, $x \leq y$ implies $(\sqrt{x})_R \leq (\sqrt{y})_R$. For property 3., let $\sqrt{\gamma_i} = (\sqrt{x})_R$ and $\sqrt{\gamma_j} = (\sqrt{y})_R$. Since $(\sqrt{x})_R < (\sqrt{y})_R$, we have $\gamma_i < \gamma_j$. We also have $\gamma_{i-1} < x \leq \gamma_i$ and $\gamma_{j-1} < y \leq \gamma_j$. Now, $\gamma_i < \gamma_j$ implies $\gamma_i \leq \gamma_{j-1}$; hence $x \leq \gamma_i \leq \gamma_{j-1} < y$ and $x < y$.

Note that $(\sqrt{x})_R \leq (\sqrt{y})_R$ does not imply $x \leq y$.

Theorem (9). Let $x, x^2 \in S(q_k)$ with $x \neq 0$. Then $(\sqrt{x^2 - y})_R = x$ if and only if $y \in [0, 2x - 2]$ (here [ ] has usual definition).

Proof. If $(\sqrt{x^2 - y})_R = x$, then $(x - 1)^2 < x^2 - y \leq x^2$. This implies that $y \in [0, 2x - 2]$. Conversely, assume $y \in [0, 2x - 2]$. Since $(x - 1)^2 = x^2 - (2x - 1) < x^2 - y \leq x^2$, we have $x = (\sqrt{x^2 - y})_R$.

Theorem (10). $\forall\, x \in S(q_k)$, $[(\sqrt{x})_R]^2 \geq x$.

Proof. Let $\sqrt{\gamma_i} = (\sqrt{x})_R$. Then, if $x \neq 0$, $\gamma_{i-1} < x \leq \gamma_i = (\sqrt{\gamma_i})^2 = [(\sqrt{x})_R]^2$. If $x = 0$, the theorem is obvious.

Theorem (11). $\forall\, x, y \in S(q_k) \ni xy \in S(q_k)$, $(\sqrt{xy})_R \leq (\sqrt{x})_R (\sqrt{y})_R$.

Proof. Let $\sqrt{\gamma_i} = (\sqrt{x})_R$ and $\sqrt{\gamma_j} = (\sqrt{y})_R$. Then, if $x \neq 0$, $y \neq 0$, $\gamma_{i-1} < x \leq \gamma_i$ and $\gamma_{j-1} < y \leq \gamma_j$. Hence, $\gamma_{i-1}\gamma_{j-1} < xy \leq \gamma_i \gamma_j$. Therefore, $(\sqrt{xy})_R \leq \sqrt{\gamma_i \gamma_j} = \sqrt{\gamma_i}\,\sqrt{\gamma_j} = (\sqrt{x})_R (\sqrt{y})_R$. Once again, if $x = y = 0$, the theorem is obvious.

8. EMBEDDING

If we wish to find another problem that gives a "better" answer to replace Problem (1), we must expand our field by embedding $GF(p_k)$ in a field $GF(p_{k'})$ where $k' > k$. Because of its greater richness $GF(p_{k'})$ can provide substitute problems that "more closely approach" Problem (1).

Let us choose $p_{k'}$ such that $p_{k'}/p_k = 100 + R$ where $R > 0$. Then we shall identify every 100^{th} element (up to $100q_k$) of $S(q_{k'})$ with the elements of $S(q_k)$, i.e. if $x' \in S(q_{k'})$ and if $x' \equiv 0 \pmod{100}$, then $\exists\, x \in S(q_k) \ni x \to x'$. Now we can pose Problem (1′) which is to find $\sqrt{x'} \in S(q_{k'})$, $(x' \leftarrow x \in S(q_k))$. Again replace Problem (1′) by finding $\gamma_{i'} \in \Gamma(q_{k'}) \ni \gamma_{i-1} < x' \le \gamma_i$. Again introduce the replacement problem and a soluable problem in $GF(p_{k'})$. Then, using the relation $x \to x'$, we associate a solution, say γ_i, with Problem (1) by the decimal version of $\frac{\gamma_i}{10}$. And if greater resolution is sought repeat this process to $GF(p_{k''})$, etc.

Example. $S(11) = \{0,1,2,\ldots,9\}$. Find $\sqrt{7}$. Replacement problem yields $(\sqrt{7})_R \to 3$. Go to richer field with $S(1109)$. Then $676 < 700 < 729 \Rightarrow (\sqrt{7})_R \to 2.7$.

In this way we have established a procedure that serves to define acceptable square roots to within any desired "resolution" or order of refinement. Let us point out that embedding is a form of replacement and the identification of 1.0 with 1.00 is a matter of pure and arbitrary convention. We could—in principle—associate 1.0 with any number, say 6.25, but that would violate standard practice. The only theoretical requirement is that every element in the coarse field be mapped nonambiguously onto an element of the finer field.

9. ALTERNATIVE REPLACEMENT TECHNIQUE

Instead of "rounding up" as we have done, one could "round to closest neighbor." This changes the form of the theorems and the triangle inequality is lost; however, there are certain aspects that are quite desirable. In this section, we shall just present the definition.

Let $x \in S(q_k) \ni x \notin \Gamma(q_k)$. Then $\exists\, i \in \{0,1,\ldots,n-1\} \ni \gamma_i < x < \gamma_{i+1}$. Form the differences $d^+ = \gamma_{i+1} - x$ and $d^- = x - \gamma_i$. Clearly d^+ and $d^- \in S(q_k)$ so are unambiguously comparable with our order relation. Let us designate the replacement square root of x by $(\sqrt{x})_R$. Then, the following will serve as the definition of $(\sqrt{x})_R$. Let $\sqrt{x}$ designate the positive Galois field square root.

<u>Definition</u>. If $d^+ > d^-$, then $(\sqrt{x})_R = \sqrt{\gamma_i}$; if $d^+ < d^-$, then $(\sqrt{x})_R = \sqrt{\gamma_{i+1}}$.

Since $(N + 1)^2 - N^2 = 2N + 1$, there can be no $x \in S(q_k)$ such that $d^+ = d^-$ and we have an unambiguous formulation. If $x \in \Gamma(q_k)$, then $(\sqrt{x})_R = \sqrt{x}$.

In the subsequent development we shall restrict our attention to the computationally simple "rounding up". However, we must first demonstrate that this choice does not unnecessarily prejudice the conclusions. Thus let us show that the three possible replacement techniques lead to essentially equivalent results.

See Appendix I.

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10. GALOIS FIELD GEOMETRY

A vector space defined over a Galois field[12] cannot have an inner product with all of the customary properties because of the lack of transitive order in $GF(p)$. However we shall generalize this notion to what will be called a Galois product in the hopes of introducing a concept of direction.

<u>Definition (3)</u>. Let V be a vector space of columns defined over $GF($ Let $[x,y] = x^T y$ define a Galois product $\forall\, x, y \in V$. If $[x,y] = 0$ we shall call x,y orthogonal.

<u>Theorem (12)</u>. The Galois product satisfies the following conditions:

1. $[x,y] \in GF(p) \quad \forall\, x,y \in V$;
2. $[x,y] = [y,x] \quad \forall\, x,y \in V$;
3. $[x,\alpha y + \beta z] = \alpha\,[x,y] + \beta\,[x,z] \quad \forall\, x,y,z \in V,\ \forall\, \alpha,\beta \in GF(p)$.

Proof.

Let $x = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix}$, $y = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{bmatrix}$, $z = \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_n \end{bmatrix}$ where

the α_i, β_i, $\gamma_i \in GF(p)$. Then $[x,y] = \sum_{i=1}^{n} \alpha_i \beta_i$, etc. Hence 1. follows from closure of GF(p) under multiplication and addition. Similarly 2. follows from commutativity of multiplication in GF(p). Finally 3. follows since multiplication is also distributive in GF(p).

Let us now study the relationships between linear independence and the Galois product.

Theorem (13). If $[x,x] \neq 0$ and $\alpha x = y$, $\alpha \neq 0$ where $x,y \in V$ and $\alpha \in GF(p)$, then $[x,y] \neq 0$. Thus linear dependence implies a nonzero Galois product.

Proof. $[x,y] = [x,\alpha x] = \alpha[x,x] \neq 0$.

Theorem (14). If $[x,y] = 0$ and $[x,x] \neq 0$, $[y,y] \neq 0$, $x,y \in V$, then x and y are linearly independent.

Proof. Let us seek two scalars $\alpha,\beta \in GF(p)$ such that $\alpha x + \beta y = 0$. Operate on this equation with x^T to obtain $\alpha x^T x + \beta x^T y = \alpha [x,x] + \beta [x,y] = \alpha [x,x] = 0$. Hence, since $[x,x] \neq 0$, $\alpha = 0$. Operate similarly with y^T to find $\beta = 0$. Therefore x and y are linearly independent.

11. NORM AND METRIC

Let us now combine the above results and define a region over which an inner product and norm can be identified. Let $E(q_k) = \text{Toss}\,(0, q_k)$ be the transitive ordered subset of $GF(p)$. Let V be an n-dimensional vector space defined over $GF(p)$. Define a subset of $E(q_k)$ as

$$F(q_k) = \{\alpha\colon \alpha \in E(q_k) \text{ and } n\,\alpha^2 < q_k\}.$$

Define a region of V by

$$F = \{x\colon x \in V,\ x = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix},\ \alpha_1, \alpha_2, \ldots, \alpha_n \in F(q_k)\}.$$

<u>Theorem (5)</u>. $\forall\, x \in F$, $[x,x] \geq 0$; $= 0$ iff $x = 0$.

<u>Proof</u>. Let $x = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix}$; $\alpha_1, \alpha_2, \ldots, \alpha_n \in F(q_k)$. Then we have

$[x,x] = \sum_{i=1}^{n} (\alpha_i)^2$. Let $F^+(q_k) = \{\alpha\colon \alpha \in F(q_k) \text{ and } \alpha \geq 0\}$.

Clearly $(\alpha_i)^2 \in F^+(q_k)$ $(i = 1,2,\ldots,n)$. Since $n(\alpha_i)^2 < q_k$, the sum of $\sum_{i=1}^{n} (\alpha_i)^2$ is still in $F^+(q_k)$, i.e. transitivity holds and we can sum the inequalities $0 \leq n(\alpha_i)^2 < q_k$ to obtain the theorem.

The set F is not a subspace of V because it is not closed under vector addition or scalar multiplication. Thus when formulating certain theorems additional restrictions are needed to insure that operations do not carry beyond the limits of F into the set V-F. Thus, for example, the condition $[\alpha x,x] = \alpha[x,x]$ is valid over V and GF(p) but the condition $[\alpha x,\alpha x] \geq 0$ is valid only for those $\alpha \in GF(p)$, $x \in F \ni \alpha x \in F$.

<u>Theorem (16)</u>. If $\alpha \in GF(p)$, $x \in F$ and $\alpha x \in F$, then $[\alpha x,\alpha x] \geq 0$; $= 0$ iff $x = 0$ or $\alpha = 0$.

<u>Proof.</u> Follows immediately from Theorem (1) with αx replacing x.

<u>Theorem (17)</u>. If we restrict ourselves to operations involving elements of F that do not produce results out of F, then the Galois product becomes an inner product over F.

<u>Proof.</u> From Theorem (12), we know that the Galois product satisfies all but the condition that $(x,x) \geq 0$, $= 0$ iff $x = 0$ of the definition of an inner product. Theorems (15) and (16) insure that this condition is also satisfied.

Let $H(q_k) = \{\alpha\colon \alpha \in E(q_k) \text{ and } 4n^2\alpha^4 < q_k\}$.

$$\text{Let } H = \left\{x\colon x \in V,\ x = \begin{bmatrix}\alpha_1\\ \alpha_2\\ \vdots\\ \alpha_n\end{bmatrix},\ \alpha_1,\alpha_2,\ldots,\alpha_n \in H(q_k)\right\}.$$

<u>Theorem (18)</u>. If $x,y \in H$, then $[x,y]^2 \leq [x,x][y,y]$.

<u>Proof</u>. Let $x = \begin{bmatrix}\alpha_1\\ \alpha_2\\ \vdots\\ \alpha_n\end{bmatrix}$, $y = \begin{bmatrix}\beta_1\\ \beta_2\\ \vdots\\ \beta_n\end{bmatrix}$, $\alpha_i,\beta_i, \in H(q_k)$ $(i = 1,2,\ldots,n)$.

We have that $0 \leq \sum_{i=1}^{n} \sum_{j=1}^{n} (\alpha_i \beta_j - \alpha_j \beta_i)^2$. This inequality holds because by the definition of $H(q_k)$, each term is nonnegative and their sum stays within $E(q_k)$. We may expand this inequality to obtain

$$\left[\sum_{i=1}^{n} \alpha_i \beta_i\right]^2 \leq \sum_{i=1}^{n} (\alpha_i)^2 \sum_{i=1}^{n} (\beta_i)^2 .$$

In terms of x and y, this is equivalent to $[x,y]^2 \leq [x,x]\,[y,y]$.

Theorem (19). If $x,y \in H$, then $[x,y] \leq (\sqrt{[x,x]})_R \; (\sqrt{[y,y]})_R$.

Proof. From theorems (8), (9), and (18), we find $[x,y] \leq (\sqrt{[x,x][y,y]})_R$ and from theorem (8) $(\sqrt{[x,x][y,y]})_R \leq (\sqrt{[x,x]})_R \; (\sqrt{[y,y]})_R$.

Definition (14). Define a mapping from F into GF(p) as follows: $\forall\, x \in F,\ \| x \| = (\sqrt{[x,x]})_R$.

Theorem (20). $\| x \| \geq 0;\ = 0$ iff $x = 0,\ \forall\, x \in F$.

Proof. The proof follows from theorem (15) and the definition of $(\sqrt{})_R$.

Theorem (21). $\| \alpha x \| \geq | \alpha | \cdot \| x \| \quad \forall\, x \in F,\ \forall\, \alpha \in GF(p) \ni \alpha x \in F$.

Proof. $\| \alpha x \| = (\sqrt{[\alpha x, \alpha x]})_R = (\sqrt{\alpha^2 [x,x]})_R \geq (\sqrt{\alpha^2})_R (\sqrt{[x,x]})_R$

$= | \alpha | \cdot \| x \|$.

Theorem (22). $\forall\, x,y \in H \ni x + y \in F,\ \| x + y \| \leq \| x \| + \| y \|$.

Proof. $(\| x \| + \| y \|)^2 = (\sqrt{[x,x]})_R^2 + (\sqrt{[y,y]})_R^2 + 2(\sqrt{[x,x]})_R (\sqrt{[y,y]})_R$

Theorem (10) $\Rightarrow\ \geq [x,x] + [y,y] + 2(\sqrt{[x,x]})_R (\sqrt{[y,y]})_R$

Theorem (11) $\Rightarrow\ \geq [x,x] + [y,y] + 2(\sqrt{[x,x][y,y]})_R$

Theorem (18) $\Rightarrow\ \geq [x,x] + [y,y] + 2(\sqrt{[x,y]^2})_R$

Theorem (9) $\Rightarrow\ = [x,x] + [y,y] + 2[x,y]$

$= [x+y, x+y]$.

Therefore, $(\| x \| + \| y \|)^2 \geq [x+y, x+y]$ which implies--from theorem (8) that

$$(\sqrt{(\|x\| + \|y\|)^2})_R \geq (\sqrt{[x+y, x+y]})_R .$$

From theorem (9) and definition (4), we have

$$\| x \| + \| y \| \geq \| x + y \|.$$

Definition (5). Let $\rho(x,y) = \| x - y \| \quad \forall\, x,y \in F \ni x-y \in F$.

Theorem (23). 1. $\rho(x,y) \geq 0$; $= 0$ iff $x = y$.

2. $\rho(x,y) = \rho(y,x)$

Proof. Property 1. follows immediately from theorem (20) with $x - y$ identified with x. Property 2. follows since $\rho(x,y) = \| x - y \| = (\sqrt{[x-y, x-y]})_R = (\sqrt{[y-x, y-x]})_R = \rho(y,x)$.

Theorem (24). $\rho(x,y) + \rho(y,z) \geq \rho(x,z) \quad \forall\, x,y,z \in H \ni x - y$, $x - z$, $y - z \in H$.

Proof. We shall use the results of theorem (22). $\rho(x,z) = \| x - z \| = \| x - y + y - z \| \leq \| x - y \| + \| y - z \| = \rho(x,y) + \rho(y,z)$.

Thus we see that $\rho(x,y)$ satisfies the definition of a metric over the set H.

12. ROTATION

In addition to the basic metrical properties of geometry that have already been presented, we shall seek a mechanism for generating a concept of rotation. There has been prior work in this direction, generally by introducing finite groups of transformations that preserve some appropriate quadratic form 13. For example, if dealing with a four-dimensional space, one can introduce a metric tensor of the form

$$g = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \qquad 0,1 \in GF(p)$$

and define a bilinear form $x \cdot y = x^T g y$. This is in direct analogy with the conventional formalism of modern physics. However, this procedure does not directly consider the problem of interpretability, especially that associated with the ordered subsets that play such an important role in our development of finite geometry. Hence we impose an additional condition that a subset of all such transformations be found that transforms vectors from $F(q_k)$ into vectors of $F(q_k)$ (we might further restrict to $H(q_k)$, depending on the context).

We have devised a finite algorithm that generates transformations that "rotate" vectors of the ordered grid into other such vectors.[14] Since it is a constructive procedure, it offers immediate insight into the structure and consequences of a finite and discrete geometry.

The rotation technique consists of adjoining integer sided right triangles[15] about some common vertex so that the hypotenuse of one and a leg of the other are colinear. The common vertex is generally considered

to be the origin. If the triangles are suitably chosen, then the vertices are always realized at points of the discrete grid. Clearly this is a necessary condition. In order to guarantee that this is satisfied, one must have the grid sufficiently rich, i.e. with sufficiently many points. The adjunction is viewed conceptually as being performed via successive embedding in richer, i.e. more "closely" packed fields. Thus, if h is the number of counts of the hypotenuse of the first triangle as seen in field F^1, then this same "segment" should be h^n counts when referred to the field F^n which required after n adjunctions. In other words, we require an ever richer field to perform every subsequent rotation. By repeated application of this procedure, we may generate rational expressions of arbitrary rotations. If we let our "unit" triangle be thin, i.e. if the ratio of the legs is small, then we can approach any "angle" of ordinary rotation by repeated adjunction of this one triangle. In this case when the same triangle is used, then the sum of the squares of the coordinates, when referred to the richest field, is a conserved quantity. This case fits into the format of the above described systems. Hence we have generated a subset of the formal and global definitions of rotation. As has so often happened in the development of finite field geometry, one can introduce interpretable objects locally, not globally.

Another interesting and potentially far-reaching point to mention is the following. It is found that each successive adjunction requires a richer field if one is to refer the results back to the original orientation. Obviously, this process can continue only so long before one exhausts his capacity to further enrich the field. At this stage, one must either cease or drop the requirement of remembering exactly what the original orientation was. In the latter case, one can either eliminate the record of the original state entirely or introduce a probabilistic formulation that enables one to go further, although without a deterministic description. The probabilistic method does enable one to further extend his capabilities. However, both solutions ultimately lead to complete renunciation of strict determinism in description; hence, the predictative capability is likewise lost. It is conjectured that this failure to achieve a purely and exhaustively deterministic description might be the source of the quantum mechanical behavior so well known in the realm of atomic phenomena.

When the numerical capacity is exceeded, there are ways to retain some control and information by reducing the resolution requirements. This can be done by introducing a hierarchy of counting that no longer carries the lowest decimal. For example, an automobile odometer can register more than 10^5 miles if, after reaching 99,999, we change the gear ratio by a factor of ten. We forgo knowledge of the tenths place but obtain capacity to count hundred thousands. And this hierarchical embedding can be repeated.

13. MATHEMATICAL INDUCTION AND PASSAGE TO THE LIMIT

One of the many implications of the local ordering concept is the distinction between "for any" and "for every." We can declare an origin at any point in the space and do geometry locally; however, this does not imply that we can do geometry at every point referred to this one origin. Mathematical induction asks if the validity of $P(n + 1)$[16] follows from the assumed validity of $P(n)$ and the demonstrable validity of $P(1)$ $P(n)$ assumed true and implying the validity of $P(n + 1)$ is a local

demonstration that can be performed anywhere, i.e., at any n. However, from this local property the conventional assumption is global validity. On the other hand, since this entire process is highly similar to our local order concept, we are led to inquire whether mathematical induction is also limited by the local vs. global distinction. If so, then the principle of mathematical induction must be reevaluated to incorporate the results of a local ordering relation in a finite field.

We have found that any demonstration (from the finite context of the view taken in this paper) of the validity of mathematical induction requires an additional axiom regarding the existence of a "continuous" field. This is consistent with the findings in the early 20th century about the necessity of an axiom of infinity.[17]

In order to develop these ideas more fully, we must first examine an extralogical requirement.

This paper begins with a primitive commitment to finitism and we have attempted to demonstrate the theoretical possibility of performing certain operations wholly within a finite context. Now we must invoke another primitive commitment and can only briefly motivate its introduction. Procedural invariance is an extralogical requirement (see reference 18) that is essentially a generalization of the Einstein principle requiring invariance of physical laws under appropriate transformations. A rational system that leads to a prediction or prescription that is not invariant under the arbitrary procedures of analysis and computation is inherently ambiguous, i.e., the system should not have its results depend upon the computational path that is chosen. The choice of convention should not determine the answer. If it does then

the results cannot be unique. In general, the preservation of consistency under alternative, arbitrary procedures is a categorical requirement, i.e., a system which does not preserve consistency under arbitrary procedural convention is a fortiori inadmissible as a rational paradigm.

We shall now consider "passage to the limit" and mathematical induction to see what effects the demand for procedural invariance brings. In general, there are different limiting procedures that are not in agreement because a discrete grid, no matter how fine, is qualitatively different from a continuous line. There is no gradual transition which transforms all of the properties of the finite system smoothly into the properties of a continuous system; some of the properties of the continuum appear abruptly only when the embedding reaches the ultimate transfinite stage.

Consider three alternative conventions to govern mathematical induction. Let $P(n_p, p)$ be a proposition that is consistent with a set of axioms, G, where $n_p \in GF(p)$. Let $n_{max}(p)$ denote the largest count in $GF(p)$, i.e., starting with 1, $n_{max}(p)$ is the element such that the successor to $n_{max}(p)$ is zero, i.e., p - 1. Let $n_{max}(Toss(p))$ be the number of elements or size of Toss(p). We shall look at $P(n_p, P)$ as n_p and p increase.

(1) Conventional or Customary Mathematics: Let $p \rightarrow$ "∞" $\Rightarrow \forall p$, $n_p < n_{max}(Toss(P))$. This corresponds to the construction of a continuum by indefinite embedding and generates a countably infinite set of transitively ordered numbers.

The validity of the proposition P is then investigated by conventional logic in the context of continuous space. This procedure generates the well-known (and sometime counter intuitive) results of mathematics.

(II) Fixed Cognitive Agent: Keep $n_p \in GF(p)$ but let $n_p > n_{max}(Toss(p))$. In this case induction on $P(n_p, p)$ leads to results which are not interpretable within the context of the fixed cognitive agent i.e., the results are *relatively* indeterminably and chaotic. We describe this result by $P(n_p, p) \to X$ where X is some unexpected proposition not necessarily consistent with G.

(III) Indefinite Finite Embedding: Let n_p and p increase so that $n_p \in Toss(p)$; then perform induction. In this case the resultant proposition is determinable and consistent with G. Unfortunately, this procedure is limited to the resources that can generate ever larger p's. Hence, when the "largest" p is reached, i.e., when the capacity of the system is exhausted, then case (III) $\to$ case (II). We observe that prediction in any substantive system (including that of the physical universe) ultimately exhausts its numerical resource.

Of these procedures, case (I) is the conventional one; case (II) is more appropriate to any actual finite system, and case (III) is arbitrarily constrained to remain within system of adequate numerical resources and thereby investigate only the *determinable* properties of mathematics.

Illustrative Examples

Example A: Convergence to a Point

Consider the function $P(m) = 1/m$. We desire to define the limiting process designed by

$$\lim_{m \to} P(m) \quad ,$$

where $m \to$ indicates that m increases under that appropriate condition of the respective procedure.

Under procedure I we have

$$\lim_{m \to \infty} P(m) = 0;$$

Under procedure III we have a two-stage process

$$\lim_{m \to n(\mathrm{Toss}(p))} P(m) = \epsilon(p)$$

$$\lim_{p \to} \epsilon(p) = \dot{\delta} > 0$$

We note that δ moves arbitrarily close to zero i.e., $\delta < \frac{1}{M}$, where M is any number from any Toss(p); however great.

We may say that "δ convergences toward zero" and may be made to lie within any arbitrarily small neighborhood of zero, i.e., the limit is not a member of the sequence (cf. definition of a Banach Space and the closure requirement).

Under procedure II

We execute the first stage as above in procedure II; then carry the limiting process through the transitivity ordered numbers:

$$\lim_{n \to n_{max}(\mathrm{Toss}(p))} p(m) = \epsilon\,(n_{max})$$

During the limiting process $\epsilon(n)$ decreases to a minimum value $1/n_{max}(\text{Toss}(p))$. Next we execute the second stage by replacing n by the successors.

Then "increase" $n_{max}\text{Toss}(p))$ by successive steps defined by the process:

$$n' = n + 1$$

Since $n + 1$ and subsequent numbers are outside of Toss(p), $\epsilon(n')$ suddenly escapes the neighborhood of zero. If limited to the numerical resources of a fixed Toss(p) of GF(p) the value of $\epsilon(n')$ is undeterminable and may be any number. Here the limiting process, as n increases, behaves in a determinable manner and the value is restricted to a smoothly decreasing neighborhood until n exceeds n_{max}. The value then takes on unpredictable values including some of which are not interpretable.

Example B: Relativistic Properties of a Random Walk in Finite Space*

Consider a one-dimensional discrete space and a point executing a random walk. The probability of moving one space position to a contiguous position of higher index is p, converse q, $p + q = 1$. Let k designate the index of the point, and let n be the number of steps. Let $P(k,n)$ be the probability that the point is at the k^{th} position after the n^{th} transition.

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* This example is taken from an earlier work by one of us (LMS) reference 19 and is a simplified version of a more general viewpoint in which the embedded discrete space points are implicit. In order to preserve consistence under a velocity formation it was demonstrated in the reference that it was necessary to introduce an imaginary component of transition probability in order to achieve 6-dimensional rotational transformations (one time dimension for each space dimension). The resulting transformation was shown to be the Lorentz-Fitzgerald transformation of relativistic physics.

Given

$P(0,0) = 1$, we may show that

$$P(n,k) = \begin{bmatrix} \frac{n+k}{2} \\ \\ \frac{n-k}{2} \end{bmatrix} p^{\frac{n+k}{2}} q^{\frac{n-k}{2}} \quad ; \quad \sum_k P(n,k) = 1,$$

is a binomial distribution which extends $\pm$ n either side of $k = 0$.

We may also show that the first moment, $\bar{k} = \Sigma kP(k,n)$ is:

$$\bar{k} = n(p - q); \tag{1}$$

and that the variance is

$$\sigma^2(k,n) = \sum_k (k - \bar{k})^2 = 4npq. \tag{2}$$

Furthermore, since $p - q =$ constant $\equiv \beta$ and $p + q = 1$, we have

$$\sigma^2(k,n) = n(1-\beta^2); \tag{3}$$

the ratio of $\sigma(k,n)$ for $\beta \neq 0$ to $\sigma_0(k,n)$ for $\beta = 0$ is

$$\frac{\sigma}{\sigma_{00}} = (1-\beta^2)^{\frac{1}{2}}. \tag{4}$$

We interpret the <u>transition</u> to result in the change of one space quanta and the corresponding time to change one time quanta. In terms of measures from some much finer embedding, one space quanta represents a change in a distance of Λ, and of time, τ. The <u>mean</u> position of the point is given by

$$\bar{x} = \bar{k}\Lambda = n\beta\Lambda$$

and the time t, by

$$t = n\tau.$$

The speed of the expected position, v, is given by

$$v = \frac{\bar{x}}{t} = \frac{\beta\Lambda}{\tau} \; ; \text{ and } \sigma^2(x) = n(1-\beta^2)\Lambda^2.$$

The random walk exhibits relativistic properties, under the interpretation that $\bar{k}$ determines the position of $P(k,n)$ and that σ determines its size. We note that the speed is limited to a maximal quantity, and that the size contracts in the direction of motion as in the Lorentz-Fitzgearld contraction. The maximum velocity, v_{max}, is given by

$$\beta = 1, \text{ i.e., by } v_{max} \equiv c = \frac{n\Delta}{n\tau} = \frac{\Delta}{\tau}$$

and the size, σ, becomes 0 at $\beta = 1$, for all n.

We now examine these relativistic properties as the embedding of finite spaces becomes a continuum by permitting $\Delta \to 0$. This is not a uniquely determinable procedure. We can at most, preserve three properties (since P is a function of k, n, and β) of the distribution, $P(k,n)$. It is not possible to preserve all of its properties. The properties we select for presentation are:

(a) the "size" σ is maintained finite and nonzero;

(b) the time measure (and space measure) are held constant, and

(c) the speed v, is held constant.

Condition (a) is satisfied if for $\sigma^2 = n\Delta^2(1-\beta^2)$ we require $n\Delta^2 = n_0\Delta_0^2$ (where the subscript refers to the values in the initial discrete space).

From above we have

$$n = \frac{n_0\Delta_0^2}{\Delta^2} \qquad \text{(A)}$$

From condition (b) we have $t = n\tau = n_0\tau_0 = \text{const.}$ On combining with Condition(a) we have

$$\tau = \frac{\tau_0\ \Delta^2}{\Delta_0^2}. \qquad \text{(B)}$$

From condition (c) we have

$$v = \frac{\beta\Delta}{\tau} = \frac{\beta_0\Delta_0}{\tau_0} = \text{const} = \frac{\beta}{\beta_0}\frac{\Delta_0}{\Delta}\frac{\beta_0\Delta_0}{\tau_0} = \frac{\beta}{\beta_0}\frac{\Delta_0}{\Delta} v$$

or

$$\frac{\beta\,\Delta_0}{\beta_0\,\Delta} = 1,$$

$$\beta = \beta_0\frac{\Delta}{\Delta_0}. \quad \text{(c)}$$

We now let $\Delta \to 0$, having required v and t to remain constant.

$$\lim_{\Delta \to 0} \beta = 0$$

$$\lim_{\Delta \to 0} \sigma = n_0\Delta_0^2 = \sigma_{00}$$ (i.e., the initial standard deviation for $\beta = 0$, i.e., zero velocity).

$$\lim_{\Delta \to 0} c = \lim_{\Delta \to 0} \frac{\Delta}{\tau} = \lim_{\Delta \to 0} \frac{\Delta_0^2}{\tau_0\Delta} = \infty .$$

We note that the Lorentz-like contraction is lost; that the maximum velocity increases without bound—in short, that the natural relativistic properties of the discrete space are lost in going to the continuum as a limiting process. Having destroyed these formal properties in going to the continuum, we may reintroduce them as additional restraints appropriate to the substantitive problem encountered. For example, in the special theory of relativity one imposes the constancy of the velocity of light. The point made herein is that this property is a natural formal property of the discrete space—and that finite cognitive agents are constrained to cognize in the context of discrete spaces. Hence any admissible model of substantive finite space will, perforce, have the relativistic properties.

Example C: <u>Numerousness of the Even, Odd Numbers</u>

In conventional number theory following process I it is shown that the even (odd) numbers are equally as numerous as the transtitive-ordered set of all numbers.

Following procedure III we note that for any finite model of size n the evens are as numerous as $\frac{n}{2}$, n even (or as $\frac{n-1}{2}$ if n is odd) and the odds as numerous as $\frac{n}{2}$, n even or $\frac{n+1}{2}$, n odd. As n is increased this ratio of evens to n remains, or converges toward $\frac{1}{2}$—for any n. Thus the limiting ratio is not one of equally numerous to the total set of integers (unless one maintain that $\frac{1}{2}$ of ∞ is "as numerous" as ∞).

Procedure II conforms to procedure III until n_{max}(Toss(p)) is exceeded—at which time the odds and evens appear in random order and statistically the ratio becomes one-half.

Example D: <u>Trisecting the Angle</u>

In conventional geometry it is agreed that by using an idealized compass and ruler, any given angle may be bisected, but that it is not possible to trisect an angle. It is explicitly forbidden, under the terms of the exercise, to permit infinite interative algorithms even though they may converge to a trisection of an angle. However, by the nature of the exercise, an infinite algorithm has been implicitly admitted by the supposition that one may place the idealized point of the compass exactly on top of a given point on the idealized paper. One can devise algorithms which enable one point to be placed within any ε-neighborhood of a given point with a finite number of operations; however, they are of the nature explicitly forbidden.

The result of these observations is that if infinite procedures are ruled out an angle can neither be bisected nor trisected. On the other hand, if infinitely converging algorithms are admitted, then one may construct, within any tolerable variance, the bisected and the trisected angle. Under procedure III, the variance is decreased indefinitely. Under procedure II [and the ultimate fate of procedure III] the variance can be decreased progressively until the transitively ordered numbers are exhausted; further operations result in random results added to the initial determinable results.

The behavior of determination under case II is more nearly in conformity to actual physical behavior. We are led to surmise that this property—as a natural property of these finite spaces—may be more appropriately associated with the finiteness of the finite cognizing agent itself.

One may regard procedure III as an interim one admitting of embedding one finite field in a larger one (i.e., greater p) in such a manner as to preserve pro tem the determinable character of calculation. This process may be iteratively advanced until some secondary requirement is met (i.e., the uncertainties are balanced or the error is admissible)—or until the process must halt for lack of additional numerical resources. Procedure II identifies the resulting characteristic when such resources are exceeded—and any fixed system must necessarily sooner or later face the consequent introduction of indeterminancy (i.e., all systems are finite).

The use of continuous mathematics to represent finite space-time under the problem conditions imposed here is admissible as an expedient only if it is kept in mind that (1) some inconsistency may inadvertently be introduced, and (2) it may be necessary to introduce additional side

constraints ostensibly as "properties" of the substantive problem in order to preserve some of the characteristics of the finite spaces which have been lost in the conventional limiting processes (e.g., the constant finite speed of light in vacuo).

If one does not admit the existence of a continuous space, even as an additional axiomatic input, he is led to define a concept "infinity" and a "limit" in terms of algorithms which are in every case necessarily truncated by limiting the sequential process to a finite number of operations. Statements about the continuum are then shorthand statements of more exactly definable finite procedures. Some such statements are inadmissible (e.g., "all complex numbers have square roots").

Let us also point out that there are profound generic differences between finite fields and infinite fields and the one does not gradually grow into the other. So long as a field is finite, no matter how large p becomes, it is categorically different from an infinite set. The transition occurs not during the finite approach to the limit, but abruptly "over the horizon" when the limit is reached (e.g., the relative numerousness of evens and odds cannot be understood via a gradual transition from finite to infinite sets). Thus, we must reexamine our understanding of limits, etc., in light of these results. We as finite cognitive agents, are constrained to reach all of our conclusions by finite procedures. Hence, since we can infer an infinite set as a finite sequence of finite sets, one must ask about the status of these infinite sets. Or, if we assume their existence, how do we work with them since they are not finitely attainable or realizable?

APPENDIX I

Given $x \notin \Gamma(g_k)$ we know there exist two perfect squares between which x lies, say

$$(N_0)^2 < x < (N_0 + 1)^2 \ . \tag{1}$$

Hence $(\sqrt{x})_R$ will equal N_0 or $N_0 + 1$, depending on whether we assume round-up, round-down, or round nearest. To increase resolution, we may add a decimal place to the root by forming

$$(\sqrt{x})_R + \frac{1}{10}(\sqrt{100x})_R \ . \tag{2}$$

Now, inequality (1) implies

$$100(N_0)^2 < 100x < 100(N_0 + 1)^2 \ . \tag{3}$$

<u>Theorem (1)</u>. There are nine perfect squares between $100N^2$ and $100(N + 1)^2$, not counting the end points.

<u>Proof</u>. Consider $[10N + \alpha]^2$ where α is a nonnegative integer. Clearly $[10N + \alpha]^2 \in [100N^2, 100(N + 1)^2]$ for $\alpha \in [0,10]$.

Using Theorem (1) on inequality (3), we see that 100x lies between two perfect squares as follows:

$$(10N_0 + \alpha_1)^2 < 100x < (10N_0 + \alpha_1 + 1)^2 \qquad \alpha_1 \in [0,9] \tag{4}$$

Let us define $N_1 = 10N_0 + \alpha_1$ and rewrite (4) as

$$(N_1)^2 < 100x < (N_1 + 1)^2 \tag{4'}$$

Let us now consider still another embedding which requires

$$100(N_1)^2 < 10^4x < 100(N_1 + 1)^2 \ . \tag{5}$$

As before, we use Theorem (1) to find $\alpha_2 \in [0,9]$ such that

$$(10N_1 + \alpha_2)^2 < 10^4 x < (10N_1 + \alpha_2 + 1)^2 \tag{6}$$

and so forth. After n embeddings we have

$$(N_n)^2 < 10^{2n} x < (N_n + 1)^2. \tag{7}$$

where $N_n = 10N_{n-1} + \alpha_n = 10(10N_{n-2} + \alpha_{n-1}) + \alpha_n$

$$= 10(10[10N_{n-3} + \alpha_{n-2}] + \alpha_{n-1}) + \alpha_n$$

$$= 10^n N_0 + \sum_{i=0}^{n-1} 10^i \alpha_{n-i} . \tag{8}$$

Define $$(\sqrt{x})_{R_n} = \frac{1}{10^n} (\sqrt{10^{2n} x})_R . \tag{9}$$

Clearly $(\sqrt{10^{2n} x})_R \in [N_n, N_{n+1}]$ holds for any of the three replacement alternatives.

Thus from Equation (9) we obtain

$$(\sqrt{x})_{R_n} \in \left[\frac{N_n}{10^n}, \frac{N_n + 1}{10^n}\right] . \tag{10}$$

From Equation (10) we obtain by squaring that

$$\left((\sqrt{x})_{R_n}\right)^2 \in \left[\left(\frac{N_n}{10^n}\right)^2, \left(\frac{N_n + 1}{10^n}\right)^2\right] \tag{11}$$

Define $$\left[\left(\frac{N_n}{10^n}\right)^2, \left(\frac{N_n + 1}{10^n}\right)^2\right] = \Delta_n . \tag{12}$$

From inequality (7) we find

$$x \in \Delta_n \tag{13}$$

Consider the length of Δ_n, viz. $|\Delta_n|$. We have

$$|\Delta_n| = \frac{2N_n + 1}{10^{2n}} = \frac{2\left(10^n N_0 + \sum_{i=0}^{n-1} 10^i \alpha_{n-i}\right) + 1}{10^{2n}} . \tag{14}$$

Thus for large n, Δ_n behaves according to $2N_o/10^{\sigma}$. In the limit[20] as $n \to \eta$, Δ_n becomes infinitesimal and we represent this symbolically as

$$\lim_{n \to \eta} \Delta_n = 0 . \tag{15}$$

From Equations (11) and (13), we see that both x and $((\sqrt{x})_{r_n})^2$ are in Δ_n and from (15) we see that $\Delta_n \to 0$. Hence

$$\lim_{n \to \eta} ((\sqrt{x})_{r_n})^2 = x \tag{16}$$

and we see that the embedding procedure converges to the appropriate limit to justify its definition and claim for acceptance. Note, this formulation is valid for all three replacement techniques.

Finally, it must be mentioned that the above proofs presuppose that all values be within the appropriate region of some Toss. As the embedding becomes richer and therefore more demanding of resources, the size of the Toss and—at an exponential rate—the size of the GF(p) become increasingly large. There are serious questions about the limiting size of these fields before they become so large as to violate our primitive commitments regarding numerousness and scope, etc.

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Appendix: Notes and References

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3. For a full disclosure of philosophical commitments, see foregoing Chapter 9: The Finite Cognitive Agent: Philosophical Implications and Chapter 10: Evolutionary Systems Philosophy.

4. For the detailed development of a Galois Field, see E. Beltrametti and A. Blasi, Journal of Mathematical Physics, 9 (1968) 1027;

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5. Irreflexivity is required to avoid ambiguity.

6. This notion of an order relation is based upon that developed by Kustaanheimo, et. al. We have generalized the concept in order to incorporate more fully the principle of local ordering. Let us point out that there are alternative methods for defining an irreflexive binary relation and we have not yet fully developed the rationale for choosing among them. For instance, one could locally order by letting $x > y$ if it requires fewer "counts" to reach y by successive unit subtractions than by successive unit additions. (Such a convention fails to order the reciprocals, however.) Nevertheless, the geometry is still locally defined for $(p + 1)/2$ consecutive elements. For a general discussion of ordering a set, both finite and infinite, see G. Cantor, Contributions to the Founding of the Theory of Transfinite Numbers, Dover Publications, Inc., New York, 1915; P. Halmos, Naive Set Theory, D. Van Nostrand Company, Inc., Princeton, N. J., 1960; F. Hausdorff, Set Theory, second edition Chelsea Publishing Company, N. Y., 1957; E. Kamke, Theory of Sets, Dover Publications, Inc., New York, 1950.

7. This follows from Euler's Criterion because all primitive roots in GF(p), $P \neq 2$, are odd. See J. Uspensky and M. Heaslet, Elementary Number Theory, McGraw-Hill Book Company, Inc., New York, 1939.

8. To see a discussion of this topic in its more general algebraic framework, see A. Albert, Fundamental Concepts of Higher Algebra, The University of Chicago Press, Chicago, 1956, and B. van der Waerden, Modern Algebra, Volume I, Frederick Ungar Publishing Co., New York, 1949.

9. should read "for any." The distinction between "for every" and "for any" will play an important role in our examination of mathematical induction.

10. Beltrametti, E., and A. Blasi, Journal of Mathematical Physics, 9 (1968) 1027.

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12. See reference 8 for a proof that a vector space can be defined over a Galois Field.

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14. Smith, N., and D. Reisler "On the Generation of Pythagorean Numbers," in preparation.

15. One must introduce a purely finite field formulation of plane geometry. For instance we may define a triangle as follows. Let V be a two-dimensional vector space over GF(p). <u>Definition</u>. Let $x^1, x^2, x^3 \in V$ be a set of pairwise linearly independent vectors. Define the "object" determined by the three differences $x^3 - x^2$, $x^2 - x^1$, $x^3 - x^1$—called sides—to be a triangle. Denote this triangle by $\Delta(x^1, x^2, x^3)$. In this way one can develop a plane geometry with many familiar theorems. For example, if $\Delta(x^1, x^2, x^3)$ is a triangle with no side of zero "length," then there can be at most one orthogonal intersection of sides.

16. Here P(n) denotes some proposition at the n^{th} iteration.

17. Fraenkel, A., and Y. Bar-Hillel, <u>Foundations of Set Theory</u>, North-Holland Publishing Company, Amsterdam, 1958, and A. Fraenkel, <u>Abstract Set Theory</u>, North-Holland Publishing Company, Amsterdam, 1961.

18. Procedural invariance, a generalization of the Einstein principle of relativity, is required in order to avoid ambiguity of convention in the most general sense.

19. Smith, N., <u>Behavioral Science</u>, <u>1</u> (1956) 111.

20. Here the "number" η will represent the size or scope of a given cognitive system. In some broad sense it is the largest number of counts that can be conceived of by cognitive agents from within this system. Traditional mathematics has used the symbol as in an unrestricted and unqualified sense that does not take the capabilities of the system into account. Thus, η is the largest count not the largest number.